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Atomically Thin Semiconductor Emit Single Photons, High-Dimensional Hyperentanglement Through A Biphoton Frequency Comb, Efficient Compression Of Quantum Information, Heralded Qubit Transfer, Messages With A Quantum Seal, Quantum Digital Signatures, Single Dot Confabulates With Single Ion, Trapped Ions Make Impeccable Qubits, Searching For A Needle In A Haystack, Fault-Tolerant Quantum Error Correction, Quantum Algorithms Without Initializing The Auxiliary Qubits And Other Editorials And Representations Of Natural Interest: Gesamtkunstwerk; Complete Work Of Art: Gotterdammerung Gradus Ad Pamassum: A Step To Peak Models

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ABSTRACT

Abstract: **Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose** identify signatures of chaos in the dynamics of discord in a multiqubit system collectively modelled as a quantum kicked top. Evolution of discord between any two qubits is quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity is lost. As the initial wave function is varied from the regular regions to the chaotic sea, a contour plot of the time-averaged discord remarkably reproduces the structures of the classical stroboscopic map. **Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose** also find surprisingly opposite behavior of two-qubit discord versus entanglement of the two qubits as measured by the concurrence. Results provide evidence of signatures of chaos in dynamically generated discord. http://dx.doi.org/10.1103/PhysRevE.91.032906©2015 American Physical Society **Signatures of chaos in the dynamics of quantum discord Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose Phys. Rev. E 91, 032906 – Published 10 March 2015 STOP.** How do closed quantum many-body systems driven out of equilibrium eventually achieve equilibration? And how do these systems thermalize, given that they comprise so many degrees of freedom? Progress in answering these—and related—questions has accelerated in recent years—a trend that can be partially attributed to success with experiments performing quantum simulations using ultracold atoms and trapped ions. **J. Eisert, M. Friesdorf & C. Gogolin** provide an overview of this progress, specifically in studies probing dynamical equilibration and thermalization of systems driven out of equilibrium by quenches, ramps and periodic driving. In doing so, they also address topics such as the eigenstate thermalization hypothesis, typicality, transport, many-body localization and universality near phase transitions, as well as future prospects for quantum simulation. **Quantum many-body systems out of equilibrium J. Eisert, M. Friesdorf & C. Gogolin Nature Physics 11, 124–130 (2015) doi: 10.1038/nphys3215 Received 11 August 2014 Accepted 03 December 2014 published online 03 February 2015 STOP.** Quantum teleportation is one of the most important protocols in quantum information. By exploiting the physical resource of entanglement, quantum teleportation serves as a key primitive in a variety of quantum information tasks and represents an important building block for quantum technologies, with a pivotal role in the continuing progress of quantum communication, quantum computing and quantum networks. **Stefano Pirandola, Jens Eisert, Christian Weedbrook, Akira Furusawa, and Samuel L. Braunstein** review the basic theoretical ideas behind quantum teleportation and its variant protocols. They focus on the main experiments, together with the technical advantages and disadvantages associated with the use of the various technologies, from photonic qubits and optical modes to atomic ensembles, trapped atoms, and solid-state systems. Analysing the current state-of-the-art, we finish by discussing open issues, challenges and potential future implementations. Cite as: arXiv: 1505.07831 [quant-ph] (or arXiv: 1505.07831v1 [quant-ph] for this version) **Advances in Quantum Teleportation Stefano Pirandola, Jens Eisert, Christian Weedbrook, Akira Furusawa, and Samuel L. Braunstein (Submitted on 28 May 2015) STOP. Toby Cubitt, David Elkouss, William Matthews, Maris Ozols, David Pérez-García & Sergii Strelchuk** show that this is not the case: for any number of uses, there are channels for which the coherent information is zero, but which nonetheless have capacity. Transmitting data reliably over noisy communication channels is one of the most important applications of information theory, and is well understood for channels modelled by classical physics. However, when quantum effects are involved, we do not know how to compute channel capacities. This is because the formula for the quantum capacity involves maximizing the coherent information over an unbounded number of channel uses. In fact, entanglement across channel uses can even increase the coherent information from zero to non-zero. **Toby Cubitt, David Elkouss, William Matthews, Maris Ozols, David Pérez-García & Sergii Strelchuk** study the number of channel uses necessary to detect positive coherent information. In all previous known examples, two channel uses already sufficed. It might be that only a finite number of channel uses are always sufficient. **Toby Cubitt, David Elkouss, William Matthews, Maris Ozols, David Pérez-García & Sergii Strelchuk** show that this is not the case: for any number of uses, there are channels for which the coherent information is zero, but which nonetheless have capacity. **Unbounded number of channel uses may be required to detect quantum capacity Toby Cubitt, David Elkouss, William**

Matthews, Maris Ozols, David Pérez-García & Sergii Strelchuk Nature Communications 6, Article number: 6739 doi:10.1038/ncomms7739 Published 31 March 2015 STOP. Characterizing the behaviour of strongly coupled quantum systems out of equilibrium is a cardinal challenge for both theory and experiment. With diverse applications ranging from the dynamics of the quark–gluon plasma to transport in novel states of quantum matter, establishing universal results and organizing principles out of equilibrium is crucial. **M. J. Bhaseen, Benjamin Doyon, Andrew Lucas& Koenraad Schalm** present a universal description of energy transport between quantum critical heat baths in arbitrary dimension. Current-carrying non-equilibrium steady state (NESS) is a Lorentz-boosted thermal state. In the context of gauge/gravity duality this reveals an intimate correspondence between far-from-equilibrium transport and black hole uniqueness theorems. Authors provide analytical expressions for the energy current and the generating function of energy current fluctuations, together with predictions for experiment. **Energy flow in quantum critical systems far from equilibrium M. J. Bhaseen, Benjamin Doyon, Andrew Lucas& Koenraad Schalm Nature Physics 11, 509–514 (2015) doi: 10.1038/nphys3320: Published online 04 May 2015 STOP.** Properties of systems near quantum critical points (QCPs) have been studied extensively. A QCP is a point across which the symmetry of the ground state of a quantum system changes in a fundamental way; such a point can be accessed by changing some parameter, say λ, in the Hamiltonian governing the system. The change in the ground state across a QCP is mediated by quantum fluctuations. Unlike conventional thermal critical points, thermal fluctuations do not play a crucial role in such transitions. Similar to its thermal counterparts, the low-energy physics near a QCP is associated with a number of critical exponents which characterize the universality class of such a transition. Among these exponents, the dynamical critical exponent z provides the signature of the relative scaling of space and time at the transition and has no counterpart in thermal phase transitions. The other exponent which is going to be important for the purpose of this review is the well-known correlation length exponent ν. These exponents are formally defined as follows. As we approach the critical point at $\lambda=\lambda c$, the correlation length diverges as $\xi\sim|\lambda-\lambda c|-\nu$, while the gap between the ground state and first excited state vanishes as $\Delta E\sim\xi-z\sim|\lambda-\lambda c|z\nu$. Exactly at the critical point $\lambda=\lambda c$, the energy of the low-lying excitations vanishes at some wave number k 0 as $\omega\sim|k-k0|z$. The critical exponents are independent of the details of the microscopic Hamiltonian; they depend only on a few parameters such as the dimensionality of the system and the symmetry of the order parameter. These features render the low-energy equilibrium physics of a quantum system near a QCP truly universal. **Quantum Quenching, Annealing and Computation Lecture Notes in Physics Volume 802, 2010, pp 21-56 Date: 08 Mar 2010 Non-equilibrium Dynamics of Quantum Systems: Order Parameter Evolution, Defect Generation, and Qubit Transfer S. Mondal, D. Sen, K. Sengupta STOP Matthias Christandl, Nilanjana Datta, Artur Ekert, and Andrew J. Landahl** propose a class of qubit networks that admit the perfect state transfer of any quantum state in a fixed period of time. Unlike many other schemes for quantum computation and communication, these networks do not require qubit couplings to be switched on and off. When restricted to N-qubit spin networks of identical qubit couplings, they show that 2log3N is the maximal perfect communication distance for hypercube geometries. Moreover, if one allows fixed but different couplings between the qubits, then perfect state transfer can be achieved over arbitrarily long distances in a linear chain. **Perfect State Transfer in Quantum Spin Networks Matthias Christandl, Nilanjana Datta, Artur Ekert, and Andrew J. Landahl Phys. Rev. Lett. 92, 187902 – Published 4 May 2004 STOP.** As with classical information processing, a quantum information processor requires bits (qubits) that can be independently addressed and read out, long-term memory elements to store arbitrary quantum states1, 2, and the ability to transfer quantum information through a coherent communication bus accessible to a large number of qubits3, 4. Superconducting qubits made with scalable microfabrication techniques are a promising candidate for the realization of a large-scale quantum information processor5, 6, 7, 8, and 9. (For references please see the paper). Although these systems have successfully passed tests of coherent coupling for up to four qubits10, 11, 12, 13, communication of individual quantum states between superconducting qubits via a quantum bus has not yet been realized. Here, authors perform an experiment demonstrating the ability to coherently transfer quantum states between two superconducting Josephson phase qubits through a quantum bus. This quantum bus is a resonant cavity formed by an open-ended superconducting

transmission line of length 7 mm. After preparing an initial quantum state with the first qubit, this quantum information is transferred and stored as a nonclassical photon state of the resonant cavity, then retrieved later by the second qubit connected to the opposite end of the cavity. Beyond simple state transfer, these results suggest that a high-quality-factor superconducting cavity could also function as a useful short-term memory element. The basic architecture presented here can be expanded, offering the possibility for the coherent interaction of a large number of superconducting qubits. **Nature 449, 438-442 (27 September 2007) | doi:10.1038/nature06124; Received 18 April 2007; Accepted 25 July 2007 Coherent quantum state storage and transfer between two phase qubits via a resonant cavity Mika A. Sillanpää1, Jae I. Park1 & Raymond W. Simmonds1 STOP. Chui-Ping Yang, Shih-I Chu, and Siyuan** present a scheme to achieve maximally entangled states, controlled phase-shift gate, and SWAP gate for two superconducting-quantum-interference-devices (SQUID) qubits, by placing SQUIDs in a microwave cavity. **Chui-Ping Yang, Shih-I Chu, and Siyuan** also show how to transfer quantum information from one SQUID qubit to another. In this scheme, no transfer of quantum information between the SQUIDs and the cavity is required, the cavity field is only virtually excited and thus the requirement on the quality factor of the cavity is greatly relaxed. http://dx.doi.org/10.1103/PhysRevA.67.042311 **Possible realization of entanglement, logical gates, and quantum-information transfer with superconducting-quantum-interference-device qubits in cavity QED Chui-Ping Yang, Shih-I Chu, and Siyuan Han Phys. Rev. A 67, 042311 – Published 17 April 2003 STOP. L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia** explore the capability of spin-1/2 chains to act as quantum channels for both teleportation and transfer of qubits. Exploiting the emergence of long-distance entanglement in low-dimensional systems [Phys. Rev. Lett. 96, 247206 (2006)], here **L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia** show how to obtain high communication fidelities between distant parties. An investigation of protocols of teleportation and state transfer is presented, in the realistic situation where temperature is included. Basing setup on antiferromagnetic rotationally invariant systems, both protocols are represented by pure depolarizing channels. **L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia** propose a scheme where channel fidelity close to 1 can be achieved on very long chains at moderately small temperature. **Qubit Teleportation and Transfer across Antiferromagnetic Spin Chains L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia Phys. Rev. Lett. 99, 060401 – Published 6 August 2007 STOP. David Petrosyan, Guy Bensky, Gershon Kurizki, Igor Mazets, Johannes Majer, and Jörg Schmiedmayer P** examine the possibility of coherent reversible information transfer between solid-state superconducting qubits and ensembles of ultracold atoms. Strong coupling between these systems is mediated by a microwave transmission line resonator that interacts near resonantly with the atoms via their optically excited Rydberg states. Solid-state qubits can then be used to implement rapid quantum logic gates, while collective metastable states of the atoms can be employed for long-term storage and optical readout of quantum information. **Reversible state transfer between superconducting qubits and atomic ensembles David Petrosyan, Guy Bensky, Gershon Kurizki, Igor Mazets, Johannes Majer, and Jörg Schmiedmayer Phys. Rev. A 79, 040304(R) – Published 14 April 2009 STOP.** Quantum computers have the capability of out-performing their classical counterparts for certain computational problems1. Several scalable quantum-computing architectures have been proposed. An attractive architecture is a large set of physically independent qubits arranged in three spatial regions where (1) the initialized qubits are stored in a register, (2) two qubits are brought together to realize a gate and (3) the readout of the qubits is carried out2, 3. For a neutral-atom-based architecture, a natural way to connect these regions is to use optical tweezers to move qubits within the system. **Jérôme Beugnon, Charles Tuchendler, Harold Marion, Alpha Gaëtan, Yevhen Miroshnychenko, Yvan R. P. Sortais, Andrew M. Lance, Matthew P. A. Jones, Gaétan Messin, Antoine Browaeys & Philippe Grangier** demonstrate the coherent transport of a qubit, encoded on an atom trapped in a submicrometre tweezer, over a distance typical of the separation between atoms in an array of optical traps4, 5, 6. Furthermore, we transfer a qubit between two tweezers, and show that this manipulation also preserves the coherence of the qubit. **Nature Physics 3, 696 - 699 (2007) Published online: 12 August 2007 | doi: 10.1038/nphys698 Subject Categories: Atomic and molecular physics | Optical physics | Quantum physics | Information theory and computation Two-dimensional transport and transfer of a single atomic qubit in**

optical tweezers Jérôme Beugnon, Charles Tuchendler, Harold Marion, Alpha Gaëtan, Yevhen Miroshnychenko, Yvan R. P. Sortais, Andrew M. Lance, Matthew P. A. Jones, Gaétan Messin, Antoine Browaeys & Philippe Grangier STOP. In optically controlled quantum computers it may be favorable to address different qubits using (e) light with different frequencies, since the optical diffraction does not then limit the distance between qubits. Using qubits that are close to each other enable (erb) qubit-qubit interactions and gate operations that are strong and fast in comparison to (e&eb) qubit-environment interactions and decoherence rates. However, as qubits are addressed in frequency space, great care has to be taken when designing the laser pulses, so that they perform (e&eb) the desired operation on (eb) one qubit, without (e) affecting other qubits. Complex hyperbolic secant pulses have theoretically been shown to be excellent for (e) such frequency-addressed quantum computing [I. Roos and K. Molmer, Phys. Rev. A 69, 022321 (2004)]—e.g., for use in quantum computers based on optical interactions in rare-earth-metal-ion-doped crystals. The optical transition lines of the rare-earth-metal-ions are inhomogeneously broadened and therefore the frequency of the excitation pulses can be used to selectively address qubit ions that are spatially separated by a distance much less than a wavelength. , **Lars Rippe, Mattias Nilsson, Stefan Kröll, Robert Klieber, and Dieter Suter demonstrate experimentally** frequency-selective transfer of qubit ions between qubit states using complex hyperbolic secant pulses. Transfer efficiencies better than 90% were obtained. Using the complex hyperbolic secant pulses it was also possible to create two groups of ions, absorbing at specific frequencies, where 85% of the ions at one of the frequencies were shifted out of resonance with the field when ions in the other frequency group were excited. This procedure of selecting interacting ions, called qubit distillation, was carried out in preparation for two-qubit gate operations in the rare-earth-metal-ion-doped crystals. The techniques for frequency-selective state-to-state transfer developed here may be also useful also for other quantum optics and quantum information experiments in these long-coherence-time solid-state systems. **Experimental demonstration of efficient and selective population transfer and qubit distillation in a rare-earth-metal-ion-doped crystal Lars Rippe, Mattias Nilsson, Stefan Kröll, Robert Klieber, and Dieter Suter Phys. Rev. A 71, 062328 – Published 23 June 2005 STOP. Martin Leijnse and Karsten Flensberg** propose a method to coherently transfer quantum information, and to create entanglement, between topological qubits and conventional spin qubits. Suggestion uses gated control to transfer an electron (spin qubit) between a quantum dot and edge Majorana modes in adjacent topological superconductors. Because of the spin polarization of the Majorana modes, the electron transfer translates spin superposition states into superposition states of the Majorana system, and vice versa. It is also shown how a topological superconductor can be used to facilitate long-distance quantum information transfer and entanglement between spatially separated spin qubits. **Quantum Information Transfer between Topological and Spin Qubit Systems Martin Leijnse and Karsten Flensberg Phys. Rev. Lett. 107, 210502 – Published 18 November 2011 http://dx.doi.org/10.1103/PhysRevLett.107.210502 STOP. Gogyan** proposes a method that enables efficient conversion of the quantum information frequency between different regions of a spectrum of light based on recently demonstrated strong parametric coupling between two narrow-band single-photon pulses propagating in a slow-light atomic medium [N. Sisakyan and Yu. Malakyan, Phys. Rev. A, 75, 063831 (2007)]. It is shown that an input qubit at telecom wavelength is transformed into another at a visible domain in a lossless and shape-conserving manner while keeping the initial quantum coherence and entanglement. These transformations can be realized with quantum efficiency close to its maximum value. http://dx.doi.org/10.1103/PhysRevA.81.024304 **Qubit transfer between photons at telecom and visible wavelengths in a slow-light atomic medium A. Gogyan Phys. Rev. A 81, 024304 – Published 25 February 2010 STOP. N. Cleland and M. R. Geller** propose a quantum computing architecture based on (e) the integration of nanomechanical resonators with (e&eb) Josephson-junction phase qubits. Resonators are GHz-frequency, dilatational disk resonators, which couple to the junctions through a piezoelectric interaction. System is analogous to a collection of tunable few-level atoms (the Josephson junctions) coupled to one or more electromagnetic cavities (the resonators). Architecture combines desirable features of solid-state and optical approaches and may make quantum computing possible in a scalable, solid-state environment. **Superconducting Qubit Storage and Entanglement with Nanomechanical Resonators A. N. Cleland and M. R. Geller Phys. Rev. Lett 93, 070501 –**

Published 10 August 2004 STOP. D. N. Matsukevich, A. Kuzmich report on the coherent quantum state transfer from a two-level atomic system to a single photon Entanglement between a single photon (signal) and a two-component ensemble of cold rubidium atoms is used to project the quantum memory element (the atomic ensemble) onto any desired state by measuring the signal in a suitable basis. Atomic qubit is read out by stimulating directional emission of a single photon (idler) from the (entangled) collective state of the ensemble. Faithful atomic memory preparation and readout are verified by the observed correlations between the signal and the idler photons. These results enable implementation of distributed quantum networking. **Science 22 October 2004: Vol. 306 no. 5696 pp. 663-666 DOI: 10.1126/science.1103346 Quantum State Transfer Between Matter and Light D. N. Matsukevich, A. Kuzmich STOP. Giulia Gualdi, Vojtech Kostak, Irene Marzoli, and Paolo Tombesi** investigate the most general conditions under which a finite ferromagnetic long-range interacting spin chain achieves unitary fidelity and the shortest transfer time in transmitting an unknown input qubit. A deeper insight into system dynamics, allows us to identify an ideal system involving sender and receiver only. However, this two-spin ideal chain is unpractical due to the rapid decrease of the coupling strength with the distance. Therefore authors propose an optimization scheme for approaching the ideal behavior, while keeping the interaction strength still reasonably high. The procedure is scalable with the size of the system and straightforward to implement. **Perfect state transfer in long-range interacting spin chains Giulia Gualdi, Vojtech Kostak, Irene Marzoli, and Paolo Tombesi Phys. Rev. A 78, 022325 – Published 18 August 2008 STOP. L Fedichkin, M Yanchenko and K A Valiev** investigated the use of quantum bits (qubits) - semiconductor quantum dots containing one electron and each consisting of two tunnel-connected parts - as basic elements of the quantum computer. Numerical solution of a Schrödinger equation taking account of the Coulomb field of adjacent electrons shows that in such structures the realization of a full set of basic logic operations, which are necessary for fulfillment of quantum computations, is possible. Durations of one- and two-qubit operations versus qubit geometry are obtained. Decoherence rates due to spontaneous emission of phonons and acoustic phonons (both piezoelectric and deformation) are evaluated. Analysis of these rates shows the proposed qubit to be coherent enough to work for an unlimited time. **L Fedichkin et al 2000 Nanotechnology 11 387 doi:10.1088/0957-4484/11/4/339 Coherent charge qubits based on GaAs quantum dots with a built-in barrier L Fedichkin, M Yanchenko and K A Valiev ©2015 American Physical Society STOP.** In recent years **S. Lorenzo, T. J. G. Apollaro, S. Paganelli, G. M. Palma, and F. Plastina P have** investigated the fidelity of the quantum state transfer (QST) of two qubits by means of an arbitrary spin-12 network, on a lattice of any dimensionality. Under the assumptions that the network Hamiltonian preserves the magnetization and that a fully polarized initial state is taken for the lattice, they obtain a general formula for the average fidelity of the two qubits QST, linking it to the one- and two-particle transfer amplitudes of the spin excitations among the sites of the lattice. **S. Lorenzo, T. J. G. Apollaro, S. Paganelli, G. M. Palma, and F. Plastina P** apply this formalism to a 1D spin chain with XX-Heisenberg type nearest-neighbour interactions adopting a protocol that is a generalization of the single qubit one proposed in Paganelli et al. [Phys. Rev. A 87, 062309 (2013)]. They report that a high-quality two qubit QST can be achieved provided one can control the local fields at sites near the sender and receiver. Under such conditions, they obtain an almost perfect transfer in a time that scales either linearly or, depending on the spin number, quadratically with the length of the chain. **Transfer of arbitrary two-qubit states via a spin chain S. Lorenzo, T. J. G. Apollaro, S. Paganelli, G. M. Palma, and F. Plastina Phys. Rev. A 91, 042321 – Published 16 April 2015 http://dx.doi.org/10.1103/PhysRevA.91.042321 ©2015 American Physical Society STOP.** Combining techniques of cavity quantum electrodynamics, quantum measurement, and quantum feedback, Norbert **Kalb, Andreas Reiserer, Stephan Ritter, and Gerhard Rempe** realize the heralded transfer of a polarization qubit from a photon onto a single atom with 39% efficiency and 86% fidelity. The reverse process, namely, qubit transfer from the atom onto a given photon, is demonstrated with 88% fidelity and an estimated efficiency of up to 69%. In contrast to previous work based on two-photon interference, our scheme is robust against photon arrival-time jitter and achieves much higher efficiencies. Thus, it constitutes a key step toward the implementation of a long-distance quantum network. Journal reference: Phys. Rev. Lett. 114, 220501 (2015) DOI:10.1103/PhysRevLett.114.220501 Cite as: arXiv:1503.06709 [quant-ph] (or arXiv:1503.06709v2

[quant-ph] for this version) **Heralded Storage of a Photonic Quantum Bit in a Single Atom Norbert Kalb, Andreas Reiserer, Stephan Ritter, Gerhard Rempe 2 Jun 2015 STOP.** Current high-throughput DNA sequencing technologies enable acquisition of billions of data points through which myriad biological processes can be interrogated, including genetic variation, chromatin structure, gene expression patterns, small RNAs and protein–DNA interactions. **Mark A Urich, Joseph R Nery, Ryan Lister, Robert J Schmitz & Joseph R Ecker** describe the MethylC-sequencing (MethylC-seq) library preparation method, a 2-d protocol that enables the genome-wide identification of cytosine DNA methylation states at single-base resolution. The technique involves fragmentation of genomic DNA followed by adapter ligation, bisulfite conversion and limited amplification using adapter-specific PCR primers in preparation for sequencing. To date, this protocol has been successfully applied to genomic DNA isolated from primary cell culture, sorted cells and fresh tissue from over a thousand plant and animal samples. **NATURE PROTOCOLS | PROTOCOL MethylC-seq library preparation for base-resolution whole-genome bisulfite sequencing Mark A Urich, Joseph R Nery, Ryan Lister, Robert J Schmitz & Joseph R Ecker Nature Protocols 10, 475–483 (2015) doi:10.1038/nprot.2014.114 Published online 18 February 2015 STOP.** Starting from a product initial state, equal-time correlations in nonrelativistic quantum lattice models propagate within a lightcone-like causal region. The presence of entanglement in the initial state can modify this behavior, enhancing and accelerating the growth of correlations. **Michael Kastner** gives a quantitative description, in the form of Lieb-Robinson-type bounds on equal-time correlation functions, of the interplay of dynamics vs. initial entanglement in quantum lattice models out of equilibrium. **Michael Kastner** tests the bounds against model calculations, and also Cite as: arXiv: 1507.00529 [quant-ph] (or arXiv: 1507.00529v1 [quant-ph] for this version) **Entanglement-enhanced spreading of correlations Michael Kastner (Submitted on 2 Jul 2015) STOP.** A scheme based on coherent tunneling by adiabatic passage (CTAP) of exchange-only spin qubit quantum states in a linearly arranged double quantum dot chain is demonstrated. Logical states for the qubit are defined by adopting the spin state of three electrons confined in a double quantum dot. The possibility to obtain gate operations entirely with electrical manipulations makes this qubit a valuable architecture in the field of quantum computing for the implementation of quantum algorithms. Effect of the external control parameters as well as the effect of the dephasing on the coherent tunneling in the chain is studied. During adiabatic transport, within a constant energy degenerate eigenspace, the states in the double quantum dots internal to the chain are not populated, while transient populations of the mixed states in the external ones are predicted. **Coherent tunneling by adiabatic passage of an exchange-only spin qubit in a double quantum dot chain E. Ferraro, M. De Michielis, M. Fanciulli, and E. Prati Phys. Rev. B 91, 075435 – Published 26 February 2015 STOP.** Following system is studied and the concomitant properties investigated thereof.Colloidal Quantum Dot Spectrometer, Light Spectrum Based On The Wavelength Multiplexing Principle, Localized Surface Plasmon Resonances, Atomically Thin Semiconductor Emit Single Photons, Atomically Thin Semiconductor That Emit Single Photons, Quantum Light From An Atomically Thin Semiconductor, High-Dimensional Hyperentanglement Through A Biphoton Frequency Comb, Insulator-To-Metal Transition , Out Of Equilibrium Systems, Hyperentanglement Or High-Dimensional Entanglement Has Been Separately Proposed For High Data Capacity And Error Resilience Of Quantum Entanglement, Hong–Ou–Mandel Quantum Revival, Efficient Compression Of Quantum Information, Saving Space With Quantum Information, Efficient Quantum Circuits, Quantum Data Compression, Quantum Information Generation, And The Density-Matrix Renormalization-Group Method, Quantum Source Compression with a Quantum Helper, Heralded Qubit Transfer, Roles Of The Atom And Photon Could Be Reversed, Messages With A Quantum Seal, "Private Key" , Unitarity And Intrinsic Randomness, Quantum State Elimination Measurements On The Sequence Of Public Key States, Universal Quantum-Mechanical Manipulation, Securely Distribution Of Quantum Public Keys, Quantum Digital Signatures, Quantum Digital Signatures Offer Information-Theoretic Security Based On Laws Of Quantum Mechanics, Single Dot Confabulates With Single Ion, Natural Atoms Are "Cleaner" Systems And Their Electrons Can Be Prepared In Quantum States That Have Long Coherence Times, Single-To-Single" Interface That Couples A Single Artificial Atom To A Single Natural Atom, Excitation In A Single Quantum Dot Can Be Transferred To An Excitation In A Single Ion By Means Of A Single Photon, Quantum State Can Be Transferred Between The

Quantum Dot And The Ion, Mølmer–Sørensen-Type Gate Operation Entangling Ions, Trapped Ions Make Impeccable Qubits, Laser-Pumping Techniques That Are Well Established For Atomic Systems, Optical Excitation Would Bring Qubit To A Metastable State, "Randomized Benchmarking", Semiconductor-Atomic Interface, Photonic Coupling Of A Semiconductor Quantum Dot And A Trapped Ion, Chaotic Exciton-Polariton Billiard, Solid State Quantum Devices At The Edge Of Quantum Chaos, Dispersion Mismatch, Atom Recoil, And Multiphoton Scattering, Trapped Ions Make Impeccable Qubits, Hilbert Space Structure In Quantum Gravity, Entanglement And State Manipulation Of Two Spin Coupled Qubits, Searching For A Needle In A Haystack, Quantum-Accelerated Fractal Image Compression, Quantum Circuit Representation Of Grover's Algorithm

Key words: Single Dot Confabulates With Single Ion, Natural Atoms Are "Cleaner" Systems And Their Electrons Can Be Prepared In Quantum States That Have Long Coherence Times, Single-To-Single" Interface That Couples A Single Artificial Atom To A Single Natural Atom, Excitation In A Single Quantum Dot Can Be Transferred To An Excitation In A Single Ion By Means Of A Single Photon, Quantum State Can Be Transferred Between The Quantum Dot And The Ion, Mølmer–Sørensen-Type Gate Operation Entangling Ions, Trapped Ions Make Impeccable Qubits, Laser-Pumping Techniques That Are Well Established For Atomic Systems, Optical Excitation Would Bring Qubit To A Metastable State, "Randomized Benchmarking", Semiconductor-Atomic Interface, Photonic Coupling Of A Semiconductor Quantum Dot And A Trapped Ion, Chaotic Exciton-Polariton Billiard, Solid State Quantum Devices At The Edge Of Quantum Chaos, Dispersion Mismatch, Atom Recoil, And Multiphoton Scattering, Trapped Ions Make Impeccable Qubits, Hilbert Space Structure In Quantum Gravity, Entanglement And State Manipulation Of Two Spin Coupled Qubits, Searching For A Needle In A Haystack, Quantum-Accelerated Fractal Image Compression, Quantum Circuit Representation Of Grover's Algorithm

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Acknowledgements: I have made concerted efforts, sustained struggle and protracted endeavours for the inclusion of every one and every source either in references or cross references. In the eventuality of any act of omission and commission, I make a sincere entreat, earnest beseech and fervent appeal to kindly pardon me. By nature, the work called for browsing and collating and coordinating thousands of various documents. Please excuse me in case of any prevarication from rules, or miss classification of the protagonist or antagonist laws is based on the characterstics and pen chance, predilection, proclivity and propensities of the systems under investigation. We acknowledge in unmistakable and unambiguous terms the help of Stanford encyclopedia, Kants writings, and Deleuze's Logic Of Sense, Penrose and Hawking's nature of space and time, Penrose's Emperor's New Mind, Shadows of mind, Ken Wilber's spectrum of consciousness etc., great men seem to be endless… ad infinitium….. One feels humbled and the least……...Google search and Wikipedia. Concerted and orchestrated efforts are made to put on recordial evidence the names of all people either under references list or mostly by cross references so that no one is missed. If there be any act of omission or commission, it is my sincere entreat, earnest beseech, fervent appeal to kindly pardon me and the error is absolutely inadvertent and in deliberate. Let not any sensibilities, susceptibilities, and sentimentalities be hurt. Parson's pattern variables provide a way of categorizing the types of choices and forms of orientation for individual social actors, both in contemporary society and historically .I want to put on record with humble gratefulness the help by American Physical society, nature and other Noetic institutes of US who sent lot of alerts for my reference which were very valuable, and could not have been found by me despite assiduous and fervent search. Most important type is that concerning the stability of solutions near to a point of equilibrium. This may be discussed by the theory of Lyapunov. In simple terms, if all solutions of the dynamical system that start out near an equilibrium point x_e stay near x_e forever, then x_e is Lyapunov stable. More strongly, if x_e is Lyapunov

stable and all solutions that start out near x_e converge to x_e, then x_e is asymptotically stable. The notion of exponential stability guarantees a minimal rate of decay, i.e., an estimate of how quickly the solutions converge. The idea of Lyapunov stability can be extended to infinite-dimensional manifolds, where it is known as structural stability, which concerns the behavior of different but "nearby" solutions to differential equations. Input-to-state stability (ISS) applies Lyapunov notions to systems with inputs. von Neumann stability is necessary and sufficient for stability in the sense of Lax–Richtmyer (as used in the Lax equivalence theorem): The PDE and the finite difference scheme models are linear; the PDE is constant-coefficient with periodic boundary conditions and have only two independent variables; and the scheme uses no more than two time levels (See Wikipedia) Von Neumann stability is necessary in a much wider variety of cases. It is often used in place of a more detailed stability analysis to provide a good guess at the restrictions (if any) on the step sizes used in the scheme because of its relative simplicity. Albeit forwarded in nine module systematizations, the entire gamut is to be seen in a single shot, and the presentation of nine schedule twenty seven storey models is to circumvent typing of hundreds of superscripts and subscripts. In fact the statement is made inclusive of all previous models, and the variables are definitely different for each schedule, which again is reinstated due to typing of corresponding variables millions of systems, a fastidious and fussy work again. I beg pardon for any inconvenience caused to the readers due to such utilization of convention. I am grateful to Professor Chadralekha MD PhD. Tagore medical College, Chennai for deliberations and discussions on Medicine. To discussions on Physics topics credit goes to Dr. A.S. Krishna Prasad, Former Director DRDO, and Bangalore Chapter. Prof. Sunita MSc. PhD., of MS Ramaiah University helped me with valuable suggestions on Aerodynamics and propellant chemistry. Sir KVB Pantulu, former Chairman of NALCO, ESSAR Steels helped in formatting process and project management advices.

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PREFACE

Note: Some of the following concepts are modelled; some are not. For those which are not modelled kindly see one of the papers in the series. This note has the axiomatic predication to give a holistic view of the study. Here we talk of the characterstics of systems which satisfy the condition of cosmological constant or any other for that matter or variable or axiomatic predication. . There are lots of zeroes corresponding and concomitant to the second law of black holes. Infact as many as that of extant and existential blackholes exist. At the outset, it is to be stated that there are hadrons in every system. Supersymmetry between forces and matter, with both open and closed strings; no tachyon; group symmetry is SO (32) and its axiomatic predications, predicational anteriorities, character constitution shall be extant and existential in very many systems, and the characterstics are taken in to consideration in the classification scheme. Many systems have such fundamental instabilities like that of quantum gravity and characterstics of those systems form the citadel and fulcrum, bulwark and manor, mainstay and reinforcement, alcazar and chateau theory has a fundamental instability on which the classification schémas are valid. There are lots of systems which follow the axioms of string theory. It is the characterstics of this system which are taken in to consideration in the classification scheme. "All institutionalization involves common moral as well as other values. Collectivity obligations are, therefore, an aspect of every institutionalized role. But in certain contexts of orientation-choice, these obligations may be latent ...." (Parsons, 1951, p. 99). There are various systems that have the same bastion, support system, stylobate and sentinel as that of the Deleuzean terms and predications and phenomenological methodologies systemized. Look at this beautiful passage whose stability analysis would be eye

opener. To "deconstruct" is not the same as to destroy. Deconstruction attempts to undo logical contradictions, to overturn rigid conceptual oppositions while releasing new concepts and meanings that could not be included in the old system. At the heart of Western metaphysics, for example, Derrida finds the opposition between "speech" and "writing." This binary logic functions in an illicit way to establish speech as the means of giving "presence" to the world, while writing is deemed derivative and inferior In Derrida's sense of "grammatology," however, all production of meaning is writing and subject to the infinite play of signification. By taking away the transcendental signified and advancing the concept of "differance" (language organized around difference and deferred, or mediated, understandings), Derrida, like Nietzsche, wants to leave us without transcendental illusions, metaphysical unities, and foundations that constrain thought and creativity. **THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html** Each and every system has electrons, neutrons and protons and for that matter quarks. There shall be strong nuclear force and weak nuclear force. There are many systems that satisfy the criterion specified by the equation, principle or statement in question. Characterstics of the investigating systems form the bastion for the classification scheme and doxa thereof. Systemic differentiation is conducted. Despite gravity being constant, there exists gravity between two objects, and this could be taken as a system. Depending upon some parametric representationalities, functionalities, advantageousness, appropriateness, benefit, facilitation, fittingness, helpfulness, instrumentality, merit, practicality, serviceability, suitability, and usefulness, utility, these systems could be classified in to various categories. In respect of an equation, there shall be many systems that satisfy the given equation. Equations themselves could be by the utilization of the model solved term by term as has been exemplified and illustrated many time in the previous papers. There is lot of systems that could be brought in to the orbit of and gamut of the theory in question which the investigatable systems satisfy the axiomatic predications and postulation alcovishness of the systems in question. Towards the end of classificational consummation, consolidation, corporation and concatenation we take the characterstics of the systems, the predicational interiorities, ontological consonance and primordial exactitude, accolytish representations, functional topology, apocryphal aneurism and atrophied asseveration, event at contracted points, and other parameters as the bastion and stylobate of the stratification purposes. Such totalistic entities would have easy paradigm of relational content, differentiated system of expressly oriented actions with primary focus and locus of homologues receptiveness and differentially instrumental activity, variable universalism and particularism, imperative compatibilities and structural variabilities, interactional dynamical orientation, institutionalization and internalisation of pattern variables common attitudinal orientation of constituionalisation of internalized dispositions, and a qualitative gradient of structural differentiation and ascribed particularistic solidarity abstraction or interactional dynamics, internal differentiation, structural morphology, formal characterization, concept formulation, phenomenological methodologies, constituent structure, transformational minimal condition, paradigmatic feasibilities, programmatic plausibilities, comparative variability, normative aspect of expectational prediction, projection and prognostication as consideration of the investigatory systems. Any scale can be used that is convenient to the classification scheme. It is important to note that the scheme of classification and the stratification doxa must not be adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrogressive, inimically inverse, violatively unsimilar, diversely dissimilar, antipathetically anti thetical, conflictingly combative, obstructively pugnacious, inimically obstructive, repellently restrictive, disputatiously gainsaying or conformingly pugnacious to the axiomatic predications and postulation alcovishness of the theory in question or the equation representative or constitutive thereof. Sole intention, main objective and primary aim is twofold. One is towards the end of circumvention of the extra equation and the concomitant and corresponding variable therein. Second is avoidance of clustered congest, swarmed huddle, mustered pack, sardine squash, and swamp throng in the scheme of classification. Only thing that is sought out is the consonance in the entire diaspora and body fabric of the systems under study. This statement is true and holds unmistakably true for all the papers and I sincerely entreat readers to remember the statement and read the paper against this background. When we write A+B we mean by that B is being added to A or vice versa. It is like adding milk to water and water

to milk. There may or may not be a time gap. As said earlier there may be many systems that satisfy the conditions of the equations and those systems that are investigatory or investigatable are taken in to consideration based on their characterstics in the classification doxa. When there are more than two entities, we can take logarithm and find the value of that factor to be taken with anti log to obtain prediction and projected values of the model. In case of A-B, we are removing B from A and that means B is eating up A. These factors are taken in to consideration in the application of the model to equations. Log (ab) and Log (a+b) is well defined. Model stands out as universal testament and template for application to each sentence and equation what with the quantification process done and the correlations well defined. In the eventuality of non existence of any connection at some phase, the model would have the accentuation and attrition coefficients and detritions coefficients as zero rendering the equations of concatenation simpler. Projection formula which incorporates in its diaspora the initial values provide autheticative determination, unimpeachable validation, incontrovertible establishment , apodictic evidence, reliable roll out, unfailing cinquecento quattrocento trecento, incontrovertible indication of the final finale, notwithstanding appellation, appellative, brand, cognomen, compellation, designation, flag, handle, identification, label, moniker, nomen, slot, style, surname, tab, tag, term, title designation, appellation, appellative, class, classification, denomination, description, epithet, of the classification scepter, scenario, scimitar, schottische. There is pure and impure consciousness in every one. Gratification producing and deprivation producing one's can be easily classified from individual general ledgers. Similar analogy holds for collective general ledger and cosmic general ledger or nature's general ledger. It is also to be noted that while dealing with equations towards the end of consummation of the measure, it is necessary that the two variables are to be classified in to three sections and each one would have the adventitious and decidedly stated relationship whereby the fundamental equations are drawn up and the analysis made. All the parametric representationalities, conditionalities, orientationalities remain unequivocal as stated in the variables stated in to consideration section and are different from module to module. In essence the paper is to be read as holistic one with the sole intention, primary objective and raison d' etre to build a TOE. Towards the end of circumvention of typing hundreds of superscripts and subscripts which would be a sardine squash and the concomitant operational difficulties, model is presented in piece meal of nine modules. Logarithms are to be taken in respect of those which incorporate more than one variable in bra-ket. Values of log (ab) and log (a+b) are readily available. Anti logarithm shall be taken at the end to predict, project, prognosticate the value of the variable. This is true for tensors, vectors and other variables too. Affirmational assertion, and explanation justification, statement of vindication, annotational commentary, and explicational glossary for each and every system changes and physical interpretation of results is one thing that is to be with earnest endeavour and feverish and febrile expediency. Editions never ever mean the same and identicality of parameters and this has been explicitly and unmistakably stated in the model apriori itself. Akin and analogous, cognate and concurrent, correspondent and congruent, comparable and complementary, synonymous and duple, tantamount and agnate, commensurate and correlative representation is only to highlight the importance and subterfuge the replication of work. It is my fervent solicitation to kindly bear with me for any lapses, notwithstanding the orchestrated efforts for a paper without any minor errors. Postulation predication, conclusive presumption, differential presuppositions, underscored decidedly axiomatic statement of the statement, equation form the bye word or the watch word in the aggrandizement-amplification, caricature and crock, understatement- unembellishment, elocution-emphasis, enunciation-inflection, announcement-argument, articulation, assertion, asseveration, choice of words, commentary and communication, declaration and definition, delivery-diction, elucidation-emphasis, enunciation-execution, explanation, exposition-formulation, idiom, interpretation and intonation tone and tenor of stratification. Now, how do find the reaction of systems to these singularities. You do the same thing a boss does for you. "Problematize" the events and see how you behave. I will resort to "pressure tactics". "intimidation of deriding report", or "cut in the increment" to make you undergo trials, travails and tribulations. I am happy to see if you improve your work; but may or may not be sad if you succumb to it and hang yourself! We do the same thing with systems. systems show conducive response, felicitous reciprocation or behave erratically with inner roil, eponymous radicality without and with blitzy conviction say like a solipsist nature of bellicose and blustering particles, or for that matter coruscation, trepidiational motion in fluid

flows, or seemingly perfidious incendiaries in gormandizing fellow elementary particles, abnormal ebullitions, surcharges calumniations and unwarranted(you think so but the system idoes not!) unrighteous fulminations. Amount of emotion or affect that is appropriate or expected in an given form of interaction. Particular individuals and diffuse obligations (see c and d) are associated with affectivity, whereas contacts with many individuals (universalistic) in a bureaucracy may be devoid of emotion and characterized by affective neutrality. Affective neutrality may refer to self discipline and the deferment of gratification (eg Weber's spirit of capitalism). In contrast, affectivity may be associated with expressing emotions. Adams and Sydie also refer to affective neutrality being associated with ego control (p. 15). (Parsons: Wikipedia). So the point that is made here is "like we problematize the "events" to understand the human behaviour we have to "problematize" the events of systems to understand their behaviour. This statement is made in connection to the fact that there shall be creation or destruction of particles or complete obliteration of the system (blackhole evaporation) or obfuscation of results. Some systems are like "inside traders" they will not put signature atoll! How do you find they did it! Anyway, there are possibilities of a CIA finding out as they recently did! So we can do the same thing witn systems to. This is accentuation, corroboration, fortification, .fomentatory note to explain the various coefficients we have used in the model as also the dissipations called door. In the bank example we have clarified that various systems are individually conservative, and their conservativeness extends holisticallytoo.that one law is universal does not mean there is complete adjudication of nonexistence of totality or global or holistic figure. Total always exists and "individual" systems always exists, if we donot bring Kant in to picture! For the time being let us not! Equations would become more ensorcelled and frenzied..... philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". In consciousness research, two rival sets of theories can be recognized: (A) Scientific material interpretations of consciousness are based on axioms that view consciousness in the context of highly advanced intentional processing of information in which subject-object relations evolve, and (B) humanistic interpretations of consciousness are based on axioms that view consciousness in the context of, say, "centered pulsations" that enable a conscious agent to act from his or her center of awareness. In this paper I will argue for the selection of axioms that favor humanistic interpretations of consciousness. The Pursuit of Autonomy Interdisciplinary Observations to Human Consciousness: Wautischer, Helmut (1993) the Pursuit of Autonomy. Interdisciplinary Observations to Human Consciousness: Social Neuroscience Bulletin, 6 (4). pp. 52-56.Everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum; it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for very hum of voices; or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring; if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sense fully. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality; **CONSCIOUSNESS AND ITS UNIQUENESS June 12, 2012 at 8:40pm (See Deleuze Logic of sense, Wikipedia and Stanford encyclopedia for more details)** Atrocious contrivance and device, gratificational primogeniture, calamitous dodge and expedient, depraved, destructive, disastrous, execrable

gambit and gimmick, great solace and succor, iniquitous, injurious, loathsome, low, maleficent, malevolent machination and maneuver, promethaleon of candor, frankness, honesty, honor, ingenuousness, innocence, openness, reality, sincerity, truthfulness, spiteful, stinking, ugly, unpleasant, unpropitious play and ploy, progenitor of jurisprudence and circumspection, racket and ruse, savvy, scam, stratagem, subterfuge, tactic and wile, forthrightness, honesty, truthfulness embodied and personified is the structural predisposition and dispensation of the complex. Nobody need have to take the Brahman-Anti Brahman agency to be a recalcitrant, repugnant, and refractory proposition. God gives and also takes away. That is the point made. And of course lesser gods, and God doth follow them and they follow God. That is why they are lesser gods. And wield the power on lesser mortals like us. When we mean gratification and deprivation we mean celestial Brahman Anti Brahman and Terrestial Brahman and Anti Brahman agencies producing such happiness or sadness by any means such as (blending- amalgamation, adulterant-adulteration, amalgam-amalgamation, blend- combination, composite- compound, debasement-denaturant, fusion-hybrid, intermixture-reduction, amalgam- mixture, composite- compound, fusion-mishmash, aggregate- alloy, amalgamation-blend, commixture-composite, composition-compost, conglomerate-fusion, goulash-medley, mishmash-stew, blend-coalition, commixture-compound, federation-heating, integration-junction, melting-merger, synthesis-unification, assortment- combination, adulteration-alloy, assimilation-association, batter-blend, brewed combine, composite compound, concoction confection, conglomeration cross of money, orientation(dis), penury creation, murder, mayhem plunder, pillage, apocalypse, Armageddon in various forms and permutation and combination of Manichaean artifice, agathokakological malevolence, agley chicane fourberie, fraud, furtiveness, gambit, awry and bad ,flagitious mechanizations, cacodemonic gambit, hanky-panky, intrigue, machination, deprecatory and diabolic underhandedness and wiles, energumenical skullduggery, goetic chicanery, lenocinant stratagem, malefic and maleficent strategy, malominous sell, sellout, sham subterfuge, peccable, humbug, imposture, misrepresentation peccant fraud, fraudulence, hoax, qued , scelerate pretense, ruse, sell, sellout, sham stratagems' of maladroit manouevures , sinistral ,tortious and venal sophistry, contingency enterprise, endangerment emprise, jeopardizing adventure, jejune jeremiad, avoidable inertia, latent passiveness: with each action accounted and showing balance in the nature's general ledger as also actionlessness:e). All these are recorded in Nature's general ledger. Neuron DNA encapsulates all the actions passions, actions, interactions, transactions or the lack of it at various levels of individual, collective and cosmic levels. In celestial Brahman Anti Brahman ledger there is cessation of dualities. One more point to be made is if 5=3+2, then we can say 5 gormandizes 3 or 5 gobbles up 2. Quintessentially such a statement arises out of the difference of LHS and RHS term wise. It must be remembered despite all theoretical abstractions and generalizations be it proton mass or Higgs Boson, or topology of a space, it is notwithstanding the fundamentality of nature is just a number. Just everything in Physics and mathematics is just a number. Let us assume I buy rice in Bangalore (District) worth 5k.g. I can always divide 5k.g.s of rice in categories of Bangalore (South), Bangalore (west), Bangalore (East) and Bangalore (South). On similar basis, there are billions of neutrons, protons and concomitant interactions which would satisfy the given numerical value. Example of gravity above makes things clear. Everything atleast is a function of time and someplace or some other coordinates, subordinates or superordinates. Mass of proton is 2.47 does not mean total mass of all protons is 2.47. It is this argument that is used in the classification scheme. By the very nature, self the unmonitoring but the witness consciousness agency identifies itself with something, be it a profession, an actor on the screen, when it does not have a identity. Such examples are legion, a Charlie Chaplin in "The Kid" or Raj kapoor in "Jagte Raho". These are people who just wanted to live and living without being put under surveillance every moment is the greatest part of their life. This is "Absolute Subjectivity" of Shiva. Such a state is always craving for identification for in pure consciousness all identities are lost. It is this "identitylessness" or having very minimal identity is what pervades most in India and this is due to expansion of individual consciousness. When that happens, be it education, or wealth, the "identity' becomes so well entrenched, they become the exact opposite of "pure consciousness"," impure consciousness" (Shakti). This is a state of "Absolute or relativistic objectivity". There was this case of a lady who went to such an extent of planning, she was sure that whatever she does must work out. When it did not, she lapsed in to "identity crisis". Such thing as this might happen in well developed countries where

the competition is extremal and to be achieved at any cost. Realisation that one is not the doer but the witness consciousness is just like watching a movie in which one has acted. It is in this sense "advaita", "non duality' seems to be of cardinal and preeminent importance for the analytical treat ment of "identity' and "identity lessness", both happen in space time. "Multiverses" in this sense is also like a different film of an actor released at different theaters and it is in this sense we state that that every object in space time is a multiverse. You must have totally immersed in doing a "string theory" problem or watching a "film", oblivious of surroundings. Essentially creation (space time) also is what is happening in mind and honed in to you billion times. Identification of anything with a designation or signification is one that leads to association within spacetime, which would only have pathological consequences and detrimental ramification as stated above or positive implications of "happiness" because of the identity itself. Search or 'show" has to go on! It gives a sense of consummation. a watching pot never boils; donot go to check nature; you are disturbing it; it will never show its true colours bhakti and Virakthi, gratification and deprivation, raga and dwesha is essential to live in space. It is the quantum interest thereof that produces energy to live .gratification and deprivation which form the bastion, pillar, post and stylobate and sentinel of all human actions depends on tams, rajas, and sattva of individual or insentient object. Verily tamas, rajas and satva are the characterstics of space itself, nay the behavioural pattern and attitudinal orientation. There are peaceful places like Vienna or Sringeri and there are violent places like others. Space is warped and woofed by the Akshara, the quantum information of primordial syllable Om. cosmic general ledger is the one vibration of which is brahman; quantum information possessions thereof form the sentinel of all transactions (Gargi to Yajnavalkya: my interpretation)adhocism, nonseqientiality of events, anti sequentiality of events, simultaneity of phenomenon, concurrent happenings , improbability of phenomenon happening itself are characterstics of brahman-anti brahman (Terrestial and celestial) are the cause of non predictability of happenings like in black hole, or in the areas of operation of brahman-anti brahman (Terrestial and celestial). brahman AntiBrahman actions, interactions, decisions, circumspection, adhocism, satisfies sensitive to initial conditions; topologically mixing; dense periodic orbits, the axiomatic predications and postulation alcovishness of chaos theory. Like gravity is a characteristic of space, so is tamas, rajas, sattva, with spaces knotted together by quantum information (for models see below) .accidents are preplanned actions of Terrestial and celestial Brahman AntiBrahman. All actions are perpetrated by them, be it train on fire or death. Belief systems play an important role in the determination of tamas, rajas, and satva state of human beings. There is neither cause nor effect. All actions are performed by Terrestial and celestial Brahman anti Brahman and it is off the record. psychic energy is negative equivalent to –mc^2, so that second law of thermodynamics is satisfied on a holistic basis vis-à-vis material energy ParaBrahaman is infinite individual, collective and cosmic consciousnesses are infinite (concomitant general ledgers) for they are in the evolution mode with respect to each other in that order. Later emanates from the former as transactions emanate from the conservative state of assets and liabilities. Is not zero true for infinite conservative systems inside space and time? zero is the only consummation, consolidation and conservation. if fifty out of seventy members come out of the room at whatever time you come out, talk about the grand limbo of oblivion and hibernation of lifeless world you are going to and if you are not a celebrity, then it is not coincidence, it is coordination. Nature is playing the same trick on you. That a generalised law is there and you can measure does not make things less good albeit recalcitrant, refractory and rogue particles occur time and again, by and large. Entering in to all living beings through his constant part on the new moon night, he is born therefrom next morning. Absolute ParaBrahaman (zero) projects himself again and again on the screen of individual consciousness which appears of depairing despondent world. Some where the balancing act is done on karma. cosmic general ledger reflects upon itself to keep all transactional ties conservative (Brihadarnkayaka Upanishad: 1.5 14) for models see next paper through its divine power (i interpret man creates god; it is attribution and ascription of various Shaktis would the formation of god takes place like space and time) the self assumes (eb) the form of a deity man can contemplate and venerate, even though siva, the pure subject, can never in fact be an object of meditation (adhyeya). Until we realise our true identity with Sankara, he is worshipped and conceived to be a reality alien to ourselves. While we are in the realm of creation, he too is a creation or mode or appearing of the absolute, manifest to us in meditation, through his freedom as an

eternal, omniscient being. There is no gulf between the created and the uncreated creator: nothing in reality, although an object of knowledge, ceases to be Siva: this is the reason why meditation [on this or that aspect] of reality bestows its fruit. The world of the senses and mind appears to the well awakened (suprabuddha) as a theophany, an eternal revealing of god in his creation. The doctrine of vibration declares that "there is no state in word, meaning or thought, either at the beginning, middle or end, which is not Siva." 59 to utter any word is, in reality, to intone a sacred formula.every act is a part of Siva's eternal cosmic liturgy, every movement of the body a ritual gesture (mudras), and every thought, god's thought. By what path are you not attainable? What words do not speak of you? in which meditation are you not an object of contemplation? What indeed are you not, o lord? 61 spandashakti, which accounts for the appearing of all things, is also the means by which deity in its many varied forms appears to man. Ksemaraja concludes: the ultimate object of worship of any theistic school differs not from the Spanda principle. The diversity of meditation is due solely to the absolute freedom of Spanda. 62 Sankara is not only the supreme object of devotion; as the static polarity of the absolute, he is the inner reality which holds together its Siva and Sakti outer manifestations. Phenomena are patterns of cognitions projected onto the surface of self-luminous siva-consciousness. There they become apparent, directly revealed to consciousness according to their manifest form. Siva is accordingly symbolized as the ground or surface of awareness, smooth and even like a screen (samabhittitalopama). 64 inscribed on this screen (kutfya) are the countless manifest forms which appear within it rendering it as diverse and beautiful as a fossil ammonite (falagrama). 65 Siva is the sacred ground upon which the cosmic mandalas are drawn, the absolute surface of inscription which bears the mark (cihna) of the universe. Abhinava writes: the variety of this world can only be manifest if the highest lord, who is essentially the pure light of consciousness, exists; just as a surface is necessary for a picture. if external objects were perceived in isolation then, because 'blue' and 'yellow', etc., are self-confined and the perceptions [we have of them] refer to their objects alone and so are insentient, mute and dumb in relation to one another . . . how would it be possible to be aware that an object is variegated? but just as depths and elevations can be represented by lines on a smooth wall, and we perceive [a female figure and think], similarly it is possible to be aware of differences in the variegated (contents of experience) only if all the diverse perceptions are connected together on the one wall of the universal light of consciousness. 66 Siva is the perfect artist who, without need of canvas or brush, paints the world pictures. the instant he imagines it, it appears spontaneously, perfect in every respect. The colours he uses are the varying shades and gradations of his own Spanda energy and the medium his own consciousness. Intractable disaffect, Mutinous obdurate, Rebellious recalcitrant, Seditious refractory, Insubordinate insurgent, antagonistically antipodean, discrepant incompatible, Contumacious coadjutant, Conflicting converse, Unyielding obdurate, the antagonistic and anachronistic dispositional ties of Anti Brahman are in fact the sine qua non of Brahman only (See Vishwaroopa of Mahavishnu, which contains the polarised positives on RHS and negatives on LHS). The universe is coloured with the dye of its own nature (svabhava) by the power of Siva's consciousness (a//). Rajanaka rama says: homage to him who paints the picture of the three worlds, thereby displaying in full evidence his amazing genius {pratibha)\ to Sambhu who is beautiful with the hundreds of appearances laid out by the brush of his own unique, subtle and pure energy. 68 analogously, at the microcosmic level, all the cognitions and emotions, etc., which make up the individual personality form the outward flow of essentially introverted consciousness. They are specific pulsations (vise$aspanda) or aspects of the universal pulsation (samanyaspanda) of pure t consciousness. at the lower level, within the domain of maya, they represent the play in the fettered soul of the three primary qualities (gunas) or feeling-tones' which permeate to varying degrees his daily experience. These are: 1) sattva — the quality of goodness and luminosity which accompanies blissful experience both aesthetic and spiritual. 2) Rajas — the passion or agitation which oscillates between the extremes of 'light' and 'darkness' and characterises inherently painful experiences. 3) Tamas — the torpor and delusion which accompany states of inertia and ignorance. 69 the liberated soul recognises that these three are the natural and uncreated powers of pure consciousness. for him they are manifest respectively as: 1) Sankara 's power of knowledge (jnana) — the light of consciousness (prakasa); 2) the power of action (kriya) — the reflective awareness of consciousness (vimarsa)\ 3) the power of maya — which does not mean here the world of diversity, but the initial subtle distinction which appears between subject and object in

pure consciousness. 70(doctrine of vibration: mark S.G. Dyczkowski) .to "deconstruct" is not the same as to destroy. Deconstruction attempts to undo logical contradictions, to overturn rigid conceptual oppositions while releasing new concepts and meanings that could not be included in the old system. at the heart of western metaphysics, for example, derrida finds the opposition between "speech" and "writing." this binary logic functions in an illicit way to establish speech as the means of giving "presence" to the world, while writing is deemed derivative and inferior in derrida's sense of "grammatology," however, all production of meaning is writing and subject to the infinite play of signification. By taking away the transcendental signified and advancing the concept of "differance" (language organized around difference and deferred, or mediated, understandings), derrida, like Nietzsche, wants to leave us without transcendental illusions, metaphysical unities, and foundations that constrain thought and creativity. The postmodern turn in philosophy: theoretical provocations and normative deficits by Steven best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html. you cannot see beyond consciousness for that is the limit of your storage of information or the basis for understanding further information. Turiya is empty bucket; it is from which prajna the measure of arises; turiya measures the number of quantum information stores in other three states of waking, dreams, and dreamless deep sleep (Manduka Upanishad; my interpretation for models next paper).let there be path where angels tread: it is a force never born never dies the brahman an anti brahman verily there are parallel universes in (eb) in the ambit of space and time; the society of sinners and the sinned with the grammar of evil; parallel universes contain(e) multiverses in them; all physics, philosophy, break down(e) in these universes; astronomical gratification ,deprivation, glorification, mortification, projections, axes, dimensions, rotations, foldings, concurrences, motions, shocks, are the axiomatic predications of multiverses and parallel universes; death is when self (the witness consciousness; register of individual general ledger) leaves the body; ostensibly parallel seem to have (e) an objective; parallel universes donot (e) have objectives; their paradigm of relational content, differentiated system of expressly oriented actions with primary focus and locus of homologues receptiveness and differentially instrumental activity, variable universalism and particularism, imperative compatibilities and structural variabilities, interactional dynamical orientation, institutionalization and internalisation of pattern variables common attitudinal orientation of constituionalisation of internalized dispositions, and a qualitative gradient of structural differentiation and ascribed particularistic solidarity abstraction or interactional dynamics, internal differentiation, structural morphology, formal characterization, concept formulation, phenomenological methodologies, constituent structure, transformational minimal condition, paradigmatic feasibilities, programmatic plausibilities, comparative variability, normative aspect of expectational prediction, affection- appreciation, awareness- discernment, emotion- feeling, gut reaction, heart insight, intuition, judgment, keenness, perceptiveness, rationale, sensation, sense, sensitiveness, sensitivity, sentiment, susceptibility, taste, vibes, admiration, aesthetic sense, affection, appraisal, assessment, attraction, awareness, cognizance, commendation, comprehension, enjoyment, esteem, estimation, grasp, high regard, knowledge, liking, love, perceptional realization, recognitional regard, relish, respect, responsiveness, sensibility, sensitiveness, sensitivity, sympathy, understanding, valuation, acknowledgment, airing, baring, , confessional defenselessness, denudation-denunciation, disclosure, display, divulgence-exhibition, hazard, introduction, jeopardy, laying open, liability, manifestation, nakedness, openness, peril, presentation, publicity, revelation, risk, showing, susceptibility , susceptiveness,susceptivity, unfolding, unmasking, unveiling, vulnerability, vulnerableness, action, affection, appreciation and ardor, behavior, capacity, compassion, concern, cultivation and culture, delicacy, discernment and discrimination, emotion, empathy, faculty, fervor, fondness, heat, imagination, impression and intelligence, intensity and intuition, judgment, keenness- palpability, passion, pathos, pity, reaction, refinement, sensibility, sensitivity, sentiment, sentimentality , sharpness, spirit, sympathy, tangibility, taste, tenderness, understanding, warmth, fond memories, hearts and flowers, homesickness, longing, pining, reminiscence, ,schmaltz, sentimentality wistfulness, yearning satiation is only ostensible; their job is to get the job done; their business is to mind other's business ;all laws of physics and finance breakdown; it is antilaw,piscatorial Piratish and bubonic bucaneerishness; there is free transactions between in multiverses; when reflected on the screen of consciousness they appear as extant existential universe. Transactions are astronomical and gratification deprivation, glorification and

mortification incalculable; they have no barriers of region, religion, language which they create for others. theirs objectives are: corruption criminality, debasement debauchery, degradation evil immoral lewdness licentiousness, perversion sinfulness, vice viciousness, vileness wickedness, improbity corruption, atrocity decadence, degeneration degradation, depravity evil, immorality impurity, infamy iniquity, looseness lubricity, perversion profligacy, sinfulness turpitude, vice viciousness, vulgarity wickedness, disgrace bad reputation, abasement abuse, baseness black eye, blemish blur, brand comedown,contempt contumely, corruption culpability, debasement debasing, defamation degradation, derision disbarment, discredit disesteem, disfavor dishonor, disrepute disrespect, humbling humiliation ignominy ill repute, infamy ingloriousness, meanness obloquy,odium opprobrium, pollution ,put-down reproach, scandal scorn, slander slight, slur spot, stain stigma, taint tarnish, turpitude venality, enormity Horribleness , abomination atrociousness, atrocity crime and the positive aspectionalities thereof .let there not be any incursions where evil mercenaries in the garb of mendicants thrive with adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrogressive, inimically inverse, violatively unsimilar, diversely dissimilar, antipathetically anti thetical, conflictingly combative, obstructively pugnacious, inimically obstructive, repellently restrictive, disputatious gainsaying or conformingly pugnacious Manichaean artifice, agathokakological malevolence, agley chicane fourberie, fraud, furtiveness, gambit, awry and bad ,flagitious mechanizations, cacodemonic gambit, hanky-panky, intrigue, machination, deprecatory and diabolic underhandedness and wiles, energumenical skullduggery, execrable and goetic chicanery, lenocinant stratagem, malefic and maleficent strategy, malominous sell, sellout, sham subterfuge, peccable, humbug, imposture, misrepresentation peccant fraud, fraudulence, hoax, qued , scelerate pretense, ruse, sell, sellout, sham stratagems' of maladroit manouevures , sinistral ,tortious and venal sophistry, contingency enterprise, endangerment emprise, jeopardizing adventure, jejune jeremiad, avoidable inertia, latent passiveness . statement for Mephistophelean mercenaries philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum; it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for very hum of voices; or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring; if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sense fully. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality. Negative energy stabilising wormholes are considered candidates for darkmatter. An analysis of negative energy might be an useful ingredient of the thoughts of travel at speeds more than the velocity of light. Peirce explains that an "imperfect continuum" possesses "topical singularities, or places of lower dimensionality" which break continuity, while "the parts of a perfect continuum have the same dimensionality as the whole." (CP 4.642) Circular continua without singularities are time and space. (CP 6.210-212) By using math as his lens, Peirce envisioned singularities or lower dimensional holes in the imperfect cosmologic continuum, while maintaining that a probabilistic law of chance exists that entropy decreases **(Wikipedia).** On the side of philosophy**,** quite explicitly, Deleuze provides a

critique of Husserl's "sleight-of-hand" change from a static categorical phenomenology to one supposedly more dynamically constitutive of reality. This a clear cut **chagrined circumvention**, **contraventional curbing**, **defeated disgruntlement**, **dissatisfactional downer**, **dragged failure**, **fizzled foil**, **grievance hindrance**, **impedimental irritation**, letdown, nonfulfillment, nonsuccess, obstruction, old one-two, **resent mental setback**, unfulfillment of the earlier theories makes Sartre contend that human existence is a conundrum whereby each of us exists, for as long as we live, within an overall condition of nothingness (nothing-ness)—that ultimately allows for free consciousness. But simultaneously, within our being (in the physical world), we are constrained to make continuous, conscious choices. It is this dichotomy that causes anguish, because choice (subjectivity) represents a limit on freedom within an otherwise unbridled range of thoughts. Subsequently, humans seek to flee our anguish through action-oriented constructs such as escapes, visualizations, or visions (such as dreams) designed to lead us toward some meaningful end, such as necessity, destiny, determinism (God), etc. Thus, in living our lives, we often become unconscious actors—Bourgeois, Feminist, Worker, Party Member, Frenchman, Canadian or American—each doing as we must to fulfill our chosen characters' destinies. However, Sartre contends our conscious choices (leading to often unconscious actions) run counter to our intellectual freedom. Yet we are bound to the conditioned and physical world—in which some form of action is always required. This leads to failed dreams of completion, as Sartre described them, because inevitably we are unable to bridge the void between the purity and spontaneity of thought and all-too constraining action; between the being and the nothingness that inherently coincide in our self. Deleuze insists on the absolute non-resemblance between what conditions (metastable, pre-individual plane of singularities) and what is conditioned (actualized or the individuated 'thing'). Only then do "the conditions of the true genesis become apparent," Deleuze writes (Logic of Sense 105). Without addressing this issue, Deleuze's borrowings from Simondon, Lautman and structuralism remain untethered. Because of Bowden's decision to completely sidestep Deleuze's overt negotiations with phenomenology, his otherwise sophisticated commentary suggests a pedagogical narrowness. If, the omission of phenomenology strengthens Bowden's focus on the ontological priority of the event over substantial ontologies via the genealogy of figures and movements Deleuze outlines, it also incompletely articulates the problem of the sense-event as Deleuze formulates it. We have lost any idea of whom Deleuze takes to be his primary adversary, against whom he stakes out his position. This worry is most clearly manifested in Bowden's description of Deleuze's ontological prioritizing of events as offering a "transcendental ontology." For Bowden Deleuze's transcendental ontology defines events that "are ontologically prior to substances, 'all the way down'" (82). But Bowden provides a second definition: it is that the priority of events over worldly individuals means that the world of events "is not something external to the conditions of knowledge" (69). **SEAN BOWDEN the Priority of Events: Deleuze's Logic of Sense Sean Bowden, the Priority of Events: Deleuze's Logic of Sense, Edinburgh University Press, 2011, 296pp., $40.00 (hbk), ISBN 9780748643646. Reviewed by David Scott, Coppin State University Weltschmerz, agony, apprehension, blues, depression, dread, mid-life crisis, misgiving, nervousness, uneasiness, angst were expressed with such effulgent words…… aggrandizement-amplification, caricature and crock, understatement- unembellishment, elocution-emphasis, enunciation-inflection, announcement, argument, articulational assertion, asseverational choice of words, commentary and communication, declaration and definition, delivery, diction, elucidation, emphasis, enunciation-execution, explanation, exposition-formulation, idiom, interpretation and intonation about the human predicament is brought out in Being and Nothingness. Being and Nothingness offers a critique of Sigmund Freud's theory of the unconscious, based on the claim that consciousness is essentially self-conscious. Sartre also argues that Freud's theory of repression is internally flawed. According to Sartre, in his clinical work, Freud encountered patients who seemed to embody a particular kind of paradox—they appeared to both know and not know the same thing. In response, Freud postulated the existence of the unconscious, which contains the "truth" of the traumas underlying the patients' behavior. This "truth" is actively repressed, which is made evident by the patients' resistance to its revelation during analysis. Yet what does the resisting if the patients are unaware of what they are repressing? Sartre finds the answer in what Freud calls the "censor". "The only level on which we can locate the refusal of the subject," Sartre**

writes, "is that of the censor. The Holographic Principle, as conceived in current physics, applies to fields, and perhaps, even to more elementary entities, called strings and branes. These more elementary entities remain hypothetical at the time of this writing. There are many layers between the level of fields and that of neurons. We know that neurons are surrounded by a surface phospholipid membrane, which supports an electrochemical process that is fundamentally necessary to human experience. As the information supported by the membrane surface according to the Holographic Principle is proportional to its area, we should expect to find that the neural correlates of consciousness most clearly associated with those parts of the neuron that have the have a high proportion of neuronal surface area, and, in particular, receptive surface area. Dendrites account for an average of about 90% of the neuron is receptive surface area (Wong, 2002), and so would be expected to be the most important neural structures in information processes. We find this to be true **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science (Wikipedia).** Discordant-discrepant, dissonant-hard-line, implacable, incompatible, incongruous- inconsistent, inexorable-inflexible, inharmonious-intransigent, opposed, reluctant, unappeasable, uncompromising, unfriendly some of these expatiations and enumerations might sound, a more generalised interpretations is given by a Vedantin. Look at this arresting passage: The teacher continued to be silent. When addressed a second and third time he said: "I am teaching, but you do not follow. The Self is silence." The undetermined and unthinkable character of the Brahman is a consequence of the absolute's eternal and immutable nature. To concede the existence of a real universe is, from the Vedantin's point of view, to posit the existence of a reality apart from the Brahman. Nor can we simply identify a real universe with the absolute unless we are prepared to compromise its unchanging, absolute status. The criterion of authenticity is immutability. Reality never changes; only that which is less than real can appear to do so. Reality is constant in the midst of change. What this means essentially is that there is change although nothing changes. This impossible situation is reflected in the ultimate impossibility of change itself. That which does not exist prior to its changing and at the end, after it has changed, must be equally non-existent between these two moments. Although the world of change appears to be real, it cannot be so. Change, according to the Vedantin, presupposes a loss of identity. Reality cannot suffer transformation; if it were to do so, it would become something else and the real would be deprived of its reality. The immortal can never become mortal, nor can the mortal become immortal. The ultimate nature of anything cannot change. Change of any sort is merely apparent (vivarta); the world of change and becoming is a false super-imposition (adhyaropa, adhyasa) on the absolute. In cosmic terms, the mistake (bhranti) consists of the supposition that the real Brahman is the unreal universe and the unreal universe is the real Brahman. In microcosmic terms, it is the mistake of falsely conceiving the body, mind or even one's personality to be the Self. In the same way as the image of a snake is falsely superimposed on a rope, similarly the universe is falsely projected onto the real substratum, the Brahman. Ignorance is not merely a personal lack of knowledge, but a cosmic principle. As such it is called "Maya," the indefinable factor (anirvacaniya) that brings this mistake in identity about. The reality status of this cosmic illusion is also indefinable: on the one hand it is not Brahman, the sole reality; on the other hand it is not absolutely non-existent like a hare's horn or the son of a barren woman. Brahman is the source of world appearances only in the sense of being their unconditioned ground or essential nature. The universe is false not because it has no nature of its own but because it does have one. Just as the illusion of a snake disappears when one sees that it is nothing but a rope, similarly cancellation (badha) of the empirically real occurs when the absolute reality of the Brahman is realised. Thus, according to Vedanta, appearance implies the real, while the real need not imply appearance. To appear is essentially to appear in place of the real, but to be real is not necessarily to appear. All things -exist because the absolute exists. It is their Being. Thus the very existence of phenomena implies their non-existence as independent realities. When they are known to be as they are, in the fullest sense of their existence, their phenomenal nature disappears leaving the ground of Being naked and accessible. This approach was validated by a critique of experience. The Vedanta established that space, time and the other primary categories of our daily experience can have no absolute existence. It was therefore necessary to make a distinction between relative truths — that accepted by the precritical common man — and an absolute truth discovered at a higher level of consciousness. The Saiva absolutist 17 rejects any theory that maintains that the universe is less than real. From his

point of view a doctrine of two truths, one absolute and the other relative, endangers the very foundation of monism. The Kashmiri Saiva approach is integral: everything is given a place in the economy of the whole. It is equally wrong to say that reality is either one or diverse. Those who do so fail to grasp the true nature of things which is neither as well as both. "We do not" says Abhinavagupta, "base our contention that [reality] is one because of the contradictions inherent in saying that it is dual. It is your approach {paksa) that accepts this [method] While, if [duality and oneness] were in fact [to contradict each other], they would clearly be two [distinct realities]." The Vedantin, who maintains that non-duality is the true nature of the absolute by rejecting duality as only provisionally real, is ultimately landed in a dualism between the real and illusory by the foolishness of his own excessive sophistry (vacafadurvidya). Oneness is better understood as the coextensive unity (ekarasa) of both duality and unity. They are equally expressions of the absolute. Gopinath Kaviraj says: According to Sarikara, Brahman is truth and Maya is inexplicable (anirvacaniya). Hence the [Advaitin's] endeavour to demonstrate the superiority of Advaita philosophy is turned against his own system. It tarnishes the picture of its philosophical perfection and profundity. He cannot accept Maya to be a reality, therefore his non-dualism is exclusive. The whole system is based on renunciation and elimination and thus is not all-embracing.... By accepting Maya to be Brahman (brahmamayi), eternal (nitya) and real (satyarupa), Brahman and Maya [in the Tantra] become one and coextensive. **The Doctrine of Vibration: An Analysis of the Doctrines and Practices of Kashmir Shaivism Mark S. G. Dyczkowski. Concept of being from Shaivite point of view:** Solutional behaviour and stability analysis of such systems is of paramount importance, notwithstanding the assumptions made which are not farfetched, in that it throws light on the fact whether they could be candidates for yet unsolved problems, providing Authetícative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication for the wide amplitudinal ramificatory usage of the thesis propounded. The world is now the other person's world, a foreign world that no longer comes from the self, but from the other. The other person is a "threat to the order and arrangement of your whole world...Your world is suddenly haunted by the Other's values, over which you have no control"(Satre: being and Nothingness) Sartre states that many relationships are created by people's attraction not to another person, but rather how that person makes them feel about themselves by how they look at them. This is a state of emotional alienation whereby a person avoids experiencing **their subjectivity by identifying themselves with "the look" of the other**. The consequence is **conflict**. In order to maintain the person's own being, the person must control the other, but must also control the **freedom of the other "as freedom**". These relationships are a profound manifestation of "bad faith" as the for-itself is replaced with the other's freedom. The purpose of either participant is not to exist, but to maintain the other participant's looking at them. This system is often mistakenly called "love", but it is, in fact, nothing more than emotional alienation and denial of freedom. In essence we are always simulating and selling ourselves at the cost of our freedom. Both as a script writer and participant consciousness, human beings experienceâ complete subjective feelings and sometimes absolute subjective feelings, which if remains unchecked might lead to pathological ramifications. (italics mine) Perspicuous forbearance, Sophisticated seasoning, Partcipational observation, Background actuality, Existential worldliness, Existential strife, Predicational anteriorities, Character consonance, Ontological consonance, Primordial exactitude, Phenomenological correlates, Accolytish representation, Atrophied asseveration. Anamensial alienisms, Anchorite aperitif, Arcadian Atticism all delineated in the conditionalities and functionalities and orinetatilities of the system which incorporate the rules and regulations, axiomatic predications and postulation alcovishness of the foregoing state form the bastion, pillar, post, stylobate and sentinel of the classification scheme and doxa. appropriate and diagnostic, differentiating-discriminating, distinctive-distinguishing, emblematic- especial, essentially exclusive, fixed, idiosyncratic-inborn, inbred, indicative and individual, individualistic-inherent, individualizing and ingrained, inherent, innate, local, marked, native, normal, original, particularly- peculiar, personal, private, proper, regular, representative-singular, special, specific, symbolic, symptomatic- unique these form the classification and stratification measure that is suitable notwithstanding the constancy in many a case, as in gravity, which depends on the mass and distance for two given objects**.** Like Kant and Bergson, Deleuze considers traditional notions of space and time as unifying forms imposed by the subject. He therefore concludes that pure

difference is non-spatio-temporal; it is an idea, what Deleuze calls "the virtual". (The coinage refers to Proust's definition of what is constant in both the past and the present: "real without being actual, ideal without being abstract.") Proust, Le Temps Retrouvé, ch. III: see the fourth line from the bottom of this page, or, in English translation, the thirteenth paragraph here: "I began to discover the cause by comparing those varying happy impressions which had the common quality of being felt simultaneously at the actual moment and at a distance in time, because of which common quality the noise of the spoon upon the plate, the unevenness of the paving-stones, the taste of the madeleine, imposed the past upon the present and made me hesitate as to which time I was existing in. Of a truth, the being within me which sensed this impression, sensed what it had in common in former days and now, sensed its extra-temporal character, a being which only appeared when through the medium of the identity of present and past, it found itself in the only setting in which it could exist and enjoy the essence of things, that is, outside Time. [...] **Nothing but a moment of the past Much more perhaps; something which being common to the past and the present, is more essential than both. [...] a marvellous expedient of nature had caused a sensation to flash to me—sound of a spoon and of a hammer, uneven paving-stones**—simultaneously in the past which permitted my imagination to grasp it and in the present in which the shock to my senses caused by the noise had effected a contact between the dreams of the imagination and that of which they are habitually deprived, namely, the idea of existence—and thanks to that stratagem had permitted that being within me to secure, to isolate and to render static for the duration of a lightning flash that which it can never wholly grasp, a fraction of Time in its pure essence. When, with such a shudder of happiness, I heard the sound common, at once, to the spoon touching the plate, to the hammer striking the wheel, to the unevenness of the paving-stones in the courtyard of the Guermantes' mansion and the Baptistry of St. Mark's, it was because that being within me can only be nourished on the essence of things and finds in them alone its subsistence and its delight. It languishes in the observation by the senses of the present sterilised by the intelligence awaiting a future constructed by the will out of fragments of the past and the present from which it removes still more reality, keeping that only which serves the narrow human aim of utilitarian purposes. But let a sound, a scent already heard and breathed in the past be heard and breathed anew, simultaneously in the present and in the past, real without being actual, ideal without being abstract, then instantly the permanent and characteristic essence hidden in things is freed and our true being which has for long seemed dead but was not so in other ways awakes and revives, thanks to this celestial nourishment"While Deleuze's virtual ideas superficially resemble Plato's forms and Kant's ideas of pure reason, they are not originals or models, nor do they transcend possible experience; instead they are the conditions of actual experience, the internal difference in itself. "The concept they [the conditions] form is identical to its object."[Desert Islands, p. 36.] A Deleuzean idea or concept of difference is therefore not a wraith-like abstraction of an experienced thing, it is a real system of differential relations that creates actual spaces, times, and sensations. [See "The Method of Dramatization" in Desert Islands, and "Actual and Virtual" in Dialogues. In fact it not as if the style and substance of science and philosophers have been entirely different. look at the conformality and consonance in the Gaussian Theory and Deleuze's statements on identity and the loss of it in the following passage…….That is, identity is a continuous process that finds its limit in exhaustion. As Deleuze will write, "…the logical relation of causality is inseparable from a physical process of signalling, without which it would not be translated into action. By 'signal' we mean a system with orders of disparate size, endowed with elements of dissymmetry; by 'sign' we mean what happens within such a system, what flashes across the intervals when a communication takes place between disparates. The sign is indeed an effect, but an effect with two aspects: in one of these it expresses, qua sign, the productive dissymmetry; in the other it tends to cancel it" (dr, 20). An actualized entity just is such a signal-sign system, remaining in perpetual communication with the virtual multiplicity it actualizes, constituting its own elements and identity. Such is Deleuze's account of what Badiou refers to as the operation of the "count-for-one"… an operation that is immanent to being itself and energetic in character. yet how are we to discern the local nature of ontological situations, the manner in which there is no global, overarching global situation in which all local situations are embedded as if parts of a whole? in part the above passage already responds to this question. if series must be brought into relation to communicate in order for causality to occur, and if the resonance of series is always a matter of chance,

of a throw of the dice, like Lacan's objet a where no series enjoys primacy of model over copy, then all we have are local situations without a total situation. Unlike Heidegger's analysis of Dasein where Dasein is always being-in-the-world, Deleuze's simulacra or actualizations are always divergent, enjoying only local relations. Heidegger argues that in order to understand the being of Dasein we must understand the manner in which Dasein is a being-in-the-world. By contrast, Deleuze's concept of multiplicity provides us with the principle of a new structuralism, of a new local ontology, that allows us to understand the immanent organization of a multiplicity without referring it to an embedding global space. as DeLanda so beautifully puts it in relation to the mathematician gauss, "…when gauss began to tap into these differential resources, a curved two-dimension surface was studied using the old Cartesian method: the surface was embedded in three-dimensional space complete with its own fixed set of axes; then, using those axes, coordinates would be assigned to every point of the surface; finally, the geometric links between points determining the form of the surface would be expressed as algebraic relations between the numbers. but gauss realized that the calculus, focusing as it does on infinitesimal points on the surface itself (that is, operating entirely with local information), allowed the study of the surface without any reference to a global embedding space. basically, gauss developed a method to implant the coordinate axes on the surface itself (that is, a method to implant the coordinate axes on the surface itself (that is, a method of 'coordinatizing' the surface) and, once points had been so translated into numbers, to use differential (not algebraic) equations to characterize their relations. as the mathematician and historian Morris Kline observes, by getting rid of the global embedding space and dealing with the surface through its own local properties, 'gauss advanced the totally new concept that a surface is a space in itself'" (intensive science and virtual philosophy, 11-12).according to the old Cartesian method, we can only outline the properties of a space by relativizing it to a global space in terms of which it is then mapped. by contrast, gauss is able to explore a space in terms of its intrinsic metric and organization as a local space, without referencing it to a whole of which it is conceptualized as a part. **The Image of Thought**

"Image of thought "permeates both popular and philosophical discourse. According to this image, thinking naturally gravitates towards truth. Thought is divided easily into categories of **truth and error**. The model for thought comes from the educational institution, in which a master sets a problem and the pupil produces a solution which is either true or false. This image of the subject supposes that there are different faculties, each of which ideally grasps the particular domain of reality to which it is most suited. In philosophy, this conception results in discourses predicated on the argument that "**Everybody knows..."** the truth of some basic idea. Descartes, for example, appeals to the idea that everyone can at least think and therefore exists. Deleuze points out that philosophy of this type attempts to eliminate all objective presuppositions while maintaining subjective ones. Deleuze maintains, with Artaud, that real thinking is one of the most difficult challenges there is. Thinking requires a confrontation with stupidity, the state of being formlessly human without engaging any real problems. One discovers that the real path to truth is through the production of sense: the creation of a texture for thought that relates it to its object. Sense is the membrane that relates thought to its other. Accordingly, learning is not the memorization of facts but the coordination of thought with a reality. "**As a result, 'learning' always takes place in and through the unconscious, thereby establishing the bond of a profound complicity between nature and mind".** Deleuze's alternate image of thought is based on difference, which creates a dynamism that traverses individual faculties and conceptions. This thought is fundamentally energetic and signifying: if it produces propositions, these are wholly secondary to its development. Philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". Everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; even tum tan tum;it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance

and ramification of its disjunction; the vorticity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoin der for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for every hum of voices or a single silence for all the silences; a single vacuum for all the vacuousness consciousness should not be said without conjuring if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. The attributable to all states of bodies and states of affairs and the expressible of every proposition So univocity of consciousness means the identity of thé noematic attribute and that which is expressed linguistically and sense-fully. Univovity.We mean that it does not allow consciousness to be subsisting in aquasi state and but expresses in all pervading reality. (Reference: LOS: Deleuze) Deleuze sometimes spoke of getting up behind an author and "creating a monster", as if his reading methodology somehow consisted of a distortion of the philosopher's or author's thought. Certainly there is a case to be made for this with regard to the process of deterritorialization, where something is wrested from a territory and deterritorialized upon a new territory, like the animal paw that is deterritorialized from the earth and reterritorialize on the branch. however, it seems to me that the more interesting aspect of Deleuze's approach to other philosophers and art is not so much his "monstrous creations" (i seldom find them particularly monstrous), but rather their Gaussian or Riemannian style, where he explores them in terms of their own internal organization and metric, without reducing them to something alien such as history, society, biology (reductivism), or the signifier. What we find in Deleuze's approach to phenomena is a Gaussian technique. for instance, take Deleuze's books on cinema, his book on Francis bacon, or his study of sacher-masoch. In the first instance, Deleuze's carefully separates cinema from narrative and the signifier, studying it in terms of its specific organization pertaining to the production of images. in the case of bacon, Deleuze doesn't look for an underlying narrative or "meaning", but instead studies the manner in which bacon composes and organizes his images and lines, both in terms of their production and actuality, so as to liberate a logic of sensation. finally, in approaching sacher-masoch, Deleuze allows Sacher-Masoch's novels to speak for themselves in terms of their desire and relation to pain, stalwartly refusing to reduce masochism to the complement of sadism. in each case, we have a local exploration of a "space" of multiplicities that is extremely precise. Sean Bowden the priority of events: **Deleuze's logic of sense sean Bowden, the priority of events: Deleuze's logic of sense, Edinburgh university press, 2011, 296pp., $40.00 (hbk), isbn 9780748643646 Reviewed by David Scott, Coppin state university and Gaussian spaces and multiplicities a note on Gaussian spaces and multiplicities posted by larval subjects under uncategorized.** What you "see" is not always what you "get." Many people mistakenly take their own visions literally without "seeing through" the various possibilities that are outside their belief system, knowledge or skill base. Deeper reality is not remote in the physical sense but in a psychological sense. The archetypes of the collective unconscious are arrayed behind the scenes of current worldwide conditions, of crisis and confusion. They mirror our own states back at us, whether we perceive them as such or interpret them plausibly or not. The noise of ordinary consciousness and beliefs drowns out the signal. Unconsciousness is the background of our ordinary awareness. Our organism is very much at the center of such effects. The organismic source is our human bodies and the focus of human consciousness. The fantasy principle dethrones reality, but can be dissociative or compensatory. The human mind is a meme-Scape. Pre-conceived concepts vie with structures, concepts with images. Like scientists who ignore assumed truths, we leapfrog over our beliefs and personality deficits, claiming idiosyncratic imagination is literal reality. It couldn't be further from the truth and symbolism is utterly lost. The metaphor that might heal us enslaves us. Perhaps images like the holographic universe have an implicate order. Can we have a sense of the cosmos in the world without projecting myriad fantasies on it that we embrace literally? Has the world become so horrible it is unreasonable to be realistic? We may need to look at our drives and wishes, rather than the fantasy content. Psyche constructs reality. Our experience of so-called reality is always mediated by our image of it. Even if all the contents of the psyche are real, that doesn't mean they are realistic. That psyche is real is still a radical proposition, but psychic politics certainly color the self-image and ideas of everyone. We observe and

participate with images It is not a question of nature or nurture (genes alone or experience alone). Rather, everything is both. We inherit the structures that make our experience what it is. But the structure itself is "empty," and each human culture "fills" it with its own specific adaptations. It is difficult to define an archetype and set boundaries that distinguish it from others. In a hologram each part contains all the information but in lower resolution. Archetypes have this holographic quality. There are patterns within patterns within patterns. Some overlap with others, and some are nested inside others. Archetypal realities, passed on through DNA, are expressed in distinctive neuronal tracts in the brain. They include customs and laws regarding property, incest, marriage, kinship, and social status or roles; myths and legends; beliefs about the supernatural; gambling, adultery, homicide, schizophrenia, and the therapies to deal with them. A mythic and visionary language of immediate experience encompasses themes of deepest, highest, and ultimate concern. Most fantasy-based individuals are at a complete loss to coherently explain their own conventional behavior much less anomalous events and their deep meaning, much less the cultural unconscious or mythological unconscious matrix. But they try, and become utterly entrenched in their belief that they are right about the nature of the world and reality. We have pseudo-memories about our personal lives. Why not more so for our collective life? The subject matter often revolves around catastrophe, creation and the mythopoeic forces of mankind. Ignorant of such dynamics, interpretive mistakes and displaced psychic contents proliferate into errors of fact. Propaganda, media distortions, memes, and disinformation compound the social problem of misapprehension further. Shameless self-promotion by personalities of such ideas leads to cults. They make up myths about the myths of by-gone eras. Roiling unconscious images can be fatally confusing. Thought illusions culminate in projections and projections of mythology. Jung suggested symbols live only as long as they are pregnant with meaning. Philosophy arose from criticism of myth, from discussing and challenging it. In science, we criticize, reject and eliminate theories. At the edge of the abyss of the unknown, new signs and symbols emerge. Credible theories and paradigms must include biology, physics, and neurophysiology. One of the reasons people "see God", or a guru, or anomalies may be because our brains are constructed to see reality through the eyes of others. There are heaps of mirror neurons which are there to make us feel the 'other'. Mirror neurons do for psychology what DNA did for biology. They provide a unifying framework and help explain a host of mental abilities. As in the psychochemical processes of empathy or falling in love, a complex feedback loop sustains a state of mind. But when we empathically transpose ourselves into someone else's position, we expose ourselves to that reality -- cognitively and emotionally. The unconscious complicates empathy, both ways. Mirror neurons might well play a role in bonding, language and self-awareness. Naively, we take too much as self-evident. But 'seeing' does not always 'believe', though many make this error or leap in logic and formulate their choices and future accordingly. Yet, there is only one way to learn what consciousness is. Experience. But we have no satisfactory explanatory edifice for consciousness. Would such a theory release in each of us our own inner knowledge of the creativity of our own consciousness, and its infinite possibilities? The problem is trying to define a verb, a dynamic, as if it were a noun. But we do recognize the effect of consciousness. It functions to mediate states of consciousness, high and low psychobiological arousal. Consciousness is the subconscious lifted up by the physical body. When the body fails, the consciousness collapses back into the subconscious. All our thoughts come from the subconscious which can see our intentions but not our world. This relates somehow to intention being imaginary and not of the physical frictionized world (King). Gerald Edelman postulates that the flows of information in the brain are mediated through're-entrant' feedback loops. As evolution provides new cognitive functions, new re-entrant loops are established. Even language itself is an archetype -- a chaotic field of dynamic associations. A subtle net of tropes, grammar, symbols, and meaning, the program language begins in limbic resonance. Some phenomena generate their own language patterns, nomenclature, and internal coherence of meaning and representation. n a holographic universe, even time and space could no longer be viewed as fundamentals. Because concepts such as location break down in a universe in which nothing is truly separate from anything else, time and three-dimensional space would also have to be viewed as projections of this deeper order. At its deeper level reality is a sort of super hologram in which the past, present, and future all exist simultaneously. This suggests that given the proper tools it might even be possible to someday reach into the super holographic level of reality and pluck out scenes from the

long-forgotten past. Or not. A fantasy of such penetration or phenomenon inside the head is not the same as that penetration. **Jung in the 21st Century: Synchronicity and science By John Ryan Haule Mind Control Countermeasures.** Adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar some statements in Vedanta, it is not superfluous or redundant. It is evolutionary. Autheticative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication of the evolutionary process towards understanding is ever present in all, its thematic and discursive form. it is the relentless holding on to a theory this is despicable albeit the theory is not. Look at this paragraph. It is the 'I am the body/mind' belief that gives rise to the 'I am not (e) the world' belief. These two beliefs are (=) co-created."Consciousness projects (e&eb) the appearance of the mind, body and world by taking the shape of thinking, sensing and perceiving." "Attention is Consciousness with an object. When the object vanishes, attention simply remains what it always is, Consciousness.""There is (=) no purpose to (e) meditation. The purpose is already accomplished." "Everything that is experienced is experienced by, through, in and as Consciousness." "The seen cannot be separated from seeing and seeing cannot be separated from (e) Consciousness." "The Reality of any experience is not hidden in the appearance; it is expressed by the appearance." "Once we see that everything is Consciousness... Maya still dances, but it is a dance of love not seduction." There are some truly excellent sections such as a long one taking, as a starting point, the following quotation from the painter Cézanne: "Everything vanishes, falls apart, doesn't it? Nature is always the same but nothing in her that appears to us lasts. Our art must render the thrill of her performance, along with her elements, the appearance of all her changes. It must give us a taste of her Eternity." Rupert begins: "That statement must be one of the clearest and most profound expressions of the nature and purpose of art in our era." And he goes on for some 14 pages to elaborate on this claim, examining the nature of the 'elements' Existence, appearance and Consciousness and their relationship. "This Reality is the support or ground of the appearance. The appearance may be an illusion, but the illusion itself is real. There is an illusion. It has Reality." He refers to the rope-snake metaphor and says that: "We do not see anything new. We see in a new way." "So that we know that nature is real, that there is something present, that there is a reality to it, even if everything that appears to us is insubstantial and fleeting." **The Transparency of Things' by Rupert Spira Book Review by Dennis Waite The following is a review of Rupert's book: 'The Transparency of Things: Contemplating the Nature of Experience'. There is an essay from the book here and the essay on Cézanne may be read at the awakened eye website For models see Mitigation Of Beam-Induced Backgrounds, Multi-Particle Azimuthal Correlations, Search For Extra Dimensions, Multi-Higgs-Boson Cascade, Top Quark Pair (Tt) Production Charge Asymmetry, $\Delta Y \equiv Yt - Yt^{-}$, Reflects The Asymmetry In Tt^{-} Production, Ultra-Relativistic Electrons And Other Letters Of Interest: A Synecdochal Syncretism: Et Lux In Tenebris Lucet: Light Shines In The Darkness Models.** What then is the relation of Higgs boson and consciousness? Functionalist approaches generally assume that conscious experience appears as a novel property at a critical level of computational complexity. On the surface this would seem to deal with issues 1 and 3, however a conscious threshold has neither been identified nor predicted, and there are no apparent differences in electrophysiological activities between non-conscious and conscious activity. Regarding the nature of experience (why we are not unfeeling "zombies") functionalism offers no testable predictions. Problem 2) of 'binding' in vision and self is often attributed by functionalists to temporal correlation (e.g. coherent 40 Hz), but it is unclear why temporal correlation per se should bind experience without an explanation of experience. As functionalism is based on deterministic computation, it is also unable to account for Penrose's proposed non-computability (4), or free will (5). Something may be missing. To address these issues, various proposals have been suggested in which macroscopic quantum phenomena are connected to the brain's known neural activity. For the problem of unitary binding, Marshall (1989) suggested that coherent quantum states known as Bose-Einstein condensation occurred among neural proteins (c.f. Penrose, 1987; Bohm and Hiley, 1993; Jibu and Yasue, 1995). Pre-conscious to conscious transitions was identified by Stapp (1992) with collapse of a quantum wave function in pre-synaptic axon terminals (c.f. Beck and Eccles, 1992). In another proposal, protein assemblies called

microtubules within the brain's neurons are viewed as self-organizing quantum computers ("orchestrated objective reduction - Orch OR" e.g**. Penrose and Hameroff, 1995; Hameroff and Penrose 1996a; 1996b; c.f. Hameroff 1997; 1998a; 1998b; 1998c; 1998d). Stability analysis of such theories would certainly provide a gateway for further bringing in the "reality" in space time.** String theory suggests that the graviton particle would not be detected from a particle collision because it would have "jumped" into another dimension. So, if the graviton exists and the theory is correct, there should be a "gap" in the energy profile of all the particles ejected from the collision. To understand this, think of breaking the pack of balls in a game of pool. The total energy of all the balls after the collision will depend on the force with which the cue ball was hit. So, after the collision, the energy of all the balls is measured and added-up to make sure it is the same as the energy of the cue ball. BUT - imagine that there is some energy missing, because one of the pool balls has "gone missing" - into a higher dimension. (POSTED BY THEREALJEFFHAL**) Talbot says** "If the universe is a hologram, in some sense it suggests that there may be two very drastically different levels of reality: the concrete reality that we see when we look at [things]... and at some deep level there's a level of reality where everything dissolves into an ocean of energy that is holographically interconnected.". A theory of everything must encompass all the variations however contradictory it may be. Nor need any theory be sent in to grand limbo of oblivion and hibernation or a differentiated groundless less ness because there are some irregularities which are like audit discrepancies need to be rectified. Although the language of religion and science seem different, both are talking about the central order of nature relative to DNA and information, as is Peirce, Petrus, Bacon, and pharaonic Egypt. This path-ordering, active information suggests that the central pattern or law of our holographic cosmos may be modeled on the biophysical battleground of gene regulation in bacteriophage Lambda, one of nature's most efficient and highly evolved mechanisms identified by experimental praxis. Egyptian texts support that the Lambda lifestyles of lysogeny and lysis can be understood as two paths to two living systems, photosynthesis and its reversal chemiluminescence, the cosmic key for evolution of mind and continuity. The texts support that the Lambda genome is the world-heart of two ways. The viral lifestyle of lysogeny controlled by cI protein results in matter, classical spacetime, our world of projected shadow, photosynthesis, sexual genesis or vertical gene transfer, mind in the human body, and the imperfect continuum. In contrast, the viral lifestyle of lysis controlled by cro protein and lactose metabolism results in transformation to energy, the quantum, chemiluminescence, asexual genesis or HGT, mind in the cosmos, and the perfect continuum. Some scientists believe our cosmos is entangled with another (Chown 2007). On the holographic quantum level, perhaps this is just the competitive entanglement of cI and cro proteins cycling in perpetual motion—an inherent law of nature elucidating the assembly of things. In light of modern experiments, physicist John Wheeler (1988) speculated that the act of conscious observation functions with quantum mechanics, catalyzing a probability or outcome. So, if an observing consciousness has knowledge of the classical path and its magnetic field orientation, an act of measurement or choice might entangle space to reify the holographic quantum paths to a perfect continuum. Peirce guesses that to be drawn into a new system, a particle must have the right mass, the right velocity in the right direction, the right attraction, and it must present itself at the right point (EP 1. 270). Similarly, a finite probability exists for any particle that approaches the black hole event horizon to bounce back, dependent on the incoming particle's energy, its charge, and its projection of the orbital momentum on the axis of rotation of the black hole (Kuchiev 2003; 2004; 2004a), the same conditions in Egyptian texts (King 2004; 2006).Only an approximate conclusion remains: chance may be knowledge of the cosmos' holographic mode of operation and magnetic fields; habit breaking may result in the transformation of energy by a violation of the second kind; and evolution of mind or energy might be possible through HGT via a cosmic viral code of Lambda-genesis. "Order is simply thought embodied in arrangement;" (CP 6.490). Accordingly, the original meaning of the signs may relate to a viral biophysics of crystallized mind for human evolution via a natural chemical pathway to a perfect continuum. Again, this semiotics of evolvability and order recurs in time with the chaotic dream of reason that projects a pervasive falsification of our perceptual world. As Wallace Stevens envisions in "Connoisseur of Chaos", perhaps "the pensive man may see." **Peircean Process Metaphysics Origin of Science Quantum and Classical Laws Ancient Egyptian Evolutionary Biophysics** Adversely antagonistic, inexorably irreconcilable, antithetically antipodal,

diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar are the views of epistemologists in the analysis of raison d'etre of knowledge. Epistemology is the study of knowledge and those things closely related to it: justification, what it takes for you to be justified, the relation between knowledge and justification, whether you can have any justified beliefs at all, and if so, how you come to know (or justifiably believe) things, how you can use what you know (or justifiably believe) to come to know (or justifiably believe) other things, and whether and why it's valuable to know instead of merely having justified and true beliefs. The literature on epistemology is vast. Here's a very brief summary of some epistemological discussions. Concerning knowledge, many epistemologists think knowledge is justified true belief, where the justification you have is linked to the truth of the matter in the right kind of way, though what this way is a matter of debate; some epistemologists think knowledge can't be analyzed this way. Concerning justification's relation to knowledge, some epistemologists think we don't need to be justified to know, and some think we do need to be. Concerning what it takes to be justified, some epistemologists think that what it takes for you to be justified are only factors internal to the believer (Internalists). Others think it takes an external factor, like reliable or well-functioning cognitive faculties (Externalists). Skeptics argue that we can't have any justified beliefs at all, and many epistemologists reply to the skeptic's arguments. Concerning how we use what we know (justifiably believe) to come to know (justifiably believe) other things, some epistemologists (Foundationalists) argue that there are bedrock propositions that we know (justifiably believe), and we build our knowledge (justified beliefs) on those. Others (Coherentists) argue that there aren't bedrock propositions; rather, a set of beliefs is justified as a whole, and several beliefs can be mutually supporting. Concerning the value of knowledge, some argue that knowledge is intrinsically valuable. Others have argued that knowledge is valuable only because of the role it plays in practical reasoning, and others argue that knowledge isn't more valuable than justified and true belief, but there are other epistemic states such as understanding, that do have value above their proper subparts. **Edited y Matthew McGrath (University of Missouri, Columbia). It is not as if there is an anathema or misnomer in the philosophy or physics. Infact most of the pnysicists have used philosophical ideas towards the end of consummation of their their and their concomitant justification thereof.** The eternal return produces becoming-active. It is sufficient to relate the will to nothingness to the eternal return in order to realize that reactive forces do not return. However far they go, however deep the becoming-reactive of forces, reactive forces will not return. The small, petty, reactive man will not return. Affirmation alone returns, this that can be affirmed alone returns, joy alone returns. Everything that can be denied, everything that is negation, is expelled due to the very movement of the eternal return. We were entitled to dread that the combinations of nihilism and reactivity would eternally return too. The eternal return must be compared to a wheel; yet, the movement of the wheel is endowed with centrifugal powers that drive away the entire negative. Because Being imposes itself on becoming, it expels from itself everything that contradicts affirmation, all forms of nihilism and reactivity: bad conscience, ressentiment..., we shall witness them only once. [...] The eternal return is the Repetition, but the Repetition that selects, the Repetition that saves. Here is the marvelous secret of a selective and liberating repetition. There is no need to remind the reader that neither the image of a centrifugal movement nor the concept of a negativity-rejecting repetition appears anywhere in Nietzsche's writings, and indeed Deleuze does not refer to any text in support of this interpretation. Further, one could highlight that Nietzsche never formulates the opposition between active and reactive forces, which constitutes the broader framework of Deleuze's interpretation. For some years, Marco Brusotti has called attention to the fact that Deleuze introduced a dualism that does not exist in Nietzsche's writings. To be sure, the German philosopher describes a certain number of "reactive" phenomena (for example, in the second essay of the Genealogy of Morality, § 11, he talks about "reactive affects" [reaktive Affekte], "reactive feelings" [reaktive Gefühlen], reactive men [reaktive Menschen]); but these are nonetheless the result of complex ensembles of configurations of centers of forces that remain in themselves active. Neither the word nor the concept of "reactive forces" ever appears in Nietzsche's philosophy. **After the discovery of the two principles of thermodynamics began a debate about the dissipation of energy and the thermal death of the universe which framed the modern renewal of the debate between the linear and circular conceptions of time**. Scientists such as Thomson, Helmholtz, Clausius, and Boltzmann and-by way of Kant, Hegel and Schopenhauer-

philosophers such as Dühring, Hartmann, Engels, Wundt and Nietzsche have tried to address this problem by using the force of scientific argumentation and of philosophical discussion. Whoever believed in an origin and a final end to the motion of the universe (be it in the physical form of the gradual loss of heat, or in the metaphysical form of a final state of the "world process"), relied on the second principle of thermodynamics or on the demonstration of the thesis of Kant's first cosmological antinomy. On the contrary, those who refused to admit a final state to the universe used Schopenhauer's argument of infinity a parte ante-according to which if a final state were possible, it should already have established itself in the infinity of time past-to propose henceforth a number of alternative solutions. Scientists would propose the hypothesis that energy could have re-concentrated after a cosmic conflagration, thus reversing the tendency towards dissipation. Those belonging to the monistic and materialistic tradition relied on the first principle of thermodynamics and on the infinity of matter, space and time, and regarded the universe as an eternal succession of new forms. A certain critical agnosticism was widespread among scientists and philosophers, oftentimes through a reaffirmation of the validity of Kant's antonymic conflict, this movement avoided to take a stand on specifically speculative issues. Other German philosophers, like Otto Caspari, or Johann Carl Friedrich Zöllner, had reintroduced an organicist and pan-psychical conception of the universe, investing atoms with the ability to escape any state of balance. Indeed, it is probably one of Otto Caspari's works, The Correlation of Things (Der Zusammenhang der Dinge. Gesammelte philosophische Aufsätze (Bleslau: Trewendt, 1881)), which awakened Nietzsche's interest for all things cosmological, in that summer of 1881, in Sils-Maria. addressing Schopenhauer and Eduard von Hartmann's mystical pessimism according to which the world is the creation of a stupid and blind essence (which, after having created the world by mistake, comes to the realization that it had made a mistake and strives to return it to nothingness) Caspari stresses that it is nothing short of mystical to imagine that the world may have been borne out of a an originary and undifferentiated state. Where would it have drawn the first impulse? But, continues Caspari, even if the world had received this first impulse from some deus ex machina, there is no doubt that, in the temporal infinity of past time thus far, it would have either attained the end of the process (but this is impossible because the world would then have ended), or it would be necessarily bound to repeat indefinitely this original mistake, and the entire process that accompanies it. But then, what is the process of the world? We must now take one more step back and understand further the process of the world according to von Hartmann. Hartmann objects that the regressive movement postulated by Schopenhauer is possible only in thought: it remains nothing more than an "ideal postulate" with no real object and which "does not teach us anything about the real process of the world that unfolds in a movement contrary to this backwards movement of thought" (Hartmann, Philosophie des Unbewussten, third edition (1871): 772). Hartmann affirms that if unlike Schopenhauer one admits the reality of time and of the world process, one must also admit that the process must be limited in the past and therefore that there must be an absolute beginning. In Hartmann's mind, failure to do so would result in positing the contradictory **concept of an accomplished infinity**: "The infinity that from the point of view of regressive thinking, remains an ideal postulate, which no reality may correspond to, must, for the world, whose process is, on the contrary, a progressive movement, open up to a determinate result; and here the contradiction comes to light" (Hartmann (1871): 772). What really "comes to light" in this passage is the fact that Hartmann does not provide a demonstration but a petitio principii. Indeed, the concept of the world process analytically contains the concept of a beginning of the world. In all rigor, it is therefore impossible to demonstrate these concepts with reference to each other. Secondly, Hartmann's view that one is bound to accept the reality of the world process even if one rejects the ideality of Schopenhauer's time is mistaken. Hartmann believes that if time is real there must be a world process with both an **absolute beginning and an absolute end. Without any justification, Hartmann jumps from Schopenhauer's negated time to oriented time.** Hartmann's view is that the world process leads into a final state absolutely identical to the initial state. However, it follows from this that even as the cosmic adventures of the unconscious come to a close, we are still haunted by the specter of a new will and of another beginning of the world process. This exposes a serious internal flaw of Hartmann's system insofar as it jeopardizes the possibility of a final liberation from existence and suffering. This is why in the last pages of his work, "On the Last Principles," he painstakingly calculates the degree of probability of a reawakening of the volitional faculty of the unconscious.

Insofar as the will is entirely free, unconditioned and a-temporal, the possibility of a new volition is left to pure mathematical chance and is therefore ½. Hartmann further stresses that if the will were embedded in time, the probability of the repetition would amount to 1 and the process of the world would be bound to begin again, in an eternal return which would completely preclude the possibility of a final liberation. Fortunately, this is not the case since-according to Hartmann's remarkable logic-the world-process develops through time, but the original will is outside of time. In fact, one may even affirm, along the lines of Hartmann's peculiar theory of probability, that every new beginning gradually reduces the probability of the next beginning: let n be the number of times that the will is realized, the probability of any new realization is ½ n. "But it is clear that the probability ½ n diminishes as n increases, in a way that suffices to reassure us in practice" (Hartmann (1869): 663). In his famous work entitled **On the Conservation of Force (1847), Hermann von Helmholtz** had divided the totality of the energy in the universe between potential energy and kinetic energy and affirmed the reciprocal convertibility of the two. In 1852, William Thomson pointed out that there exists a sub-ensemble within kinetic energy, heat, which, once it has been generated, is no longer entirely convertible into potential energy-or into any other form of kinetic energy. Considering that the (partial) reconversion of heat into labor is possible only in situations that present a disparity in temperature, and that heat tends to pass from warmer to cooler bodies by spreading on an even temperature level through space, Thomson concluded that the universe tends towards a final state where any energetic transformations, every movement and every form of life will cease. Caspari's atoms (which bring to mind those in Leibniz's Monadology) resemble some sort of biological monads, endowed with internal states. For Caspari, every atom obeys the ethical imperative to participate in the conservation of the general organism and its movement does not only follow the simple physical kind of interaction but also an a priori law ensuring. If a balance of forces had been attained at any moment, this moment would still be going on: therefore, it never happened. The present state contradicts this proposition. Supposing that a certain state rigorously identical with the present state had, one day, existed, this supposition is not refuted by the present state. As one of the infinite possibilities, it is necessary that the present state had been given anyway, since until now an infinite period of time has already unfolded. **If equilibrium were possible, it must have occurred**; and if the present state has already taken place, then so too the one that preceded it as well as the one preceding that one. Therefore it has already taken place a second time, a third time and so on. And likewise it shall take place again a second time, a third time. Innumerable times forwards and backwards. This amounts to saying that all becoming occurs within a repetition of an innumerable number of absolutely identical states. [.] The immovability of forces, their equilibrium is a conceivable case, but it has not occurred. As a result the number of possibilities is greater than the number of realities. -The fact that nothing identical recurs may be explained not thanks to chance, but only thanks to an intention infiltrated within the essence of force. Indeed, supposing an enormous amount of cases, the random occurrence of the same combination is more probable than the same combination never recurring. The quantum of force in the universe is determinate and not "infinite": let us beware [hüten wir uns] from such conceptual extravaganza! Therefore the number of situations, modifications, combinations and developments of this force is doubtless enormous and practically "immeasurable," but in any case this number is determinate and not infinite. On the other hand, the time in which the universe exerts its force is infinite. That is to say, that force is eternally identical and eternally active: -until the present instant an infinity has already taken place, that is to say that all possible developments must have already taken place. Consequently, the present development must be a repetition and therefore both this that was born from it and this that shall be born from it and so on both forwards and backwards. Everything has taken place an innumerable number of times because the overall situation of all forces always recurs (FP 11 [202] of 1881). Nietzsche regards the mechanistic vision as more plausible and less anthropomorphic than organicism. However, faced with the two major **cosmological models of his time, the mechanistic model and the organic model, Nietzsche wishes to return its polymorphous, proteiform, unstructured** and chaotic character to nature of which the perfectly non-theological and non-teleological theory of eternal return is the strongest seal. This is the first of the "new battles" which come to whoever is aware of the consequences of the death of God: take any antropomorphism away from nature. In the preparatory papers, the third book of the Gay Science is entitled "Gedanke eines Gottlosen / Thoughts of a Godless

One." Aphorism 109 of this book, which immediately follows the famous aphorism against the shadows of God, **Boltzmann's theory belongs to the third phase of the debate on thermodynamics and cosmology. The publication of Thomson's brief paper "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy" of 1852 signals the beginning of the scientific controversies on the problem of the dissipation of energy and opened the first phase of the debate, which was announced in the Reflexions on the Motor Powers of Fire by Sadi Carnot and whose conclusion is represented by Clausius's recapitulative article on the concept of entropy in 1865.[44] The second phase started in 1867 when, at the forty-first congress of German scientists and doctors, Clausius gave a lecture on "The Second Principle of the Mechanistic Theory of Heat," where he applied the results of his research on thermodynamics to the universe**. It is true that in his famous lecture of 1854, Helmholtz had already presented the cosmic consequences of the second principle, but Clausius' contribution had a strong impact on German culture. This is because in this lecture he robustly rejected the possibility to consider the universe as an eternal and self-renewing circle, an ewiger Kreislauf in which force and matter are in constant transformation, as was heretofore affirmed by the materialism of the scientists and philosophers, and he did so in the name of the second principle of thermodynamics. In this way, the debate on the principles of thermodynamics gained great importance in European Culture starting in 1867. In the two first phases, it is Thomson's mechanism that predicts the thermal death of the universe. In the third phase, on the contrary, the meaning of the term mechanism changes radically.[46] In accordance with the apocalyptic climate of this period dominated by the "rebirth of idealism," the "overcoming of scientific materialism" and the "bankruptcy of science", the mechanistic paradigm which had accompanied the birth of modern science became challenged on **account of the second principle of thermodynamics**. According to the theorem of the quasi-periodicity of the motions of mechanical systems demonstrated by Poincaré as part of the problem of the three bodies (1890), a mechanical system must evolve according to a quasi-periodical movement and consequently it must always return-sooner or later-to the initial state. **An easily established theorem informs** us that a limited world obeying solely the laws of mechanics shall always pass through a state closely similar to its initial state. On the contrary, according to established experimental laws (supposing we grant them absolute value and wish to push their consequences to the end), the universe is directed towards a final state, which once it is attained, it shall not be able to escape. In this final state, which shall be like a sort of death, all material bodies shall be at rest at the same temperature. **Poincaré's theorem seems** therefore incompatible with the second principle of thermodynamics, which predicts a unidirectional movement of all natural phenomena until the whole universe is brought to a total standstill. Wilhelm Ostwald and the entire energeticist school of thought contended that the principles of thermodynamics were fundamentally new, and could not be re-incorporated to traditional physics and that they should serve as a basis for a new science that regards the qualitative diversity of energy and its tendency to degradation as its axioms. Against energeticism and in an effort to bring entropic phenomena back into the theoretical framework of mechanism, Ludwig Boltzmann introduced the concept of probability in physics, not as an instrument of calculation, but as an explicative principle. In Boltzmann's statistical thermodynamics, the increase of entropy assumed by Clausius is re-interpreted as an increase in molecular chaos. As a result, it becomes possible to explain mechanistically the evolution of closed systems endowed with increasing entropic value, without it committing us to granting absolute value to the second principle of thermodynamics. **Moreover, one no longer needs to fear the thermal death of the universe insofar as the state of equilibrium will in principle** never be complete, but rather will be attained only statistically, leaving open the possibility of fluctuations towards less probable states. Boltzmann's critics remarked that this hypothesis involved two paradoxes called the objection of reversibility (Umkehreinwand), and that of repetition (Wiederkehreinwand). I shall only address here the second one since it coincides with the theory of the eternal return. Based on Poincaré's theorem quoted above, Ernst Zermelo objected to Boltzmann that his model of the universe suggested that after a finite (if admittedly very long) time the system would return to its initial position. In his first response to Zermelo, Boltzmann avoids committing himself directly to cosmological questions and he only observes that, in the case of concrete thermodynamic systems, the time of recurrence may be extremely long. For example, in normal conditions of pressure and temperature, one-centimeter cube of gas requires 1010 years to

reach a molecular configuration identical to the original one! However, following a response by Zermelo, Boltzmann wrote a new article where he outlines a cosmological picture that he will re-use later in his conclusion to his famous Lectures on Gas Theory. In this cosmological picture, Boltzmann considers the universe as a closed system with constant entropy, within which some fluctuations occur, creating islands of negative entropy. Our solar system originates in one of these fluctuations. As Clausius correctly pointed out, the entropy of our solar system increases constantly as the solar system gets closer to the state of chaos and of the thermal death of the rest of the cosmos. However, in other zones of the universe, some new fluctuations and new islands appear, so that thermal death is never generalized. Here we are given a grand cosmic image, in which the solar system and the sparkle of life that was lit on planet earth are only a fluctuation of order from within a dominant entropic tendency. Life, and the order on which it is based are exceptions, transitory forms taking place in the realm of the shapeless, they are islands of the cosmos that will soon be re-absorbed into chaos. According to Poincaré's theorem, our island will have to be reborn, to develop and die innumerable times in a strictly identical fashion. This happened an infinite number of times during the past eternity and it shall take place again an infinite number of times in the eternity to come. **Boltzmann accepts the "paradox" of recurrence-that is the eternal return of the same-as a legitimate consequence of the probabilistic conception of thermodynamics**. It may be rejected for ethical reasons, it may be stored away as an abstract speculation or dismissed along with other cosmic fantasies, but it cannot be rejected on the basis of any rigorously scientific viewpoint. **The Eternal Return: Genesis and Interpretation BY PAUL D'IORIO(excerpts). Nietzsche found the template of a materialistic cosmology based upon the first principle of thermodynamics. During the same year, he could have found a model of an organistic solution to the problem of thermal death of the universe as well as a discussion on the conformation of space in Zöllner's book On the Nature of Comets** (Johann Carl Friedrich Zöllner, Über die Natur der Kometen. Beiträge zur Geschichte und Theorie der Erkenntnis (Leipzig, Engelmann, 1872), 299 f. and 313 f.); Assumed Association In Space Time Of Particles Or Men Leads To Detrimental Ramification Be In Terms Of Progress Of “Subject Matter” Or In The Pathological Disequilibrium Of Gratification Deprivation Complex Which Again Has Mental Turbulence In Case Collective Consciousness Approve It (Raaga) Or Pernicious Implications In Case The Theory Is Not Accepted; After All What Is A Theory: Set Of Correlations And Causations Which Are “Tested” Against A Collective Consciousness Background, Which Yields “Results’ Which Are Themselves In “Question” Due To The Very Perception And The Simulation Prospective. The Definition Of Quantum Theorists' Terms, Such As Wavefunctions And Matrix Mechanics, Progressed Through Many Stages. For Instance, Erwin Schrödinger Originally Viewed The Electron's Wavefunction As Its Charge Density Smeared Across The Field, Whereas Max Born Reinterpreted It As The Electron's Probability Density Distributed Across The Field. Although The Copenhagen Interpretation Was Originally Most Popular, Quantum Decoherence Has Gained Popularity. Thus The Many-Worlds Interpretation Has Been Gaining Acceptance Moreover, The Strictly Formalist Position, Shunning Interpretation, Has Been Challenged By Proposals For Falsifiable Experiments That Might One Day Distinguish Among Interpretations, As By Measuring An AI Consciousness Or Via Quantum Computing. More Or Less, All Interpretations Of Quantum Mechanics Share Two Qualities: They Interpret Formalism—A Set Of Equations And Principles To Generate Predictions Via Input Of Initial Conditions They Interpret A Phenomenology—A Set Of Observations, Including Those Obtained By Empirical Research And Those Obtained Informally, Such As Humans' Experience Of An Unequivocal World Two Qualities Vary Among Interpretations: Ontology—Claims About What Things, Such As Categories And Entities, Exist In The World Epistemology—Claims About The Possibility, Scope, And Means Toward Relevant Knowledge Of The World. Now How Does A Person In A Crowd Behave? They He Would Due To Herd Mentality; Ok; That Is One Theory; Simulation By Terrestial Brahman Anti Brahman Will Make Them Behave In A Highly Organised And Coordinated Manner; With Association Of Things And Individuals Already Established Resultant Orientation Of The Actions Hall Be Only Oriented Towards The Figment Of Imagination And Product Of Puerile Prognostication Formally Established By Terrestial Brahman Anti Brahman Before Such A Coordinated And Organised Execution Of Exercise Is Conducted; Analogically, If A Simulation Is Going On In A Bunch Of Particles Or Electrons, And If Some Agency Say Celestial Brahman Anti Brahman

Agency Is Simulating It, There Shall Be Coordination And Organisation So That You See “Reality” While The Truth Shall Remain Hidden. Result Is The Suppression Of Information, Delineation Of Disinformation, And The Concomitant Development Of Mushrooms Of Concocted And Fabricated Theories To That “Reality”; In The Case Of Terrestial Brahman Anti Brahman Agency Intention May Be To Bulldoze And Bowdlerize You, Baffle And Befuddle You In To Submission; In The Case Of Celestial Brahman Anti Brahman Agency Only Reason Could Be To Shield The Truth Being Disseminated; But Why?;May Be You Should Never Become God Which You Are Trying To; May Be God Is Really Playing Dice; What Does That Result In To ; You Become A Scezophreniac Associating Again And Again What You Had Been Until Now Of Particles And Other Concomitants Again And Again Leading To Flawed Inferences. Confusing The Epistemic With The Ontic, Like If One Were To Presume That A General Law Actually "Governs” Outcomes—And That The Statement Of A Regularity Has The Role Of A Causal Mechanism—Is A Category Mistake. In A Broad Sense, Scientific Theory Can Be Viewed As (=) Offering Scientific Realism—Approximately True Description Or Explanation Of The Natural World—Or Might Be Perceived With Antirealism. A Realist Stance Seeks The Epistemic And The Ontic, Whereas An Antirealist Stance Seeks Epistemic But Not The Ontic. In The 20th Century's First Half, Antirealism Was Mainly Logical Positivism, Which Sought To Exclude Unobservable Aspects Of Reality From Scientific Theory. Since The 1950s, Antirealism Is More Modest, Usually Instrumentalism, Permitting Talk Of Unobservable Aspects, But Ultimately Discarding The Very Question Of Realism And Posing Scientific Theory As A Tool To Help Humans Make Predictions, Not To Attain Metaphysical Understanding Of The World. The Instrumentalist View Is Carried By The Famous Quote Of David Mermin, "Shut Up And Calculate", Often Misattributed To Richard Feynman. Quantum Mechanical Wave Function (Absolutely Squared) Describes The Completed Interference Pattern, It Must Describe An Ensemble Interpretations Of Quantum Mechanics From Wikipedia. Philosophy Of Science, The Distinction Of Knowledge Versus Reality Is Termed Epistemic Versus Ontic. A General Law Is A Regularity Of Outcomes (Epistemic), Whereas A Causal Mechanism May Regulate The Outcomes (Ontic). A Phenomenon Can Receive Interpretation Either Ontic Or Epistemic. For Instance, Indeterminism May Be Attributed To Limitations Of Human Observation And Perception (Epistemic), Or May Be Explained As A Real Existing Maybe Encoded In The Universe (Ontic). Confusing The Epistemic With The Ontic, Like If One Were To Presume That A General Law Actually "Governs” Outcomes—And That The Statement Of A Regularity Has The Role Of A Causal Mechanism—Is A Category Mistake. Interpretations Of Quantum Mechanics From Wikipedia. Reality Is Constant In The Midst Of Change. Note The Conservation Of Individual, Collective And Cosmic General Ledgers Are What Are Being Spoken Of. Not The Changing Transactions, But The Immutable Refractory Zero. (Italics Mine) What This Means Essentially Is That There Is Change Although Nothing Changes. Like There Are Changing Transactions And Still The Assets And Liabilities Will Be Equal. This Impossible Situation Is Reflected (E&Eb) In The Ultimate Impossibility Of Change Itself. That Which Does Not Exist Prior To Its Changing And At The End, After It Has Changed, Must Be Equally Non-Existent Between These Two Moments. Although The World Of Change Appears To Be Real, It Cannot Be So. (Real)Change, According To The Vedantin, Presupposes A Loss Of Identity. Reality Cannot Suffer Transformation If It Were To Do So, It Would Become Something Else And The Real Would Be Deprived Of Its Reality. The Immortal Can Never Become Mortal, Nor Can The Mortal Become Immortal The Doctrine Of Vibration: An Analysis Of The Doctrines And Practices Of Kashmir Shaivism Mark S. G. Dyczkowski. Concept Of Being From Shaivite Point Of View: In His Famous Work Entitled On The Conservation Of Force (1847), Hermann Von Helmholtz Had Divided The Totality Of The Energy In The Universe Between Potential Energy And Kinetic Energy And Affirmed The Reciprocal Convertibility Of The Two. In 1852, William Thomson Pointed Out That There Exists A Sub-Ensemble Within Kinetic Energy, Heat, Which, Once It Has Been Generated, Is No Longer Entirely Convertible Into Potential Energy-Or Into Any Other Form Of Kinetic Energy. Considering That The (Partial) Reconversion Of Heat Into Labor Is Possible Only In Situations That Present A Disparity In Temperature, And That Heat Tends To Pass From Warmer To Cooler Bodies By Spreading On An Even Temperature Level Through Space, Thomson Concluded That The Universe Tends Towards A Final State Where Any Energetic Transformations, Every Movement And Every Form Of Life Will Cease: The Eternal Return: Genesis And

Interpretation BY PAUL D'IORIO: The Ultimate Nature Of Anything Cannot Change. Change Of Any Sort Is Merely Apparent (Vivarta). The World Of Change And Becoming Is A False Super- Imposition (Adhyaropa, Adhyasa) On The Absolute. It Appears So On The Screen Of Individual, Collective Consciousness, Albeit One Cannot Be So Sure Of Cosmic, For There Are Many Reasons That Cosmic Transactions Also Had To Be Subjected To Change By ParaBrahaman. In Cosmic Terms, The Mistake (Bhranti) Consists Of The Supposition That The Real Brahman Is The Unreal Universe And The Unreal Universe Is The Real Brahman. In Microcosmic Terms, It Is The Mistake Of Falsely Conceiving The Body, Mind Or Even One's Personality To Be The Self. Note We Have Taken Self As Witness Consciousness. Notwithstanding There Is Nothing Wrong In The Conception Of Self As A Dynamical Presupposition, In That Individual Consciousness And Collective Consciousness And Even Cosmic Consciousness Evolutes In To ParaBrahaman. In The Same Way As The Image Of A Snake Is Falsely Superimposed On A Rope, Similarly The Universe Is Falsely Projected Onto The Real Substratum, The Brahman. Ignorance Is Not Merely A Personal Lack Of Knowledge, But A Cosmic Principle. As Such It Is Called "Maya," The Indefinable Factor (Anirvacaniya) That Brings This Mistake In Identity About. The Reality Status Of This Cosmic Illusion Is Also Indefinable: On The One Hand It Is Not Brahman, The Sole Reality; On The Other Hand It Is Not Absolutely Non-Existent Like A Hare's Horn Or The Son Of A Barren Woman. Note Here The Vedantin Avoids Talking About Zero Which We Have Avidly And Assiduously Stressed About. Nothing Is The Factor That Exists. Also See the Remarks about Satre's Being And Nothingness. Brahman Is The Source Of World Appearances Only In The Sense Of Being Their Unconditioned Ground Or Essential Nature. The Universe Is False Not Because It Has No Nature Of Its Own But Because It Does Have One. The Doctrine Of Vibration: An Analysis Of The Doctrines And Practices Of Kashmir Shaivism Mark S. G. Dyczkowski. Concept Of Being From Shaivite Point Of View: It Is The 'I Am The Body/Mind' Belief That Gives Rise To The 'I Am Not The World' Belief. These Two Beliefs Are (=) Co-Created."Consciousness Projects The Appearance Of The Mind, Body And World By Taking (Eb) The Shape Of Thinking, Sensing And Perceiving.""Attention Is Consciousness With An Object. When The Object Vanishes, Attention Simply Remains What It Always Is Consciousness.""There Is No Purpose To Meditation. The Purpose Is Already Accomplished.""Everything That Is Experienced Is Experienced By, Through, In And As Consciousness.""The Seen Cannot Be Separated From Seeing And Seeing Cannot Be Separated From Consciousness." "**The Reality Of Any Experience Is Not Hidden In The Appearance; It Is Expressed By The Appearance The Transparency Of Things**' By Rupert Spira Book Review By Dennis Waite The Following Is A Review Of Rupert's Book: 'The Transparency Of Things: Contemplating The Nature Of Experience'. There Is An Essay From The Book Here And The Essay On Cézanne May Be Read At The Awakened Eye Website Nature Rejects The Naiveté That Seeks Absolute Truth. We Are Beginning To Realize, Individually And Culturally, That "Realities" Are All Human Constructions. The Task Becomes One Of "Catching Ourselves In The Act" Of Creating Our Own "Reality" From The Flow Of Events. Human Truth Is Always An Engagement Of Mind With Experience. The Challenge Of The Therapist In These Times Of Chaotic Change Is To Validate The Concept That We Don't Have To Fear The Collapse Of What We Think We Are. Strange Attractors: Transference, Holography, And An **Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003).** When we get to the topic and understanding of the Transcendental Logic, the brilliant and baffling Transcendental Deduction (TD) Recall the two movements just discussed, the one from experience to its conditions and the one from the forms of valid inference to the concepts that we must use in all judging (the Categories). This duality led Kant to his famous question of right (quid juris) (A84=B116): with what right do we apply the Categories, which are not acquired from experience, to the contents of experience? (A85=B117). Kant's problem here is not as arcane as it might seem. It reflects an important question: How is it that the world as we experience it conforms to our logic? In briefest form, Kant thought that the trick to showing how it is possible for the Categories to apply to experience is to show that it is necessary that they apply (A97).TD has two sides, though Kant never treats them separately John Marlado, accounts, attributes, and ascribes the philosophy of Nishida KitarÅ is lucid, insightful, and deeply informative despite the highly questionable argument that gives the book its structure. In comparison with other, technically more accurate accounts of Nishida's

philosophy, it is broader in scope than most but more single-minded in its approach. The analysis is for the most part limited to four books that have been translated into English, out of approximately 16 volumes of philosophical essays in Japanese, but it aims to capture the contours and most important details of Nishida's entire philosophy. To appreciate the achievement of this account, one would need only to read, in English translation or in the Japanese original, any of the volumes with their streams of repetitive, puzzling expressions and circuitous, often dead- end argumentation. Wilkinson's elucidation of details in the passages he samples makes it easier to understand why Nishida is considered Japan's greatest academic philosopher. A critical look at the frame of the book and some of its own claims reveals where, to my mind, it has gone astray. The argument that frames this book is repeated throughout: Nishida's philosophy is an attempt to articulate, in a characteristically Western philosophical manner, the Zen experience he had as a young man. Nishida first made the attempt by using the conceptual frameworks of Western philosophers, and after they failed him he developed his own, largely by recasting Buddhist insights. In the end, a philosophical articulation of the Zen worldview in Western terms or even in Eastern terms recast by conceptual, Western philosophical methods approved impossible. What Nishida attempted is incommensurable with Western philosophy. While the author first offers these statements as hypotheses (p. 2), one premise of his argument that there is such a thing as "the Zen world-view" or "the Zen experience "remains unquestioned; and another that Nishida's pivotal experience was the Zen experienceâis asserted as "beyond question" (p. 151; see also pp. 28, 48, 10, 114).Wilkinson's presentation of Zen in the first chapter is drawn largely from D. T. Suzuki, with help from Suzuki's portage Abe Masao; some comments on time, birth, and death in; and a few other sources. The result is a caricature that imagines Zen as an invariable, unitary experience. In fact, Dagan's teachings on practice as the manifestation of enlightenment do not sit well with D. T. Suzuki's emphasis on a satori experience, and Suzuki does not enlighten us about what Zen in its various practices has historically been. The author would have benefited from acquaintance with other presentations by Zen teachers, such as that of ShunryÅ Suzuki'sZen Mind Beginner's Mind, with historical accounts of Zen traditions, and with contrasts between DÅgen's view of time and Sam Â· sÄraand that of other Buddhist philosophers. A little research in contemporary Zen scholarship would have shown how implausible is the talk of "nirvana or awareness of mu" (p. 24), "the basic Zen position [that] never varies" (p. 3), "the Zen vision" (pp. 5, 7), "Zen epistemology" , "Zen experience" (p. 59), "the conception of time involved in Zen" (p. 15), and "the fundamental Zen assertion that there is no absolute time" (p. 16). Even if DÅgen's notion that time and being are nondual (p. 16) epitomized most of Zen, Nishida's early notion of "a transcendent, unchanging reality apart from time" (p. 51) would contrast sharply with it. Equally questionable is Wilkinson's contention that Nishida's "pure experience" is equivalent to Zen satori. The notion of pure experience Nishida developed in his first major work may be pivotal for his entire philosophy, but he never claims or implies that any part of his philosophy presupposes an exceptional experience like satori that grounds his convictions and renders them unverifiable to those who have not had the experience, as Wilkinson intimates (pp. 55, 159, and 161). Nishida presents his notions of pure experience and intellectual intuition, the grasp of its unity, and later... Perennialism and Constructivism is studied by Randolph T Dible : experiencer or the observer and the transcendental subjectivity itself Pure objectivity is essentially a deconstruction, apophatic and negativa The encountering of the experiencer or observer—transcendental subjectivity itself—at the foundation of the world leads inevitably to the recognition of pure objectivity as ultimate reality (which can be taken as its ultimate deconstruction, analogous to the apophatic or via negativa), from which objects derive their value, weight, significance, meaning or objectivity. In this way, pure objectivity can be seen as the supra-self-evident Axiological Axiom, so to speak, even Unconditional Love, in romantic terms. This axiology (value theory) has a structure inverse to the relationship between transcendental subjectivity as the radical unity of pure self-reference and on the other hand, the world of forms, as mere traces (representations, indications) of the unique, original "first distinction" Spencer-Brown speaks of at the foundation of his calculus. That is, all forms (i.e., distinctions, differences) would reduce to being the first distinction, also known as the marked state, which can be called penultimate reality (pure self-reference or transcendental subjectivity: the Spirit which animates us), except that forms are complimentary to their content, which is their objectivity or value, which would reduce to the unmarked state or ultimate reality. It is the incongruity of form (thoughts; Whitehead's

"negative prehensions") and value (feelings; Whitehead's "positive prehensions," or my notion of objectivity, meaning and qualia; in short, the non-formal aspects of experience) that holds forms open and keeps them from absolute reduction. This accounts for the brute, concrete persistence of the "functional illusion"-- to use a term from Dzogchen Buddhism-- of the world. Thus this system has an axiology of metaphysical objectivity grounded on the ideal of pure objectivity as the source of all value, meaning and significance, itself the very fecundity of profundity, which is the motive of drawing the distinction in the first place **(Wikipedia, Kant's writings and Logic Of Sense)** Nano rings were created by accident while intending to make quantum dots. They have interesting optical properties associated with excitons and the Aharonov–Bohm effect. Application of these rings used as light capacitors or a buffer includes photonic computing and communications technology. Analysis and measurement of geometric phases in mesoscopic rings is ongoing. Several experiments, including some reported in 2012, show Aharonov-Bohm oscillations in charge density wave (CDW) current versus magnetic flux, of dominant period h/2e through CDW rings up to 85 **μm** in circumference above 77 K. This behavior is similar to that of the superconducting quantum interference devices (seeSQUID). **Aharonov–Bohm nano rings (Wikipedia) Stability analysis of nano rings would help progressively accentuate the research about the materials division that uses nano rings.** From the profound contemplation and wisdom of the Buddha and Mahabodhisattvas, we know there is no such reality. Instead, egolessness (non-self) is the only path to understand the reality of the deluded life. All existences are subject to the law of causes and conditions. These include the smallest particles, the relationship between the particles, the planets, and the relationship between them, up to and including the whole universe! From the smallest particles to the biggest matter, there exists no absolute independent identity. Egolessness (non-self) implies the void characteristics of all existence. Egolessness (non-self) signifies the non-existence of permanent identity for self and existence (dharma). Sunyata stresses the voidness characteristic of self and existence (dharma). Sunyata and egolessness possess similar attributes. As we have discussed before, we can observe the profound significance of sunyata from the perspective of inter-dependent relationships. Considering dharma-nature and the condition of nirvana, all existences are immaterial and of a void-nature. Then we see each existence as independent of each other. But then we cannot find any material that does exist independent of everything else. So egolessness also implies void-nature! From the observation of all existences, we can infer the theory of nirvana and the complete cessation of all phenomena. From the viewpoint of phenomena, all existences are so different from each other, that they may contradict each other. They are so chaotic. In reality, their existence is illusionary and arises from conditional causation. They seem to exist on one hand, and yet do not exist on the other. They seem to be united, but yet they are so different to one another. They seem to exist and yet they do cease! Ultimately everything will return to harmony and complete calmness. This is the nature of all existence. It is the final resting place for all. If we can understand this reality and remove our illusions, we can find this state of harmony and complete calmness. **Teachings in Chinese Buddhism (6) sunyata (emptiness) in the Mahayana context (Wikipedia) Many models have been given concatenating Buddhist doctrines and Vaccum energy. These models essentially throw light on the essence of self(witness consciousness) and the cosmic general ledger (cosmic consciousness) while the imbalance in the gratification deprivation complex always remain unbalanced when it comes to individual general ledger , advocating the evolutionary theory of individual consciousness to cosmic consciousness. It is to be noted that it is not the question of knowing all the transactions but the wisdom of what lies behind the debit- Credit, Debit-Debit, or Credit-Credit principle that is most important for assimilation.** Look at this simple thesis to prove the point stated in the foregoing. Thesis: 'The World Has A Beginning In Time, And In Space It Is Also Enclosed In Boundaries.' Proof: 'For If One Assumes That The World Has No Beginning In Time, Then Up To Every Given Moment In Time An Eternity Is Elapsed, And Hence An Infinite Series Of States Of Things In The World, Each Following Another, Has Passed Away. But Now The Infinity Of A Series Consists Precisely In The Fact That It Can Never Be Completed Through A Successive Synthesis. Therefore An Infinitely Elapsed World-Series Is Impossible, So A Beginning Of The World Is A Necessary Condition Of Its Existence, Which Was The First Point To Be Proved (Kant, Critique Of Pure Reason, Tr. By R. Guyer And A. W. Wood (Cambridge: Cambridge University Press, 1998): B 454, P. 470). As Quoted In The Eternal Return: Genesis And Interpretation

BY PAUL D'IORIO"The Cyclical Hypothesis, So Heavily Criticized By Nietzsche (VP II 325 And 334), Arises In This Way." In Fact, Nietzsche Was Not Criticizing The Cyclical Hypothesis But Only The Particular Form Of That Hypothesis Presented In Vogt's Work. All Of Nietzsche's Texts Without Exception Speak Of The Eternal Return As The Repetition Of The Same Events Within A Cycle Which Repeats Itself Eternally. If Deleuze's Interpretation Holds That The Eternal Return Is Not A Circle, Then What Is It? A Wheel Moving Centrifugally, Operating A "Creative Selection," "Nietzsche's Secret Is That The Eternal Return Is Selective" Says Deleuze: The Eternal Return Produces Becoming-Active. It Is Sufficient To Relate The Will To Nothingness To The Eternal Return In Order To Realize That Reactive Forces Do Not Return. However Far They Go, However Deep The Becoming-Reactive Of Forces, Reactive Forces Will Not Return. The Small, Petty, Reactive Man Will Not Return. Affirmation Alone Returns, This That Can Be Affirmed Alone Returns, Joy Alone Returns. Everything That Can Be Denied, Everything That Is Negation, Is Expelled Due To The Very Movement Of The Eternal Return. We Were Entitled To Dread That The Combinations Of Nihilism And Reactivity Would Eternally Return Too. The Eternal Return Must Be Compared To A Wheel; Yet, The Movement Of The Wheel Is Endowed With Centrifugal Powers That Drive Away The Entire Negative. Because Being Imposes Itself On Becoming, It Expels From Itself Everything That Contradicts Affirmation, All Forms Of Nihilism And Reactivity: Bad Conscience, Ressentiment..., We Shall Witness Them Only Once. [...] The Eternal Return Is The Repetition, But The Repetition That Selects, The Repetition That Saves. Here Is The Marvelous Secret Of A Selective And Liberating Repetition. There Is No Need To Remind The Reader That Neither The Image Of A Centrifugal Movement Nor The Concept Of A Negativity-Rejecting Repetition Appears Anywhere In Nietzsche's Writings, And Indeed Deleuze Does Not Refer To Any Text In Support Of This Interpretation. Further, One Could Highlight That Nietzsche Never Formulates The Opposition Between Active And Reactive Forces, Which Constitutes The Broader Framework Of Deleuze's Interpretation. For Some Years, Marco Brusotti Has Called Attention To The Fact That Deleuze Introduced A Dualism That Does Not Exist In Nietzsche's Writings. To Be Sure, The German Philosopher Describes A Certain Number Of "Reactive" Phenomena (For Example, In The Second Essay Of The Genealogy Of Morality, § 11, He Talks About "Reactive Affects" [Reaktive Affekte], "Reactive Feelings" [Reaktive Gefühlen], Reactive Men [Reaktive Menschen]); But These Are Nonetheless The Result Of Complex Ensembles Of Configurations Of Centers Of Forces That Remain In Themselves Active. Neither The Word Nor The Concept Of "Reactive Forces" Ever Appears In Nietzsche's Philosophy. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO Some Parts Are Deleted Due To Spatial Constraints. In the midst of thoughts about the eternal return we find at least two other thematic axes On the one hand, the view of the **world as a constant flux of forces without any goal, law, or rules of becoming. A chaos sive natura de-divinized and de-anthropomorphized which constitutes the "ontological substratum**" of the whole of Nietzsche's reflexions On the other hand, an ensemble of fragments of an anthropologico-sociological character, designing a path of liberation leading to the creation of superior individuals by way of a profound transformation of their instinctual structure. This transformation must be achieved by a practice of solitude and internal struggle towards the liberation from the ancient representations of the world and from the incorporated herd values. For an analysis of these thematic perspectives, see Paolo D'Iorio, La linea e il circolo. Cosmologia e filosofia dell'eterno ritorno in Nietzsche (Genova: Pantograf, 1995): 233-322. With most of the parameters quantified, with assumptions there must be picturesque view of the self and non self, space and time, science and philosophy with the concatenational consummations. After The Chapter "On Redemption," Where Zarathustra Dares Not Expose His Doctrine, The Eternal Return Begins To Be Enunciated In Part Three Of The Work. In The First Place, It Is The Dwarf Who Formulates It In The Chapter "On The Vision And The Riddle." Facing The "Gate Of The Instant" Which Symbolizes The Two Infinities That Stretch Towards The Past And The Future, The Dwarf Whispers: "All Truth Is Crooked, Time Itself Is A Circle" The Dwarf Represents The Spirit Of Gravity, And He Embodies The Herd Morality, "The Belittling Virtue" Which Is The Title Of Another Chapter From Part III. The Dwarf Can Endure The Eternal Return Without Great Difficulties Because He Has No Aspirations; Unlike Zarathustra He Does Not Wish To Climb The Mountains That Symbolize Elevation And Solitude. In Two Unpublished Notes, From The Summer And The Fall Of 1883, Nietzsche Writes: The Doctrine Is At First Favored By The Rabble, Before It Gets

To The Superior Men. The Doctrine Of Recurrence Will First Smile To The Rabble, Which Is Cold And Without Any Strong Internal Need. It Is The Most Ordinary Of Life Instincts, Which Gives Its Agreement First. Hence, The Content Of The Doctrine Is The Same, But Whereas The Dwarf Can Endure It (Because He Interprets It According To The Pessimistic Tradition For Which "Nothing Is New Under The Sun"), Zarathustra, Who Is The "Advocate Of Life" Regards The Eternal Return As The Strongest Objection To Existence, And As The Rest Of The Dream Suggests, He Does Not Yet Succeed In Accepting It After The Vision At The Gate Of The Instant, The Chapter Is Brought To An End By The Enigma Of The Shepherd. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO .How does the memory affect the GD complex or for that matter the gratification deprivation of an individual or a collective identity? Non-Markovian local-in-time master equations give a relatively simple way to describe the dynamics of open quantum systems with memory effects. Despite their simple form, there are still many misunderstandings related to the physical applicability and interpretation of these equations. Here, we clarify these issues both in the cases of quantum and classical master equations. Further introduction of the concept of a classical non-Markov chain signified through negative jump rates in the chain configuration. **E-M Laine et al 2012 J. Phys. B: At. Mol. Opt. Phys. 45 154004 doi:10.1088/0953-4075/45/15/154004 Local-in-time master equations with memory effects: applicability and interpretation.** Supposing That There Were Indeed An "Energy Of Contraction" Constant In All Centers Of Force Of The Universe, It Remains To Be Explained Where Any Difference Would Ever Originate. It Would Be Necessary For The Whole To Dissolve Into An Infinite Number Of Perfectly Identical Existential Rings And Spheres, And We Would Therefore Behold Innumerable And Perfectly Identical Worlds COEXISTING [Nietzsche Underlines This Word Twice] Alongside Each Other. Is It Necessary For Me To Admit This? Is It Necessary To Posit An Eternal Coexistence On Top Of The Eternal Succession Of Identical Worlds? The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO. Science is usually portrayed as dehumanizing. Brave New World epitomizes this fear. "The more we understand the world, the more it seems completely pointless" (Steven Weinberg). Certainly science can seem chilling when conceived in the abstract as a metaphysical world-picture. We may seem to find ourselves living in a universe with all the human meaning stripped out: Participants in a soulless dance of molecules, or harmonics of pointlessly waggling superstrings and their braneworld cousins. Nature seems loveless and indifferent to our lives. What rights have we to be happy? **(Brave New World: Aldous Huxley).** Religion can afford to be dogmatic but science cannot. Look at this passage which counters both scientists and philosophers alike: However, Hartmann Objects That The Regressive Movement Postulated By Schopenhauer Is Possible Only In Thought: It Remains Nothing More Than An "Ideal Postulate" With No Real Object And Which "Does Not Teach Us Anything About The Real Process Of The World That Unfolds In A Movement Contrary To This Backwards Movement Of Thought" (Hartmann, Philosophie Des Unbewussten, Third Edition (1871): 772). Hartmann Affirms That If Unlike Schopenhauer One Admits The Reality Of Time And Of The World Process, One Must Also Admit That The Process Must Be Limited In The Past And Therefore That There Must Be An Absolute Beginning. In Hartmann's Mind, Failure To Do So Would Result In Positing The Contradictory Concept Of An Accomplished Infinity: "The Infinity That From The Point Of View Of Regressive Thinking, Remains An Ideal Postulate, Which No Reality May Correspond To, Must, For The World, Whose Process Is, On The Contrary, A Progressive Movement, Open Up To A Determinate Result; And Here The Contradiction Comes To Light" (Hartmann (1871): 772). What Really "Comes To Light" In This Passage Is The Fact That Hartmann Does Not Provide A Demonstration But A Petitio Principii. Indeed, The Concept Of The World Process Analytically Contains The Concept Of A Beginning Of The World. In All Rigors, It Is Therefore Impossible To Demonstrate These Concepts With Reference To Each Other. Secondly, Hartmann's View That One Is Bound To Accept The Reality Of The World Process Even If One Rejects The Ideality Of Schopenhauer's Time Is Mistaken. Hartmann Believes That If Time Is Real There Must Be A World Process With Both An Absolute Beginning And An Absolute End. Without Any Justification, Hartmann Jumps From Schopenhauer's Negated Time To Oriented Time. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO. In the last years several theoretical papers discussed if time can be an emergent property deriving from) quantum correlations. Here, to provide an insight into how this phenomenon can occur, authors present an experiment that illustrates Page and Wootters'

mechanism of "static" time, and Gambini et al. subsequent refinements. A static, entangled state between a clock system and the rest of the universe is perceived as evolving by internal observers that test the correlations between the two subsystems. They implement this mechanism using an entangled state of the polarization of two photons, one of which is used as a clock to gauge the evolution of the second: an "internal" observer that becomes correlated with the clock photon sees the other system evolve, while an "external" observer that only observes global properties of the two photons can prove it is static.Phys. Rev. A 89, 052122 (2014)DOI:10.1103/PhysRevA.89.052122 arXiv: 1310.4691 [quant-ph] **Time from quantum entanglement: an experimental illustration Ekaterina Moreva.** Time is an entanglement .When the new ideas of quantum mechanics spread through science like wildfire in the first half of the 20th century, one of the first things physicists did was to apply them to gravity and general relativity. The results were not pretty. It immediately became clear that these two foundations of modern physics were entirely incompatible. When physicists attempted to meld the approaches, the resulting equations were bedeviled with infinities making it impossible to make sense of the results. Then in the mid-1960s, there was a breakthrough. The physicists John Wheeler and Bryce DeWitt successfully combined the previously incompatible ideas in a key result that has since become known as the Wheeler-DeWitt equation. This is important because it avoids the troublesome infinites—a huge advance. But it didn't take physicists long to realise that while the Wheeler-DeWitt equation solved one significant problem, it introduced another. The new problem was that time played no role in this equation. In effect, it says that nothing ever happens in the universe, a prediction that is clearly at odds with the observational evidence. This conundrum, which physicists call 'the problem of time', has proved to be thorn in flesh of modern physicists, who have tried to ignore it but with little success. Then in 1983, the theorists Don Page and William Wootters came up with a novel solution based on the quantum phenomenon of entanglement. This is the exotic property in which two quantum particles share the same existence, even though they are physically separated. Entanglement is a deep and powerful link and Page and Wootters showed how it can be used (e) to measure time. Their idea was that the way pair of entangled particles evolves is a kind of clock that can be used to measure change. But the results depend on how the observation is made. One way to do this is to compare the change in the entangled particles with an external clock that is entirely independent of the universe. This is equivalent to god-like observer outside the universe measuring the evolution of the particles using an external clock. In this case, Page and Wootters showed that the particles would appear entirely unchanging—that time would not exist in this scenario. But there is another way to do it that gives a different result. This is for an observer inside the universe to compare the evolution of the particles with the rest of the universe. In this case, the internal observer would see a change and this difference in the evolution of entangled particles compared with everything else is an important a measure of time. This is an elegant and powerful idea. It suggests that time is an emergent phenomenon that comes about because of the nature of entanglement. And it exists only for observers inside the universe. **Time is an entanglement** (Wikipedia). Danish religious philosopher Soren Kierkegaard carried out a systematic critique of the pretensions of reason and an abstract rationalism which he believed that the modern age was nurturing. Condemning reflection as a "danger" that ensnares people in logical delays and machinations, Kierkegaard compared it to a prison. Reflection is for him a form of captivity, a bondage which "can only be broken by [passionate] religious inwardness" (1978: 81). **Reflection seduces individuals into thinking its possibilities are "much more magnificent than a paltry decision"** (1978: 82). It leads them to act **"on principle,"** to dwell on the deliberation of the context of their actions and the calculation of their worth or outcome. Kierkegaard argues that this drives away feeling, inspiration, and spontaneity, all of which are crucial for true inner being and a vital relation to God. For Kierkegaard, as Nietzsche would later agree, genuine inner being (and culture) is characterized by the tautness and tension of the soul which characterizes passionate existence. But the "coiled springs of life relationships ... lose their resilience" in reflection (1978: 78) and "everything becomes meaningless externality, devoid of [internal] character" (1978: 62). Kierkegaard thus contributes to the development of an irrationalist tradition that has echoes in some later postmodern thought. Kierkegaard might well have agreed with his contemporary Fyodor Dostoyevsky, who wrote: "An intelligent [reflective] man cannot seriously become anything ... excessive consciousness is a disease" (1974: 3, 5). In an age overtaken by rules and regulations, genuine action -- which Kierkegaard assumes to be subjective and

spontaneous -- is frustrated at every turn. Complaining that we are too "sober and serious" (1978: 71) even at banquets, Kierkegaard bemoans the fact that even suicides are premeditated (1978: 68)! "That a person stands or falls on his actions is becoming obsolete; instead, everybody sits around and does a brilliant job of bungling through with the aid of some reflection and also by declaring that they all know very well what has to be done" (1978: 73). Thus, it is passion, not reflection, that guarantees "a decent modesty between man and man [and] prevents crude aggressiveness" (1978: 62). "Take away the passion and the propriety also disappears" (1978: 64). The ambiguity in the word "passion" may cause some confusion here. To say that the age and its individuals are "passionless" is not to say there are no emotions whatsoever, but rather that there is no true spiritual inwardness and depth, no intensively motivated action and commitment. It suggests that passion exists only in a simulated, pseudo-form, "the rebirth of passion" through "talkativeness" (1978: 64). "Chattering" for Kierkegaard gets in the way of "essential speaking" and merely "reflects" inconsequential events (1978: 89-99). Hence, in “the present age,” emotions -- which in fact are all too pronounced -- have been transformed into negative forces. Anticipating Nietzsche's genealogy of the "slave revolt" in morality, Kierkegaard claims that the "enthusiasm" of the prior age of Revolution, a "positively unifying principle,” has become a vicious "envy," a "negatively unifying principle" (1978: 81), a leveling force in its own right insofar as those lacking in talent and resources want to tear down those who have them. **THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html.** In November 2011, the United States government argued before the US Supreme Court that it wants to continue utilizing GPS tracking of individuals without first seeking a warrant. In response, Justice Stephen Breyer questioned what this means for a democratic society by referencing Nineteen Eighty-Four. Justice Breyer asked, "If you win this case, then there is nothing to prevent the police or the government from monitoring 24 hours a day the public movement of every citizen of the United States. So if you win, you suddenly produce what sounds like 1984... "[65]. What is to be said in unmistakable terms is that when it comes to national security and innocent citizens there is question of compromise even it be an infringement and encroachment of so called privacy which has been made a word to subterfuge the activities that are far said to be fair. Quantum theory has provoked intense discussions about its interpretation since its pioneer days. One of the few scientists who have been continuously engaged in this development from both physical and philosophical perspectives is Carl Friedrich von Weizsäcker. The questions he posed were and are inspiring for many, including the authors of this contribution. Weizsäcker developed Bohr’s view of quantum theory as a theory of knowledge. **Harald Atmanspacher, Hans Primas** show that such an epistemic perspective can be consistently complemented by Einstein’s ontically oriented position. **Time, Quantum and Information 2003, pp 301-321 Epistemic and Ontic Quantum Realities Harald Atmanspacher, Hans Primas Robert W. Spekkens** present a toy theory that is based on a simple principle: the number of questions about the physical state of a system that are answered must always be equal to the number that are unanswered in a state of maximal knowledge. Many quantum phenomena are found to have analogues within this toy theory. These include the noncommutativity of measurements, interference, the multiplicity of convex decompositions of a mixed state, the impossibility of discriminating nonorthogonal states, the impossibility of a universal state inverter, the distinction between bipartite and tripartite entanglement, the monogamy of pure entanglement, no cloning, no broadcasting, remote steering, teleportation, entanglement swapping, dense coding, mutually unbiased bases, and many others. The diversity and quality of these analogies is taken as evidence for the view that quantum states are states of **incomplete knowledge** rather than states of reality. A consideration of the phenomena that the toy theory **fails to reproduce, notably, violations of Bell inequalities** and the existence of a Kochen-Specker theorem, provides clues for how to proceed with this research program.DOI: http://dx.doi.org/10.1103/PhysRevA.75.032110 **Evidence for the epistemic view of quantum states A toy theory Phys Rev A 75, 032110 – Published 19 March 2007 Robert W. Spekkens In such specialized and autonomous areas, it is to the eminent and erudite authors to interpret the contradistinctions and contradictions’ in the dove tailing explaining. This I feel would be yeoman service in the creation of Nature’s general ledger. M. S. Leifer and O. J. E. Maroney** examine the relationship between quantum contextuality (in both the standard Kochen-Specker sense and in the generalized

sense proposed by Spekkens) and models of quantum theory in which the quantum state is maximally epistemic. We find that preparation noncontextual models must be maximally epistemic, and these in turn must be Kochen-Specker noncontextual. This implies that the Kochen-Specker theorem is sufficient to establish both the impossibility of maximally epistemic models and the impossibility of preparation noncontextual models. The implication from preparation noncontextual to maximally epistemic then also yields a proof of Bell's theorem from an Einstein-Podolsky-Rosen-like argument. DOI: http://dx.doi.org/10.1103/PhysRevLett.110.120401 Received 25 August 2012Published 20 March 2013 © 2013 American Physical Society **Maximally Epistemic Interpretations of the Quantum State and Contextuality Phys. Rev. Lett 110, 120401 – Published 20 March 2013 M. S. Leifer and O. J. E. Maroney.** Epistemological, onitic and ontological aspects particularly of quantum mechanics have invited profound thinkers to contribute their mite to the growing literature on the subject, which includes the contributions from the subterranean realm and ceratoid dualism of philosophy like that Chaos theory of Deleuze, Derrida , Kierkegaard and others. **Andrei Khrennikov** shows that the Dirac-von Neumann formalism for quantum mechanics can be obtained as an approximation of classical statistical field theory. This approximation is based on the Taylor expansion (up to terms of the second order) of classical physical variables – maps f: $\Omega \rightarrow R$, where Ω is the infinite-dimensional Hilbert space. The space of classical statistical states consists of Gaussian measures ρ on Ω having zero mean value and dispersion $\sigma 2\ (\rho) \approx h$. This viewpoint to the conventional quantum formalism gives the possibility to create generalized quantum formalisms based on expansions of classical physical variables in the Taylor series up to terms of nth order and considering statistical states ρ having dispersion $\sigma 2(\rho) = h\ n$ (for $n = 2$ we obtain the conventional quantum formalism). **December 2005, Volume 18, Issue 7, pp 637-650 Date: 28 Nov 2005 Generalizations of Quantum Mechanics Induced by Classical Statistical Field Theory Andrei Khrennikov** It is widely accepted that consciousness or, in other words, mental activity is in some way correlated to the behavior of the brain or, in other words, material brain activity. Since quantum theory is the most fundamental theory of matter that is currently available, it is a legitimate question to ask whether quantum theory can help us to understand consciousness. Several approaches answering this question affirmatively, proposed in recent decades, will be surveyed. It will be pointed out that they make different epistemological assumptions, refer to different neurophysiological levels of description, and adopt quantum theory in different ways. For each of the approaches discussed, these imply both problematic and promising features which will be indicated. **Discrete Dynamics in Nature and Society Volume 2004 (2004), Issue 1, Pages 51-73 http://dx.doi.org/10.1155/S102602260440106X Quantum theory and consciousness: an overview with selected examples Harald Atmanspacher Copyright © 2004 Hindawi Publishing Corporation. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.** Another thought provoking and investigatory article is by Peter G. Lewis et al. Perhaps the quantum state represents information about reality, and not reality directly. Wave function collapse is then possibly no more mysterious than a Bayesian update of a probability distribution given new data. **Peter G. Lewis et al** consider models for quantum systems with measurement outcomes determined by an underlying physical state of the system but where several quantum states are consistent with a single underlying state—i.e., probability distributions for distinct quantum states overlap. Significantly, they demonstrate by example that additional assumptions are always necessary to rule out such a model. DOI: http://dx.doi.org/10.1103/PhysRevLett.109.150404 **Distinct Quantum States Can Be Compatible with a Single State of Reality Phys. Rev. Lett. 109, 150404 – Published 9 October 2012 Peter G. Lewis et al** Communication complexity of a quantum channel is the minimal amount of classical communication required for classically simulating a process of state preparation, transmission through the channel and subsequent measurement. It establishes and perpetuates a limit on the power of quantum communication in terms of classical resources. **Alberto Montina** shows that classical simulations employing a finite amount of communication can be derived from a special class of hidden variable theories where quantum states represent statistical knowledge about the classical state and not an element of reality. This special class has attracted strong interest very recently. The communication cost of each derived simulation is given by the mutual information between the quantum state and the classical state

of the parent hidden variable theory. Finally, find that the communication complexity for single qubits is smaller than 1.28 bits. The previous known upper bound was 1.85 bits. DOI: http://dx.doi.org/10.1103/PhysRevLett.109.110501 **Epistemic View of Quantum States and Communication Complexity of Quantum Channels Phys. Rev. Lett 109, 110501 – Published 12 September 2012 Alberto Montina for models see elsewhere .** Stability analysis of what? Quantum state representing reality or our knowledge of reality? Does the quantum state represent reality or our knowledge of reality? In making this distinction precise, **Nicholas Harrigan, Robert W. Spekkens** are led to a novel classification of hidden variable models of quantum theory. They show that representatives of each class can be found among existing constructions for two-dimensional Hilbert spaces. Approach also provides a fruitful new perspective on arguments for the nonlocality and incompleteness of quantum theory. Specifically, **Nicholas Harrigan, Robert W. Spekkens** show that for models wherein the quantum state has the status of something real, the failure of locality can be established through an argument considerably more straightforward than Bell's theorem. The historical significance of this result becomes evident when one recognizes that the same reasoning is present in Einstein's preferred argument for incompleteness, which dates back to 1935. This fact suggests that Einstein was seeking not just any completion of quantum theory, but one wherein quantum states are solely representative of our knowledge. Hypothesis is supported by an analysis of Einstein's attempts to clarify his views on quantum theory and the circumstance of his otherwise puzzling abandonment of an even simpler argument for incompleteness from 1927. **February 2010, Volume 40, Issue 2, pp 125-157 Date: 09 Jan 2010 Einstein, Incompleteness, and the Epistemic View of Quantum States Nicholas Harrigan, Robert W. Spekkens** statistical model of the probabilistic description of physical reality has been studied by many authors. **Andrei Khrennikov posits** a contextual statistical model of the probabilistic description of physical reality. Here contexts (complexes of physical conditions) are considered as basic elements of reality. There is discussed the relation with QM. He proposes a realistic analogue of Bohr's principle of complementarity. In the opposite of the Bohr's principle, **Andrei Khrennikov** principle has no direct relation with mutual exclusivity for observables. To distinguish his principle from the Bohr's principle and to give better characterization, he changes the terminology and speaks about supplementarity, instead of complementarity. Supplementarity is based on the interference of probabilities. It has quantitative expression trough a coefficient which can be easily calculated from experimental statistical data. There is need not appeal to the Hilbert space formalism and noncommutativity of operators representing observables. Moreover, in our model there exists a pair of supplementary observables which cannot be represented in the complex Hilbert space. There are discussed applications of the principle of supplementarity outside quantum physics. **October 2005, Volume 35, Issue 10, pp 1655-1693 The Principle of Supplementarity: A Contextual Probabilistic Viewpoint to Complementarity, the Interference of Probabilities and Incompatibility of Variables in Quantum Mechanics Andrei Khrennikov Foundations of Physics** Description of the vacuum in Yang-Mills theory and arrive at a physical interpretation of the pseudoparticle solution and the attendant violation of symmetries has been done by **R. Jackiw and C. Rebbi.** It is here vaccuum stability forms the bastion, pillar, post and stylobate of the mechanisms of yang mills Theory and action of a particle. The existence of topologically inequivalent classical gauge fields gives rise to a family of quantum mechanical vacua, parametrized by a CP-nonconserving angle. The requirement of **vacuum stability** against gauge transformations renders the vacua chirally noninvariant. DOI: http://dx.doi.org/10.1103/PhysRevLett.37.172 **Vacuum Periodicity in a Yang-Mills Quantum Theory Phys. Rev. Lett 37, 172 – Published 19 July 1976 R. Jackiw and C. Rebbi.** Three-dimensional Yang-Mills and gravity theories augmented by gauge-invariant mass terms are analyzed. These topologically nontrivial additions profoundly alter the particle content of the models and lead to quantization of a dimensionless mass-coupling-constant ratio. The vector field excitations become massive, with spin 1 (rather than massless with spin 0), and the mass provides an infrared cutoff. The gravitation acquires mass, mediates finite-range interactions, and has spin 2 (rather than being absent altogether); although its mass term is of third derivative order, there are no ghosts or acausalities. DOI: http://dx.doi.org/10.1103/PhysRevLett.48.975 © 1982 The American Physical Society **Three-Dimensional Massive Gauge Theories Phys. Rev. Lett 48, 975 – Published 12 April 1982 S. Deser et al** All scattering amplitudes in the

maximally supersymmetric N=4 super-Yang–Mills theory possess a new, dual superconformal symmetry which extends the previously discovered dual conformal symmetry of MHV amplitudes. To reveal this property we formulate the scattering amplitudes as functions on the appropriate dual superspace. Rewritten in this form, all tree-level MHV and next-to-MHV amplitudes exhibit manifest dual superconformal symmetry. We propose a new, compact and Lorentz covariant formula for the tree-level NMHV amplitudes for arbitrary numbers and types of external particles. The dual superconformal symmetry is broken at loop level by infrared divergences. However, we provide evidence that the dual conformal anomaly of the MHV and NMHV superamplitudes is the same and, therefore, their ratio is dual conformally invariant. We show this explicitly for the six-particle amplitudes at one loop. Authors conjecture that these properties hold for all, MHV and non-MHV, superamplitudes in N=4 SYM both at weak and at strong coupling. **Nuclear Physics B Volume 828, Issues 1–2, 21 March 2010, Pages 317–374 Dual superconformal symmetry of scattering amplitudes in N=4 super-Yang–Mills theory J.M. Drummonda DOI: 10.1016/j.nuclphysb.2009.11.022.** By comparison with numerical results in the maximal Abelian projection of lattice Yang–Mills theory, it is argued that the nonperturbative dynamics of Yang Mills theory can be described by a set of fields that take their values in the coset space SU(2)/U(1). The Yang–Mills connection is parameterized in a special way to separate the dependence on the coset field. The coset field is then regarded as a collective variable, and a method to obtain its effective action is developed. It is argued that the physical excitations of the effective action may be knot solitons. A procedure to calculate the mass scale of knot solitons is discussed for lattice gauge theories in the maximal Abelian projection. The approach is extended to the SU (N) Yang–Mills theory. A relation between the large N limit and the monopole dominance is pointed out. **Physics Letters B Volume 458, Issues 2–3, 8 July 1999, Pages 322–330 An effective action for monopoles and knot solitons in Yang–Mills theory Sergei V. Shabanov DOI: 10.1016/S0370-2693(99)00612-7.** It is here the quintessential formulation of consciousness occurs. Like a man standing on the threshold of infinity trying to ponder what lies beyond the veil, which separates the seen from unseen a true seeker tries to lift the individual general ledger to the cosmic general ledger. Like in Darwinian evolution, mind also has to evolve from individual consciousness to cosmic consciousness. In Kaishmiri Shaivism one withdraws from the finite to the infinite, but one also goes on an outward journey from the infinite to the finite, because both the finite and the infinite have an intimate connection. The finite is not seen as unreal, but as a symbol of the infinite. There is no real distinction between them. Those two movements constitute Spanda, a key concept in Kaishmiri Shaivism. Spanda is the pulsation of the Absolute in different phases of being. There are no opposites like subject and object, unity or duality, absolute or relative. They are just different phases of the universal vibration of the Absolute. The goal is to realize or be at once infinite and finite. One does not turn away from appearances (like in Advaita Vedanta), but one realizes that the Absolute manifests all things. Spanda, the eternal pulsation of the Absolute, oscillates between a passion to create and dispassion from the created. Through it the Absolute transform itself into all things and then returns back into the emptiness of its undifferentiated nature. In Kaishmiri Shaivism the Absolute is seen as pure consciousness (=being). The Absolute is an eternal all-pervasive principle, the highest reality, the nature of all entities eternally and blissfully at rest within its own nature. The Absolute is the nature of the Self (and thus of us all). The Absolute is divine, it is Shiva, the Lord of the Universe. It is full of conscious activity through which it generates the universe, and reabsorbs it into itself at the end of each cycle of creation. Thus we speak of monism, as everything resides within this one absolute consciousness. It sustains all things, it embraces all things, and it pervades all things. All things are appearances within the absolute consciousness, but nevertheless real (in contrast to Advaita Vedanta where appearances are seen as unreal or illusionary). All things appear external (out there, outside ourselves), but they do not have a being on their own. They do not exist as separate entities on their own. Everything is contained within consciousness. What we see as objects are manifestations of consciousness. The event which constitutes the universe is always internal events happening within consciousness because their essential nature is consciousness itself. If a physical object were totally material, and independent or external to consciousness, it could never be experienced. The universe and consciousness are two aspects of a whole. The universe is an attribute of consciousness which bears consciousness as its substance. Doctrine of Vibration: S.G.Dyczkowski What you "see" is not always what you "get." Many people

mistakenly take their own visions literally without "seeing through" the various possibilities that are outside their belief system, knowledge or skill base. Deeper reality is not remote in the physical sense but in a psychological sense. The archetypes of the collective unconscious are arrayed behind the scenes of current worldwide conditions, of crisis and confusion. They mirror our own states back at us, whether we perceive them as such or interpret them plausibly or not. The noise of ordinary consciousness and beliefs drowns out the signal. Unconsciousness is the background of our ordinary awareness. Our organism is very much at the center of such effects. The organismic source is our human bodies and the focus of human consciousness. The fantasy principle dethrones reality, but can be dissociative or compensatory. The human mind is a meme-Scape. Pre-conceived concepts vie with structures, concepts with images. Like scientists who ignore assumed truths, we leapfrog over our beliefs and personality deficits, claiming idiosyncratic imagination is literal reality. It couldn't be further from the truth and symbolism is utterly lost. The metaphor that might heal us enslaves us. Perhaps images like the holographic universe have an implicate order. Can we have a sense of the cosmos in the world without projecting myriad fantasies on it that we embrace literally? Has the world become so horrible it is unreasonable to be realistic? We may need to look at our drives and wishes, rather than the fantasy content. Psyche constructs reality. Our experience of so-called reality is always mediated by our image of it. Even if all the contents of the psyche are real, that doesn't mean they are realistic. That psyche is real is still a radical proposition, but psychic politics certainly color the self-image and ideas of everyone. We observe and participate with images It is not a question of nature or nurture (genes alone or experience alone). Rather, everything is both. We inherit the structures that make our experience what it is. But the structure itself is “empty,” and each human culture “fills” it with its own specific adaptations. It is difficult to define an archetype and set boundaries that distinguish it from others. In a hologram each part contains all the information but in lower resolution. Archetypes have this holographic quality. There are patterns within patterns within patterns. Some overlap with others, and some are nested inside others. Archetypal realities, passed on through DNA, are expressed in distinctive neuronal tracts in the brain. They include customs and laws regarding property, incest, marriage, kinship, and social status or roles; myths and legends; beliefs about the supernatural; gambling, adultery, homicide, schizophrenia, and the therapies to deal with them. A mythic and visionary language of immediate experience encompasses themes of deepest, highest, and ultimate concern. Most fantasy-based individuals are at a complete loss to coherently explain their own conventional behavior much less anomalous events and their deep meaning, much less the cultural unconscious or mythological unconscious matrix. But they try, and become utterly entrenched in their belief that they are right about the nature of the world and reality. We have pseudo-memories about our personal lives. Why not more so for our collective life? The subject matter often revolves around catastrophe, creation and the mythopoeic forces of mankind. Ignorant of such dynamics, interpretive mistakes and displaced psychic contents proliferate into errors of fact. Propaganda, media distortions, memes, and disinformation compound the social problem of misapprehension further. Shameless self-promotion by personalities of such ideas leads to cults. They make up myths about the myths of by-gone eras. Roiling unconscious images can be fatally confusing. Thought illusions culminate in projections and projections of mythology. Jung suggested symbols live only as long as they are pregnant with meaning. Philosophy arose from criticism of myth, from discussing and challenging it. In science, we criticize, reject and eliminate theories. At the edge of the abyss of the unknown, new signs and symbols emerge. Credible theories and paradigms must include biology, physics, and neurophysiology. One of the reasons people "see God", or a guru, or anomalies may be because our brains are constructed to see reality through the eyes of others. There are heaps of mirror neurons which are there to make us feel the 'other'. Mirror neurons do for psychology what DNA did for biology. They provide a unifying framework and help explain a host of mental abilities. As in the psychochemical processes of empathy or falling in love, a complex feedback loop sustains a state of mind. But when we empathically transpose ourselves into someone else's position, we expose ourselves to that reality -- cognitively and emotionally. The unconscious complicates empathy, both ways. Mirror neurons might well play a role in bonding, language and self-awareness. Naively, we take too much as self-evident. But 'seeing' does not always ‘believe', though many make this error or leap in logic and formulate their choices and future accordingly. Yet, there is only one way to learn what consciousness is. Experience. But we have no

satisfactory explanatory edifice for consciousness. Would such a theory release in each of us our own inner knowledge of the creativity of our own consciousness, and its infinite possibilities? The problem is trying to define a verb, a dynamic, as if it were a noun. But we do recognize the effect of consciousness. It functions to mediate states of consciousness, high and low psychobiological arousal. Consciousness is the subconscious lifted up by the physical body. When the body fails, the consciousness collapses back into the subconscious. All our thoughts come from the subconscious which can see our intentions but not our world. This relates somehow to intention being imaginary and not of the physical frictionized world (King). Gerald Edelman postulates that the flows of information in the brain are mediated through 're-entrant' feedback loops. As evolution provides new cognitive functions, new re-entrant loops are established. Even language itself is an archetype -- a chaotic field of dynamic associations. A subtle net of tropes, grammar, symbols, and meaning, the program language begins in limbic resonance. Some phenomena generate their own language patterns, nomenclature, and internal coherence of meaning and representation. In a holographic universe, even time and space could no longer be viewed as fundamentals. Because concepts such as location break down in a universe in which nothing is truly separate from anything else, time and three-dimensional space would also have to be viewed as projections of this deeper order. At its deeper level reality is a sort of super hologram in which the past, present, and future all exist simultaneously. This suggests that given the proper tools it might even be possible to someday reach into the super holographic level of reality and pluck out scenes from the long-forgotten past. Or not. A fantasy of such penetration or phenomenon inside the head is not the same as that penetration. **Jung in the 21st Century: Synchronicity and science By John Ryan Haule Mind Control Countermeasures.** The lower domain of experience is what we do not directly observe, the quantum realm, while the higher level is what we ordinarily observe, the classical realm. Based purely on experiment, the formulation of quantum theory initially placed the great divide between the quantum and classical domains between the measuring apparatus and the particle (Herbert, 1985). However, in the early 1930s, John von Neumann, in his rigorous mathematical treatise on quantum mechanics, found no support for such a division, compelling him to conclude that the wave for was collapsed by consciousness. Von Neumann's treatise has been called the "quantum bible" and "the most influential book on quantum theory ever written" (Herbert, 1985). The von Neumann formulation leads us to conclude that the lower realms of quantum reality form a virtually limitless array of potentials that are inherently incapable of realization without the observation of a knowing entity, which we identify with consciousness (Kafatos and Nadeau, 1990). We thus have a correspondence between quantum theory, the hierarchical structure implied by the Holographic Principle, and the higher orders of experience, leading to the full expression of the Conscious Universe. David Bohm's theory of the implicate and explicate orders involves a holographic Principle that is fully consistent with the Holographic Principle discussed here (Bohm 1980; 1986; Bohm and Hiley, 1993). According to Bohm, there is an implicate order that represents the universal, holographic subtext of reality, and which unfolds in every moment to produce the explicate order that we all observe. Thomas Germinario (2004) has equated the implicate order with the unconscious process, and the explicate order with conscious process. He emphasizes the nature of dreams within the implicate order, and the importance of the dream work for maintaining a healthy mind through integration of the implicate order and our daily lives. Allan Combs and Mark Holland (1990) connected the implicate order or holomovement (Bohm, 1980) with Carl Jung is theory of synchronicity (Jung and Pauli, 1955), with the implicate order providing a holographic medium through which apparently disconnected individuals become connected. The principle of synchronicity, the instantaneous connection of people and events beyond the senses, has been equated with the quantum-physical principle of non-locality (Combs and Holland, 1990; Germine, 1991), and has been proposed to be the fundamental mechanism of conscious process (Germine, 1991).Applying the Holographic Principle theory of mind, subjects may be connected synchronously in the manner of two individuals conversing through cellular phones, where the radio signals are transmitted through a distant satellite. The difference here is that the radio signals travel at the speed of light, and thus the there is some miniscule time lapse between the sending and receiving ends. The Holographic Principle implies a much more distant and instantaneous communication pathway, with the analogue of the satellite being the holographic boundary of the Universe, as well as a much richer communication, potentially involving a transmission

of thoughts and feelings. Our ego-consciousness seems to mask the universal relatedness implied by the Holographic Principle, and it is perhaps only through transcendence of the ego-consciousness that the higher orders of experience can become conscious. In the phenomenon of synchronicity, there seems to be a meaningful connection between individuals that breaks through the barrier of ego-consciousness. Such a connection is reported by many individuals in the course of dreams, when the ego-consciousness has been suspended, at times informing the dreamer of something that has happened in the life of a meaningfully-connected individual. **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science** Impassioned-intense Connectionist neuroscience, in an effort to attain relevance to clinical practice, has recently given us a theory of the relationship of the unconscious to consciousness which is dualistic (Viamontes and Beitman, 2007), an ad hoc theory of conclusions based on assumptions, the most erroneous of which are that representations of information are information, and that such representations elaborate themselves through a process of representations of representations in neuronal circuits (Bechtel and Abrahamsen,1991). As theory, the connectionist models are lacking heuristic value and **are inimical to the progress** of basic and clinical neuroscience. This is not to denigrate or minimize the importance of neuronal connections, but these connections are rather like the wires in a radio that plays a symphony. The radio, in and of itself, is incapable of playing the symphony. The radio doesn't do not write the symphony, nor does it broadcast the Passionate-raging symphony over the airways. Yet, if we loosen one wire, the symphony is not manifested through the radio. The brain is not subject to the same vulnerability, in that it has parallel and redundant networks of processing, and, it is in this sense that the connectivity of the brain becomes important. Connectionism, to the extent that it accepts consciousness and the unconscious at all, attributes them to separate groups of "circuits" in the brain, with consciousness-processing circuits having a limited but detailed ability to analyze information, and unconscious-processing processing more information in less detail (Viamontes and Beitman, 2007). There is an element of truth in this view in that information that has importance and meaning resonates within the brain at higher levels of experience and at higher levels of holographic recursion. This resonance involves what we know, what we feel, and what we are capable and willing to consciously realize. The Universe, as William Blake noted, can be seen in a grain of sand. Indeed, the grain of sand requires the Universe to exist. The human organism requires the Self to exist. The Self is not a simple amalgam of sensory experiences, as suggested in the connectionist model (Blinder, 2007). This amalgam view of the Self can be damaging when applied to psychotherapy, as the connectionists suggest (Blinder, 2007). The Self is ours forever and ever, from eternity to eternity. As Milton suggested, the mind is its own place. Our deepest purpose is for our minds to resonate with the supra-conscious levels of experience, much as it is purpose of the radio to play the symphony. If the symphony is not to our liking, we can turn the radio off. So, perhaps, the transpersonal consciousness can turn our own individual and collective radios off, or perhaps turn the volume down, if we are not in harmony with the Universal Mind. Physics has given us many enigmas. Quantum theory itself is an enigma. But some of the enigmas seem to involve the limitation of our own abilities of discernment. The thought experiment of "Schrödinger's Cat," as it is often interpreted (e.g. Gribbin, 1984), assumes that the cat is not conscious, and is incapable of observation in the context of collapse of the wave function. This is due, in large part, to the false dichotomy of information and experience. Our observation of other animals indicates, to those that are attuned to the animal mind, who can "feel" the animal mind, that, at the very least, all mammals are conscious. It would seem that the great neuropsychiatrist, Stanley Cobb (1948), may have been correct in his attribution of levels of consciousness to a wide variety of organisms: "...lower animals with no cerebrum appear to be conscious...even plants such as the sunflower that turns towards strong light may have a vague awareness or warmth and comfort. There are many degrees of consciousness and it is my contention that it is integrated at many levels like other important functions of the central nervous system."With respect to the theory of mind, recent mainstream thinking remains classical and mechanistic. Such views of mind are advocated by our most in prominent neuroscientists (e.g. Changeux, 1985; Gazzaniga, 1985) as neuroscience research attempts to explain all mental function on the basis of pure mechanism and the localized function of specific areas of the brain. Francis Crick, the distinguished Nobel Laureate, had a very influential second career as a neuroscientist. He concluded that we are basically soulless creatures, that all mental processes

could be reduced to identifiable, neural correlates in the brain, and that, in fact, we are nothing but a collection of neurons (Crick, 1995). Crick even went so far as to say that what we call the soul is a group of neurons located in the prefrontal lobes of the brain. Another Nobel Laureate, Gerald Edelman (1987), has written a critically acclaimed book on "neural Darwinism," which purports that the brain is "circuitry" is constructed through a developmental process of "survival of the fittest" neurons and neuronal connections. At the same time the total purposelessness and lack of direction in evolution is expounded, again to critical acclaim, by Richard Dawkins (1986). The idea that there essentially no self or soul imbued with agency and self determination was expounded by Gilbert Ryle (1949/2002) with his parody of the "ghost in the machine," and this work continues to be influential among many scholars of the mind. Philosophers such as Patricia Churchland (1986), who coined the term "neurophilosophy," take an "eliminative materialist" view of belief, free will, and consciousness. Other philosophers, such as Daniel Dennett (1991) would make experience "epiphenomenal," a by-product of brain processes with no effect whatsoever on the function of mind. However, unlike neuroscience, which is still in its infancy, the philosophy of mind has had a long history, including such notable figures as Plato, Aristotle, Descartes, Kant, Locke, Hume, Kierkegaard, and James, to name just a few. Perhaps, as noted by Ervin Laszlo (1974): "in today's world, most of the traditional functions of cognitive synthesis have atrophied and are ignored and neglected." Hopefully, what we have presented here will be a step in the right direction for what Laszlo calls a "conceptual synthesis" to help fill the need for meaningful engagement is such a world. The fundamental elements of such a synthesis have been described by Laszlo as follows: Conceptual synthesis performs at least five basic functions in the guidance of affairs. They are the mystical, the cosmological, the sociological, the pedagogical or psychological, and the editorial functions. The mystical function inspires in man a sense of mystery and profound meaning related to the universe and of himself in it. The cosmological function forms images of the universe in accord with local knowledge and experience, enabling men to describe and identify the structure of the universe and the forces of nature. The sociological function validates supports and enforces social order, representing it in accord with the nature of the universe, or as the natural or right form of social organization. The pedagogical or psychological function guides individuals through stages of life, teaching ways of understanding themselves and others and presenting desirable responses to life is challenges and trials. Finally, the editorial function of conceptual is to define some aspects of reality as important and credible and hence to be attended to, and other aspects unworthy of serious attention. **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science** Kant sums up this inversion and its spirit early in the Critique of Pure Reason: Up to now it has been assumed that all our cognition must conform to the objects; but all attempts to find something about them a priori through concepts that would extend our cognition have, on this presupposition, come to nothing. Hence let us once try whether we do not get farther with the problems of metaphysics by assuming that the objects must conform to our cognition, which would agree better with the requested possibility of an a priori cognition of them, which is to establish something about objects before they are given to us. This would be just like the first thoughts of Copernicus, who, when he did not make good progress in the explanation of the celestial motions if he assumed that the entire celestial host revolves around the observer, tried to see if he might not have greater success if he made the observer revolve and left the stars at rest. (B xvi) Onticology, like all variations of object-oriented ontology, is realist in its orientation. In defending a realist ontology onticology holds that the vast majorities of objects, actants, beings, or entities are independent of humans and are what they are regardless of whether any humans regard them or register them. In short, onticology rejects any anthropomorphic, idealist, or anti-realist thesis to the effect that to be is to be the correlate of mind, spirit, the body, the human, and language or otherwise. While it is certainly the case that knowledge is necessarily dependent on the object to which it relates, the converse does not hold true. Objects are not dependent on being known, regarded, perceived, or spoken about. As such, and to put it in Aristotlean terms, knowledge is an accident of objects, not objects an accident of knowledge. As Althusser so nicely puts it, "[n]o doubt there is a relation between thought-about-the-real and thisreal, but it is a relation of knowledge, a relation of adequacy or inadequacy of knowledge, not a real relation, meaning by this a relation inscribed in that real of which the thought is the (adequate or inadequate) knowledge" (Reading Capital, 96). Althusser goes on to remark that

"[t]he distinction between a relation of knowledge and a relation of the real is a fundamental one: if we did not respect it we should fall irreversibly into either speculative or empiricist idealism" (ibid.). Onticology categorically endorses Althusser's verdict. It is a fundamental necessity to distinguish between those relations that belong to the object and those that belong to knowledge. Contemporary philosophy continuously confuses these two very different sorts of relations. Naturally the question arises of how it is possible to surmount our relation to the object so as to determine whether objects themselves possess the properties we encounter in relating to objects. In other words, given that we can only ever relate to the object in relating to the object how is it possible to surmount this relation to get at the being of the object itself? Much more will have to be said about this later– and the answers will be surprising with respect to standard prejudices about realism –however, for the moment it can be said that onticology takes its epistemological inspiration from the transcendental realism of Roy Bhaskar. Among other things, Bhaskar sought to provide a transcendental grounding for the sciences. Insofar as onticology defends the thesis that the field of being is much more vast than the field of objects investigated by the natural sciences, it parts way with the thesis that the domain of being is exhausted by the domain of natural objects. However, the general form of Bhaskar's argument holds for our realist purposes. A transcendental argument seeks to elucidate the conditions under which certain acknowledged practices and forms of cognition are possible. Kant, for example, asked what must be the case for mathematical judgments to be possible. How is it both that we are able to extend our knowledge, as if by magic, through mathematical judgments and, more significantly, that these judgments are able to provide genuine knowledge of the world despite the fact that these forms of reasoning are not based on experience? Part of Kant's argument consisted in claiming that mind imposes the forms of space and time on the data of experience. In other words, space and time are not attributes of being itself but rather of the mind that regards being. Insofar, Kant argues, as mathematics is ultimately a rumination on the nature of space and time taken in their most abstract form and insofar as the mind imposes space and time on the manifold of sensation, it thus follows that a priori judgments about the nature of spatio-temporal relations are possible that anticipate the structure of actual-space times without directly experiencing these space-times. Why? Because any manifold of sensation must necessarily be structured by these forms imposed by intuition. **Onticology– A Manifesto for Object-Oriented Ontology Part I Posted by larval subjects under Object-Oriented Philosophy My Question Is This: You Say, "Granted There Are Deeper And Deeper And More Primitive Forms Of "Knowledge" Or Abstraction Into Mechanism, But I Wouldn't Say That The Atomic Level Brings A New Epistemic World Into View." But Doesn't Any Kind Of Knowledge, However Primitive, Imply Bringing Some Kind Of Epistemic World Into View Or Proto-View?** Joel: David, Yes Exactly. My Question Is, Just How Far Down The Complexity Gradient Can The Meaning Of 'Knowledge,' Or 'Semiotics, Or Specifically Representation (Maya) Be Stretched Without Breaking? I Can Comfortably Stretch It To The Genetic Level, As The Proto-Epistemic, Where Genetic Codes 'Represent' Phenotypic Structures Or Modes, And The Whole Of Evolution Learns From Its Mistakes And Successes As That Knowledge And History Is Encoded Into And As The Recapitulation Of The Embryogenesis Of The Organism. But An Atom, In My Model, While Certainly Reacting To Its Environment From Its Own Attractor, Essence, And 'Interiority' (Prehension) Isn't Making Representations Of It To Do So. But The Key To The Power And 'Essence' Of Representation (And Simultaneously Its Limitation) Is The Capacity Of Abstraction And Choice. Evolution Does This At A Rudimentary Level (And Anthropomorphizing A Bit) With Its Sending Into The World Its 'Random' Variations, Organismic Ideas, In A Sense. Its 'Mind' Is Literally On The Outside As Our Living World And Biosphere. A 'Choice' Is Made In The Biospheric Interiority, And Knowledge For The Evolutionary/Embryogenetic Trajectory Is Gained By The "Differential Reproductive Success" Of These Individuals Lives Through Time, As The Increasing Intelligence, Representation, And Capacity For Choice Is Directly Injected Into The Flow Through Sexual Selection. But Where Is The Encoded/Abstracted (Enfolded) Representation Of Options And A Choice Among Them Made At The Atomic Level? I Am Working With The Most Cutting-Edge And Coherent Models For The Atom (Including My Own) That I Can Find Among The Heterodoxy, And I See No Evidence Of Representation And Choice At This Level, Or Rather No Real Way To Further Stretch The Meaning Of The Terms To This Level. Sure There Is Uncertainty From Infinite Difference And Immanent Causation (See Bohm On Infinite Causation),

But Without Representation There Really Is No Choice (Although Crudely We Could Call A 'Bifurcation' A 'Choice' By The System As A Whole). So I Am Simply Saying That Representation At The Atomic Level Is Entirely Enfolded And These Are Really Just Ontic Level Phenomena. The Atomic Level, In My View, Is Pre- Not Yet Proto-Epistemic, Which I Place At The Genetic Level Where We Find The First Codes And Primitive Representation. This Just Means That There Is A Real Distinction Between Ontic And Epistemic (And A Gradient Between Them) And We Can't Collapse The Two Into Just The Ontic-Epistemic. And Yet, With The Distinction Intact, And The Gradient To Explore, They Remain Nondual, Univocal, ONE Or Ontic.What I Would Say Exists At All Levels (My Own Self-Aware Myth About The Given) Is What I Call The "Symbiogenesis Of Subject And Object". This Is The Spinozan Essence Of Dynamic Stability And Growth, And Leibnizian Prehension. Also Called The "Nucleation Of Observability" In Spinbitz. It Is From This Nucleus Of The Observer (Fuller) In A "Point-Free Geometry" (Whitehead) Of Pointless "Points Of View" As Conditions Of Boundary That Wilber's "All Is Perspective" Finds Its Ontic Grounding. And This Symbiogenesis Is Also A Rudimentary Or Deep-Level Native Or Primitive Intelligence, Evolution At The Involution Of The Subject-Object Interface. It Is An Exploration Of Creative Learning And Direct Immanent Awareness, Unmediated (Enfolded) By Representational Forms (Unfolded). It Just Is, In Waves (Cosmic Vertebrae) Before It Abstracts, Represents And Incarnates The Flesh Of What Could Be. Matter Is Enfolded Seed Of The Flower Of Its Abstraction. Maya Is This Depth And Recursion Of Brahma Into And Through The Boundary Conditions Of Abstract Relation. And Brahma Is The Span Of The Emergent Rigor Of The Cosmic Ergodic Spine That Opens The Recursions Into The Field Of Infinite Difference. In Immanent-Transcendent Waves, Literally And Empirically A Limit-Cycle In Ergodicity, It Enfolds And Unfolds From Anatom To Anatomy, From Infinite Intelligence Or Omnirelational Awareness Of Self, Through The Representational Depths Of Self As Self-Consciousness. Omni-Evolution, Trans-Dynamics, And The Anatom Posted On February 21, 2013A Discussion With Tom Huston, Part II The Following Is Part 2 Of An Excerpt From A Discussion Initiated By Tom Huston On His Facebook Page, Edited And Embellished For Clarity. Find Part One, Here. In This Part We Dig Into The Nature Of The "Anatom" And The "Ergodic Spine" Of An Electro-Fractal Cosmos, And Into The Trans-Dynamic Integration Between Being And Becoming, And The X-Interface−The Crossroads Of The Ontic-Epistemic (Brahma And Maya) And The Subject-Object Polarities. Univocal_Dynamics_V1 Tom Huston Is A Founding Member Of Integral Institute And A Former Editor Of Enlighten Next Magazine. His Webpage Is Tomhuston.Com. David Marshall Is A Writer And Editor Living In Chicago. No meaning, value, or normativity to be found in nature, that there is nothing **natural beings ought to be,** but that, rather, these judgments arise from us (See last post of larval subjects) which has led some to raise valuable questions about the coherence of these claims. The problem arises when the following three propositions are taken together: 1. there is nothing outside of nature.2. Beings have no intrinsic meaning, purpose, or value (in and of themselves, there's nothing they ought to be). 3. Value judgments about what beings and being ought to be arise from us and beings like us (bonobo apes, dolphins, institutions, birds of paradise, etc). The problem arises between proposition 1 and 3. How can it both be true that there is nothing outside of being and those normative judgments belong to us and other beings capable of making normative judgments, not nature? The problem arises from restricting these judgments to humans and beings capable of making these judgments. In making such a claim it seems as if we're saying that there's something outside of nature, something that is beyond nature, thereby violating the first thesis and potentially reintroducing the nature/culture distinction. By contrast, if we say that normative judgments are the special domain of those living beings with the proper degree of sentience to make such judgments, then we seem to reintroduce the nature/culture distinction and fall back into the sorts of problems that I outlined in my last post (and that are so nicely critiqued by thinkers such as Latour). Is there a way out of this? I don't know. Great_Chain_of_Being_2_ (lighter) ASIDE: It seems to me that this is really what the debate between realism and anti-realism, realism and socio-linguistic constructivism surrounding the new materialisms and speculative realists has really been all about. It's very easy to treat this as an abstract, academic debate: "Are you a realist or are you an anti-realist?", as if it were just a matter of what happens to be true. But it seems to me that this debate has, in reality, always been about politics. As I outlined in my last post, we have perpetually seen how appeals to the real and

natural have been used in the name of oppressive power, inscribing both the exploitation of nature and the oppression of various people in the very fabric of being itself. Theistic theology and realist ontology have perpetually been used in the name of what Deleuze called "State Philosophies" or philosophies that ontologize contingent orders of power and privilege (e.g., "the great chain of being" used to justify patriarchy, monarchy, serfdom, poverty, etc, and appeals to nature used to justify poverty and racial inequality (The Bell Curve), patriarchy (evolutionary psychology), heteronormativity, capitalism, etc). Because arrangements of power and inequality are always contingent in the sense that there's no marked difference in the capacities of peoples, power always looks for a transcendent supplement that would provide justification through ontological necessity. Antirealism– from the Greek atomists to present –became the radical and emancipatory gesture because it revealed the lie behind all of these forms of social organization or their inherit contingency or arbitrariness. Realism, by contrast, has all too often functioned as an apologetics for arbitrary power and social organizations. Here it's worth recalling what Foucault said about science: "…Even before we know to what extent something like Marxism or psychoanalysis is analogous to a scientific practice in its day-to-day operations, in its rules of construction, in the concepts it uses, we should be asking the question, asking ourselves about the aspiration to power that is inherent in the claim to being a science. The question or questions that have to be asked are: "What types of knowledge are you trying to disqualify when you say that you are a science? What speaking subject, what discursive subject, what subject of experience and knowledge are you trying to minorize when you say 'I speak this discourse, I am speaking a scientific discourse, and I am a scientist.' What theoretico-political vanguard are you trying to put on the throne in order to detach it from the massive, circulating, and discontinuous forms that knowledge can take?" (Society Must Be Defended, 10). All of these questions hold equally for claims to something being real. What is one trying to minorize when claiming something is real? What becomes privileged? What is excluded? It is these questions that have been at the heart of the realism debates, for as Spencer-Brown taught us, every distinction has a marked and unmarked space, draws attention to something to be included and pushes something into the unconscious or the domain of the invisible, hidden, or veiled. This is above all the case with evocations of the real. However, as I've tried to show antirealism leads to its own problems. First, so long as we exclude real beings from our ontological inventory, we are unable to fully understand how power functions ("such and such a set of cultural formations have been thoroughly debunked, yet people still live as if they believed them"). Not only do we not fully understand the sources of the problems due to too much focus on the discursive and semiotic, but we deny ourselves valuable sites of political intervention at the level of infrastructure. Second, we are prevented from addressing things such as cultural racism, such as that found in Heidegger's privileging of the West and the Greeks and Germans in particular. We do a good job addressing biological and theological racism, heteronormativity, and sexism by showing how it is a cultural construction, but when faced with racism such as Heidegger's where he argues that there's something "unique" about the Greek event, it's language, and about the German language, or Badiou/Zizek's racism with respect to the "Pauline Event", we really have no response. Here someone like Jared Diamond or Fernand Braud el is needed to explain global-geographical inequalities. Third, the tools of the cultural turn really do not provide us with the means of thinking the ecological as a site of the political. For this reason, I've tried to formulate a third way– which might be called "constructivist realism" or "constructivist naturalism" –that retains the insights of the cultural turn, while also allowing a robust place for the material. I don't claim to be original in this. I think that many such as Manuel DeLanda, Deleuze and Guattari, Stacy Alaimo, Karen Barad, Jane Bennett, etc., are up to something similar. It's a vast project that requires the work of a multitude of voices, especially given the way in which the culturalism pervades humanities. **The Relational and the Non-Relational: Notes towards an Immanent and Pluralist Theory of Meaning Posted by larval subjects under uncategorized k-bigpic I have deleted or reformulated some sentences for which I beg pardon attributable to spatial constraints** What is correlation, what is causation? What is coordination? What is coincidence? These thoughts have bothered me since long. I have been able to excavate some reliable material from Wikipedia which I would like to share. "All views are partial and contingent – that's the lesson of pluralism, and you say as much yourself when you say that we must be attentive to ways our own knowledge might contain superstitions. But it's not discourse that determines whether one view works

and another doesn't – it's a confrontation with **non-discursive (non-human)** agencies." I wonder if part of the issue here is that we understand pluralism differently. For me this doesn't sound like pluralism at all. Rather, it just sounds like our **epistemological condition**. We [hopefully] come to understand those portions of existence we question or investigate. The reason we come to investigate them is largely contingent. Finally, the accounts of these features of existence we give can be mistaken. All of this is perfectly consistent with a monism. I'm not sure why what he outlines above is the lesson of pluralism. It seems to me that every realist knows this. It seems to me that ontological pluralism is something quite different. Ontological pluralism is the thesis that there are many different worlds inhabited by many different entities. Thus, for example, you would have one world; say that of Lucretius, that's only inhabited by atoms and their combinations. You would have another world; say the world of the Mongolian shaman, that's inhabited by spirits that do all sorts of things. The ontological pluralist is saying that these spirits are, that they exist, that they're real. This is something quite different than merely saying that there are partial and contingent points of view on the world. It's this saying that these entities are rather than that some person or groups of people believe that they are that is the nub of the issue. Now, I think part of the issue here is that there's an ambiguity in the term ontology. Ontology can be one of two things. On the one hand, ontology is a group or persons set of beliefs as to what is. Here it's trivially true that there are pluralities of ontologies and the realist readily recognizes this. This is the whole reason there are debates over ontology. Mongolian shamans have their ontology, Europeans theirs, Christian fundamentalists theirs, materialists theirs, etc. When striving to understand and communicate with others it's vital to understand these ontologies because, as rhetoricians like Burke point out, our **beliefs about what is** are among the things that motivate our action. Despite having never seen bacteria I was my hands because I believe there are bacteria and viruses on door handles and whatnot and don't want to get sick. My belief about a particular thing existing is what motivates my action. And who knows, perhaps this belief is as superstitious as the belief that the crops failed because God was displeased with my community. On the other hand, ontology is a theory **about what is.** It is making a claim that something exists. This is where the rubber hits the road. The ontological pluralist seems committed to the thesis that every group set of beliefs about what exists is sufficient for granting the existence of those entities. This is what I find objectionable in Latour. Obviously I'm not bothered by Latour's suggestion that we should take into account the role that nonhumans like speed bumps, rivers, microbes, etc., play in the form that social assemblages take. But this is not all that Latour claims. This is precisely where the philosopher might balk. Unlike the ethnographer, the philosopher is not interested in what people believe exists, but rather philosophers– at least of the realist variant –are trying to figure out what is. In other words, the realist philosopher begins with the premise that not all of these beliefs about what is are true. So for the philosopher, recognizing that Mongolian shaman's believe in the existence of shamans would only be the first step. The next step would consist in determining whether there's good reason for thinking such entities really do exist, i.e., whether there's good reason for believing these entities have mind and culture independent reality. Lest readers think that I'm just picking on the supernatural here, we can ask similar questions about strings, subatomic particles, galaxies, etc., etc., etc. I guess the question really comes down to what exactly we mean by ontological pluralism. When we talk about ontological pluralism are we defending the thesis that people have different theories of being? If so, then ontological pluralism is trivially true. If this is what is meant, then I certainly share Jeremy's view that it's valuable to understand the different world's people believe in. Certainly when I was practicing as an analyst I didn't get in ontological debates with my patients and it was necessary to understand their theory of being or their ontology to properly attend to them. Or, when we talk about ontological pluralism are we defending the thesis that all these ontological theories are true and refer to really existing entities? That's quite a different claim and is not one I would defend or endorse. Now someone might object that "in both The Democracy of Objects and Onto-Cartography you defend the thesis that there are multiple worlds." This is true. Because I hold that not everything is related I'm led to the conclusion that there are diverse worlds. However, I also hold that however many worlds there might be, these worlds are nonetheless composed solely of material entities. Within the framework I propose I wouldn't suggest that there's one world where there are spirits and another world where there are souls and yet another where there are only material entities. My view is that there aren't spirits or souls in any of these worlds.

One of the things I keep hearing in these discussions is that somehow the realist adopts a view from nowhere. I honestly don't understand this criticism. Investigation always occurs somewhere and requires all sorts of mediations involving technologies, experiments, etc. It's that labor of gathering evidence, conducting experiments, using technologies to observe the world, etc., that gradually gives us a body of data that allows us to say there's good reason to believe that such and such a thing exists and has these powers. Another charge seems to be that the realist refuses to recognize that their claims about the world are fallible. I find this charge particularly strange because it's precisely because the realist recognizes the difference between our theories of the world or what we say about the world and the world itself that fallibility is built into the core of his position. Realism doesn't mean one holds they have special access to the world, that they know all truths, or that they have the truth in hand, only that there are truths to be known and that we can be mistaken about things. Circling **Squares Philip** has a post responding to my quandaries about how to mesh realism and pluralism. He writes: Ontologically and metaphysically the idea of realist pluralism is no longer an issue. There are (appropriately) numerous variants but the basic idea that reality is itself pluralistic is well established. The question is political-discursive. It's what Stengers and Latour are getting at with their concepts of diplomacy and cosmopolitics. They grant, first, that all entities exist and, second, that to say that someone's cherished idol (or whatever disputed entity they hold dear) is non-existent is a **'declaration of war'** – 'this means war,' as Stengers often says. They thus shunt onto-political discourse off of the terrain of knowledge/belief in the sense of existence/non-existence. Their basic claim seems to be that 'respect for otherness,' i.e. political pluralism, can only come from granting the entities that others hold dear an ontology, even if you don't 'believe' in them. You are thus permitted to say 'I do not follow that god, he has no hold over me' but you are not permitted to say 'your god is an inane, infantile, non-existent fantasy, grow up.' And it's not just a question of politeness (although there's that too). The point is to grant others' idols and deities an existence – one needn't agree over what that existence entails, over what capacities that entity has or what obligations it impresses upon you as someone in its partial presence but to deny it existence entirely is to 'declare war' – to deny the possibility of civil discourse, of pluralistic co-existence. I believe that it was Richard Rorty who once quipped something like claims to reconcile realism and idealism always seem to end with a triumph of idealism. We don't, in fact, get realism through such approaches, but rather just get a pervasive anti-realism. I think this is also the problem with the "non-controversial pluralism" advocated by Stengers and Latour that Phillip defends here. Such pluralism is not realism but is, in fact, a thoroughgoing social constructivism. I think this is the central problem with Latour's argument in Irreductions (these days I regret having ever defended it). In rejecting both Enlightenment critique and what he calls "reduction" he wants to say something like "The Pentacostal really is filled with the Holy Spirit", that for the 19th working scientist heat really is a fluid and phlogiston really is what allows things to burn, and that for the Greek lightning really is an expression of Zeus's anger. Latour tells us that we aren't to reduce or explain away the entities posited by another group's "ontology" but are to develop explanations from within that ontology. It might not sound particularly sexy– and it certainly doesn't tell us what is worth thinking –but I can't help but believe that philosophy is the critical and reflective investigation of basic concepts that guide our investigation of the world about us, how we ought to live our lives, and what form of governance might be best. Compare two figures. A scientist might ask, what causes depression? We can very well imagine a philosopher turning around and asking the scientist, what is causality? The scientist presupposes a concept of causality in her investigations. She uses this concept in her inquiry. Now she might have a sophisticated concept of causality or she might never have thought much about causality at all, using it in the sort of colloquial and unreflective way that Plato decried when, for example, people like Euthyphro talked about piety. A whole cascade of questions arise when we raise a question like "what is causality?" We can ask whether or not causality exists at all. We can ask how we distinguish between correlation and two events that merely accompany one another from genuine causation. This, for example, was Hume's question. But perhaps most importantly we can ask whether there is only one form of causality or many forms of causality. Is there only one-to-one causation; one cause and one effect? Is there many-to-one causation; or many events conspiring to produce an effect? Is there one-to-many causation; or one event producing a variety of different effects? We can even ask whether causality necessarily moves from past to present or whether there aren't

forms of causality that move from future to past! **January 23, 2014 Pluralism and Realism:** multiple-worlds Over at Struggles Forever, Jeremy Trombley has an interesting post up on "the ontological turn" in anthropology or ethnography. I've been meaning to have a discussion with him about this as I think it's an issue many of us are struggling with. For example, the core project of The Democracy of Objects– a proj ect which I think many have missed –is to somehow reconcile some version of social constructivism with a realist ontology capable of making room for ecology (which requires realist and materialist positions as there's a fact of the matter where global warming is concerned) as well as the role played by objective agencies in social assemblages such as technologies, infrastructure, features of geography, local climates, the growth cycles of plants and animals, waste, etc. Maybe we can try to organize some cross-blog event to discuss these issues. I certainly think they're close to the heart of Jeremy, Michael of Archive Fire, Arran James, and a host of others. As an aside, I'm beginning to realize how the different sites of the political I've been outlining– semiopolitics, thermopolitics, oikopolitics (political economy), geopolitics, eropolitics (the politics of sex and desire), biopolitics, and chronopolitics (and I'm sure there are other political sites!) –are drawing me away from traditional Marxism. Assuming that classical Marxism holds that economics or the conditions and relations of production are determinative of all other sites of the political, the various sites of the political that I've been outlining would lead to the conclusion that there is not one determinative base of the political. This would not require committing Marx to flames, but rather of recognizing the phenomenon of overdetermination, or of a variety of different entangled sites of the political. But I digress. First, I find myself wondering what the ontological turn means in ethnography. Is it 1) the investigation of the different ontologies held/proposed by different cultures? E.g., the Aztecs believed that reality was structured in this way, while the Greeks in that way, and the ancient Chinese this way, etc? Or 2) is it an investigation of how real entities– independent of cultural beliefs –influence cultural formations? Or is it a combination of both? A position that I would favor. January 20, 2014 Thoughts on the Social and Political Implications of Correlationism .Lion Mirror Truth be told, as my thought has evolved the issue of correlationism had fallen off the radar for me. Somehow the debate had come to seem too "philosophical" to me, too "scholastic", too remote from what interests me: understanding why social assemblages are organized as they are, how power functions in social assemblages, and what we might do to address that power and change things. Somehow the question of whether or not we can get out of the correlation between thinking and being just came to seem remote from these sorts of issues. Somehow it seemed too epistemological. ASIDE: Numerous discussions over the years have led me to believe that the debate over correlationism is poorly understood (or maybe I just don't understand it). On countless occasions I've heard people say "of course we must relate to things in order to know them." Well yeah, of course! I don't think this is what the critic of correlationism is getting at. It seems to me that correlationism is something more robust than the theses that we must relate to something to know it. Correlationism instead seems to require the theses that thought and being are indiscernible. Put more concretely, the correlationist is someone who argues that we either a) can never tell whether being is merely a construction of our thought (weak correlationism), or b) who argues that thought actually constructs being (strong correlationism). In other words, correlationism is another name for idealism. One can hold that we must relate to something in order to know it without being a correlationist. As an aside I should also add that I am a correlationist about some things. For example, I think money is something constructed by society and is therefore a strong correlationist when it comes to money. At any rate, for a long time I'd become rather indifferent to debates about correlationism and philosophies of access. I had learned the lessons of speculative realism– which I could have also learned, I think, from Deleuze and Guattari, the new materialist feminists, actor-network theorists such as Latour –and had moved on. However, occasionally you come across a tone of phrase that pitches something in a different light. In The Cut of the Real, Katerina Kolozova writes, …the political problem of contemporary philosophy identified by the 'new realists' is, in fact, the product of a more fundamental epistemic problem. In his book After Finitude, Quentin Meillassoux calls this problem 'correlationism' and identifies it as an essentially post-Kantian legacy, which continues to dominate and limit philosophy. As a matter of fact, correlationism lies at the heart of postmodern theory and consists in the premise that thought can only 'think itself,' that the real is inaccessible to knowledge and human subjectivity, and that there is nothing but

discursive constructs that fully determine thinking and that are meth0dologically accounted for all the way down. (1 – 2) Thought thinking only itself: Thought only encountering itself. In the jargon of postmodern and poststructuralist lingo, this would be the thesis of infinite semiosis, where signs ("thoughts") only ever relate to other signs. Within this framework, discursivity comes to be the hegemonic framework defining all of being. At the level of politics and social theory more generally, if the correlationist thesis is true the consequences are clear: all social phenomena are discursive and all solutions to social and political problems will be discursive. The sole sphere of the political will be the discursive and all questions of politics will be questions of speech-acts and interpretation. The problem here is not that many theorists recognize that the discursive and semiotic plays an important role in the social and the political. It does and I've repeated this tirelessly. The problem is with what happens when thought or the semiotic becomes a hegemon, an "all", foreclosing our ability to recognize other forms of power. What I've wanted to say is that not all power functions discursively. In my last post and elsewhere I spoke of some other forms of politics: Thermopolitics: The politics surrounding **energy in the form of calories and fuels such as gasoline** and coal, and how our life and our very bodies are structured by energy dependencies and by being trapped in particular distributive networks that render these forms of energy available. I'm being quite literal when I speak of energy, talking about the effects, for example, of the absence of food in certain educational environments on cognition, for example; and am generally hostile to metaphorical extensions of the concept of energy which I see as erasing the dimension of real materiality. Geopolitics: The role that features of natural and built geography such as mountain ranges, rivers, oceans, soil conditions, roads, housing design, etc., play in the form that social relations take and how they impact individual bodies. Chronopolitics: The way in which the structurations of time organize what is possible for us. For example, the structuration of the working day, how much we can say and comprehend at any given time, the impact of things like the invention of the clock, etc. **Oikopolitics: This would be the domain of political economy described so well by Marxists**. So five different types of politics: Semiopolitics (or what currently dominates critical theory), thermopolitics, geopolitics, chronopolitics, and oikopolitics. No doubt there are other sites of the political or political struggle that we could speak of, but this is a good start. Also, it should be obvious that these aren't exclusive domains, but are entangled in all sorts of important ways. For example, something might take place at the level of semiopolitics (speech, law, rhetoric, norms, and communication) that has all sorts of impact at the level of thermopolitics. Congress might decide to cut programs that fund school meal programs. This, in turn, will have a thermodynamic impact on those students that go without the calories they need developmentally and cognitively to function in a particular way. There is an entanglement here of semiopolitical and thermopolitical domains. The young student here has been constrained both at the level of **semiotic phenomena and thermodynamic structures**. The point is that if true, semiotic intervention (speech-acts, protests, interpretations, deconstructions, etc) will not be an appropriate response to all political problems because social formations are not entirely structured by the semiotic. The child in that school does not suffer from a lack of the right signs, but from a lack of calories needed to run the engine of his thought and body. Certainly semiotic interventions might be needed to render that energy available, but it is the energy itself that is at issue and the absence of that energy that forms the spider web entangling him in his position. A correlationist perspective tends to erase this as even being a site of the political. **Larval Subjects January 28, 2014 Ontological Anarché Posted by larval subjects under Uncategorized January 27, 2014 Roden on Pluralism Over at enemy industry Roden has an excellent post up on the pluralism discussion. Check it out here. January 25, 2014 Different Senses of Pluralism and Ontology Posted by larval subjects under Uncategorized Responding to one of my comments over at his blog, Jeremy Trombley:** I have deleted restricted, reformulated some sentences .Kindly bear with me. Intention is to make the discursive enucleation consistent with the subject in question**.** Recourse to epistemology of the quantum mechanics generic and fervor of string theory is a sine qua non for the further comprehension of the theory and ontology .How would the world appear to us if its ontology was that of classical mechanics but every agent faced a restriction on how much they could come to know about the classical state? We show that in most respects it would appear to us as quantum. The statistical theory of classical mechanics, which specifies how probability distributions over **phase space evolve** under Hamiltonian evolution and under measurements, is typically

called Liouville mechanics, so the theory we explore here is Liouville mechanics with an epistemic restriction. The particular epistemic restriction we posit as our foundational postulate specifies two constraints. The first constraint is a classical analog of Heisenberg's uncertainty principle; the second-order moments of position and momentum defined by the phase-space distribution that characterizes an agent's knowledge are required to satisfy the same constraints as are satisfied by the moments of position and momentum observables for a quantum state. The second constraint is that the distribution should have maximal entropy for the given moments. Starting from this postulate, authors derive the allowed preparations, measurements, and transformations and demonstrate that they are isomorphic to those allowed in Gaussian quantum mechanics and generate the same experimental statistics. They argue that this reconstruction of Gaussian quantum mechanics constitutes additional evidence in favor of a research program wherein quantum states are interpreted as states of incomplete knowledge and that the phenomena that do not arise in Gaussian quantum mechanics provide the best clues for how one might reconstruct the full quantum theory. I: http://dx.doi.org/10.1103/PhysRevA.86.012103 **Reconstruction of Gaussian quantum mechanics from Liouville mechanics with an epistemic restriction Phys. Rev. A 86, 012103 – Published 10 July 2012 Stephen D. Bartlett, Terry Rudolph, and Robert W. Spekkens.** Deleuze in fact characterizes modes of existence, with their powers and capacities. The answer is this: Deleuze approaches modes of existence, ethically speaking, not in terms of their will, or their conscious decision making power (as in Kant), nor in terms of their interests (as in Marx, for example), but rather in terms of their drives. For Deleuze, conscious will and preconscious interest are both subsequent to our unconscious drives, and it is at the level of the drives that we have to aim our ethical analysis. Here, I would like to focus on two sets of texts on the drives taken, not from Nietzsche and Spinoza, but rather from Nietzsche and Leibniz (Leibniz being one of the first philosophers in the history of philosophy to have developed a theory of the unconscious). The first set of texts comes from Nietzsche's great early book entitled Daybreak, published in July 1881. Nietzsche first approaches the question of the drives by giving us an everyday scenario: "Suppose we were in the market place one day," he writes, "and we noticed someone laughing at us as we went by: this event will signify this or that to us according to whether this or that drive happens at that moment to be at its height in us—and it will be a quite different event according to the kind of person we are. One person will absorb it like a drop of rain, another will shake it from him like an insect, another will try to pick a quarrel, another will examine his clothing to see if there is anything about it that might give rise to laughter, another will be led to reflect on the nature of laughter as such, another will be glad to have involuntarily augmented the amount of cheerfulness and sunshine in the world—and in each case, a drive has gratified itself, whether it be the drive to annoyance, or to combativeness or to reflection or to benevolence. This drive seized the event as its prey. Why precisely this one? Because, thirsty and hungry, it was lying in wait" (D 119). This is the source of Nietzsche's doctrine of perspectivism ("there are no facts, only interpretations"), but what is often overlooked is that, for Nietzsche, it is our drives that interpret the world, that are perspectival—and not our egos, not our conscious opinions. It is not so much that I have a different perspective on the world than you; it is rather that each of us has multiple perspectives on the world because of the multiplicity of our drives—drives that are often contradictory among themselves. "Within ourselves," Nietzsche writes, "we can be egoistic or altruistic, hard-hearted, magnanimous, just, lenient, insincere, can cause pain or give pleasure" (Parkes, pp. 291-292). We all contain such "a vast confusion of contradictory drives" (WP 259) that we are, as Nietzsche liked to say, multiplicities, and not unities. Moreover, these drives are in a constant struggle or combat with each other: my drive to smoke and get my nicotine rush is in combat with (but also coexistent with) my drive to quit. This is where Nietzsche first developed his concept of the will to power—at the level of the drives. "Every drive is a kind of lust to rule," he writes, "each one has its perspective that it would like to compel all the other drives to accept as a norm" (WP 481). DELEUZE AND THE QUESTION OF DESIRE: TOWARD AN IMMANENT THEORY OF ETHICS Daniel W. Smith. **In on a Possible Physical Metatheory of Consciousness Miroljub Dugic et al** show that the modern quantum mechanics, and particularly the theory of decoherence, allows formulating a sort of a physical metatheory of consciousness. Particularly, the analysis of the necessary conditions for the occurrence of decoherence, along with the hypothesis that consciousness bears (more-or-less) well definable physical origin, leads to a wider physical picture naturally involving consciousness. This can

be considered as a sort of a psycho-physical parallelism, but on very wide scales bearing some cosmological relevance. Open Systems & Information Dynamics Volume 9, Number 2, 153 (2002) arXiv: quant-ph/0212128. On a broader perspective and background, the theoretical computation of the universe is justified.IN physics and cosmology, digital physics is a collection of theoretical perspectives based on the premise that the universe is, at heart, describable by information, and is therefore computable. Therefore, according to this theory, the universe can be conceived of as either the output of a deterministic or probabilistic computer program, a vast, digital computation device, or mathematically isomorphic to such a device. Digital physics is grounded in one or more of the following hypotheses; listed in order of decreasing strength. The universe or reality: is essentially informational (although not every informational ontology needs to be digital); is essentially computable (the pancomputationalist position); can be described digitally; is in essence digital; is itself a computer (pancomputationalism); is the output of a simulated reality exercise. Following Jaynes and Weizsäcker, the physicist John Archibald Wheeler wrote the following: [...] it is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer. (John Archibald Wheeler 1998: 340) It from bit. Otherwise put, every 'it'—every particle, every field of force, even the space-time continuum itself—derives its function, its meaning, its very existence entirely—even if in some contexts indirectly—from the apparatus-elicited answers to yes-or-no questions, binary choices, bits. 'It from bit' symbolizes the idea that every item of the physical world has at bottom—a very deep bottom, in most instances—an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes–no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe. (John Archibald Wheeler 1990: 5) David Chalmers of the Australian National University summarised Wheeler's views as follows: Wheeler (1990) has suggested that information is fundamental to the physics of the universe. According to this 'it from bit' doctrine, the laws of physics can be cast in terms of information, postulating different states that give rise to different effects without actually saying what those states are. It is only their position in an information space that counts. If so, then information is a natural candidate to also play a role in a fundamental theory of consciousness. We are led to a conception of the world on which information is truly fundamental, and on which it has two basic aspects, corresponding to the physical and the phenomenal features of the world. (Wikipedia). **Participatory consciousness forms the bastion, pillar, post, stylobate and fulcrum of the redressal measures and corrective steps taken for circumvention of many problems, identified and unidentified, spread widest commonalty. Anne Baring's sententious and pithy article will be good to have a timely outlook.** I would like to open my contribution to this meeting with the words of Ken Wilber that I quoted in the first Millennium Symposium: 'The secret impulse of life is towards greater consciousness. Maybe the evolutionary sequence is from matter to body, to mind, to soul, to spirit - each transcending and including, each with a greater depth and greater consciousness and wider embrace' Ken Wilber, and with these from Rupert Sheldrake's contribution to the same meeting: "It is possible to make quite a good case that the sun could be conscious and if the sun, then why not the stars? Why not the galaxies? Why should there not be galactic minds? And if they communicate with each other then the distances are far too great for light to be the medium of transmission. We are into inter-galactic telepathy." After many decades of study, I know of only one diagram in all religious traditions that can adequately explain these statements; that can give us both a blue-print of the many levels of consciousness and a route-map to help us reach beyond our present level of understanding. I would like to introduce a diagram of the four worlds of Kabbalah to illustrate the theme that spirit has brought these four worlds into being and that we, living in the fourth and manifest world know nothing of the existence of the other three invisible ones. It is the clearest diagram I know of that can explain to us the multi-levelled structure of consciousness and our place in the great chain of being. We are indeed, as Wilbur suggests, evolving from matter to body, to mind, to soul, to spirit but where have we come from? It is possible that, as participants in the emanation of spirit into manifest life, we have also come from spirit to soul, to mind, to body, to matter. Everything, from the immense galactic energies to the minute forms of matter participates in one life, is generated from one creative spirit. There is nothing outside this life, this spirit. This diagram suggests that truth is not an article of faith but the experience of participating in the undiscovered reality of what has brought us into being. We cannot know truth until

we have entered into communion with that reality. Continuing in similar vein **Michel Bitbol dilates upon the dialectic deliberation and polemical argumentation between** hermeneutists and eliminativists. When he formulated the program of Neurophenomenology, Francisco Varela suggested a balanced methodological dissolution of the "hard problem" of consciousness. He shows that his dissolution is a paradigm which imposes itself onto seemingly opposite views, including materialist approaches. I also point out that Varela's revolutionary epistemological ideas are gaining wider acceptance as a side effect of a recent controversy between hermeneutists and eliminativists. Finally, he emphasizes a structural parallel between the science of consciousness and the distinctive features of quantum mechanics. This parallel, together with the former convergences, point towards the common origin of the main puzzles of both quantum mechanics and the philosophy of mind: neglect of the constitutive blindspot of objective knowledge. Phenomenology and the Cognitive Sciences Phenomenology and the Cognitive Sciences **Phenomenology and the Cognitive Sciences 2002, Volume 1, Issue 2, pp 181-224 Science as if situation mattered Michel Bitbol.** Caroline Williams investigated affective density of the political and its effect on our understanding of political subjectivity. Taking up Spinoza's challenge to think about affect beyond corporeal embodiment, he argues that there is a modality of affectivity that cannot simply be inscribed within the borders of subjectivity. theorising affect as an impersonal force anchored in a relational ontology that gives due recognition to the circulation of affects, as well as to their ambivalent structure in creating sites of identification, and he utilises this ontology to reflect on the dynamic of the political and the shape of political subjectivity. Williams argues that Spinoza's philosophy (through ideas of conatus and imagination) offers the conceptual resources to reconfigure the composition of affective subjectivity as a transindividual social bond and as an unconscious dynamic of ethico-political existence. **Subjectivity (2010) 3, 245–262. doi:10.1057/sub.2010.15 Affective processes without a subject: Rethinking the relation between subjectivity and affect with Spinoza Caroline Williams** Phenomenological Time bears ample evidence and infallible observatory and apodictic evidence albeit with a sense of chagrined circumvention and defeated disgruntlment circularity of both psychological and phenomenological interpretations of art which are sometimes insightful, but most often work on the logic of cyclical time and unavoidable drives and representations played out as a 'natural' order: where 'men can't avoid being boys' a circularity which valorizes a helplessness/truthfulness conundrum. One begins with a classical phenomenological fallacy with Duchamp's Fountain, does the urinal keep returning or do we keep returning to the urinal? Put in this way, it is a male question, something that phenomenology conveniently conceals when positing the "I", or indeed, the "we". But we must be careful not to simply assume that the phenomenological first person is white, heterosexual, able-bodied and male, providing a foundation upon which knowledge should be based as a universal condition. Deleuze's second critique of time is linked to Kant. This model of time extends not only to phenomenology but to the phenomenological interpretation of art and is seen as a straight line. In the Critique of Pure Reason, Kant cuts up circular time and reassembles it as a series of sensory experiences which memory processes and which constitute subjectivity. Whereas the circular model of time (and interpretation) inscribes fatalism into the subject and the world, the phenomenological method is in danger of solipsism, where the subject constitutes the world. Much of embodied philosophy and enactivist theories of cognition go down this line with a plethora of art historical analyses by the likes of Rosalind Krauss, Michael Fried and others taking the lead from Merleau-Ponty. These approaches successfully challenge the **homunculus model of a Cartesian mind-body** dualism where there is a supervisor in the mind who processes experience and even consciousness and to whom all representations are directed. But phenomenology is mainly concerned with perceptual consciousness, the processing of colors, shapes, forms, motion in art and the world. The senses and sensorimotor processes are the fundamental pathways by which the interior is connected to the exterior in a chiasm of self-constitution in the world. The problem with this is that we explore the art through our sense perceptions, and what does art reveal to us?—our sense perceptions! Such solipsistic approaches have real problems relating complex and abstract conceptual production to bodily and sensorimotor contingencies, and they nearly always posit the subject as the site and object of discovery in the world. To be fair, phenomenology has moved on to rather more sophisticated models of intersubjectivity, taking into account the specificities of different kinds of bodies and ways of being beyond the

white, male heterosexual as the common denominator, yet many naïve approaches to art perpetuate the notion that perceptual, rather than conceptual processes and evolutionary neural patterns are primary, fundamental ways to understand the art experience In fact, these aspects are only part of art production and reception, aspects which are often filtered out or made known to us in order for us to parody such trigger responses, as in much of Duchamp's work, which holds in contempt such retinal approaches to art. If we take the **subject of abstract expressionism** in the 1950s or later minimalist art, art history, teaching and interpretation still insists on putting the act of perceiving as fundamental to the constitution not only of the phenomenological first person but also of art itself. The male act of usually standing up, which consists of directing a stream with the hand(s) into a container or on a flat surface, sand or snow in order to leave patterns and traces (the usual child's prank) underlies the logic of macho 1950s Abstract Expressionist culture, especially that of Jackson Pollock, so effectively parodied by Carolee Schneeman's Vagina Scroll, 1975. Here, painting or writing is a kind of a discharge which invests the **object and space with** the phenomenological "I", disciplined by existential struggle and reflected back by the traces, signatures and gestures left behind in the work of art, rediscoverable as an element in the making of other subjectivities.It is precisely this kind of solipsism which is blind to the art event as something well in excess of re-establishing habits of subjective self-identity through the production and interpretation of art. The attempt to reinforce a phenomenological identity upon the event by personalizing sense experience is unable to see art as something which remodels the phenomenological 'I' and neural plasticity itself. We learn new technologies and redraw our neural pathways through such new and spontaneous experiences which draw us into intersubjective worlds. Duchamp's Fountain and Deleuze's Repetition and Différence Grégory Minis sale (I have made some changes) The relation of different groups to deconstruction is settled by its relation to groups. An American enthusiast--one who has had a role in the development of our affection for Derrida--notices that the motivational chains are phobically driven by concerns with social scale. Richard Rorty: "He wants to figure out how to break with the temptation to identify himself with something big. . . ."(10) And: "So I take Derrida's importance to lie in his having had the courage to give up the attempt to unite the private and the public, to stop trying to bring together a quest for private autonomy and an attempt at public resonance and utility. He privatizes the sublime, having learned from the fate of his predecessors that the public can never be more than beautiful."(11) Notice is made here of what is indeed the telling feature. Levinas is praised, for example, for his alarm when "the social will [is] sought in an ideal of fusion . . . the subject losing himself in a collective representation, in a common ideal. . . . It is the collectivity which says 'us,' and which, turned toward the intelligible sun, toward the truth, experience, the other at his side and not face to face with him. The tellingly titled D'un ton apocalyptique contains passages that are particularly excited: These people situate themselves outside the ordinary, but they have in common this: they describe themselves as having an immediate and intuitive relation with mystery. And they want to attract, to seduce, and lead others to the mystery, through mystery. This agogic function of the leader of men, of the duce, of the Führer, of the leader, places him above the crowd that he manipulates with the aid of a small numbers of adepts joined together in a sect with a secret language, a clique or a small party with its ritualized practices. The mystifiers pretend to have exclusive access to the privilege of a secret mystery. . . . The revelation or the unveiling of the secret is something that they jealously reserve for themselves. Jealousy is here a major characteristic. There is not narcissism and non-narcissism; there are narcissisms that are more or less comprehensive, generous, open, extended. What is called non-narcissism is in general but the economy of a much more welcoming, hospitable narcissism, one that is much more open to the experience of the other as other. I believe that without a movement of narcissistic reappropriation, the relation to the other would be absolutely destroyed; it would be destroyed in advance. The relation to the other--even if it remains asymmetrical, open, without reapporpriation--must trace a movement of reappropriation in the image of oneself for love to be possible, for example. Love is narcissistic. Beyond that, there are little narcissisms, there are big narcissisms. . . .(23) **Justice of the Pieces Deconstruction as a Social Psychology Douglas Collins** Sabine Wilke cogently argues for the rise of the aesthetic dimension in philosophical writing by showing how the metaphorical and poetic style of Heidegger, the principles of phrase configurations with Adorno, or the topographic writing of derrida replace through their very form something what traditionally would have been subject to discursive exposure The concern

shifts from telling to showing, showing first and foremost that something remains untold. An exemplary book in this aesthetic drive is Avital Ronell's Telephone Book where the playful layout and explicit break with traditional design interrupt the sequential logic of the paragraphs thereby performing the very argument at stake: open up new circuits of meaning and signification. The engagement with form and publishing formats is only consequential since media theory as exposed by Walter J. Ong, Eric A. Havelock, Parry Miman or Marshall McLuhan besides many others made it clear that different communication technologies condition the mode of thought of a given culture The fractals of Benoit Mandelbrot or the butterfly shaped Lorenz attractor are the popular icons of this turn in the sciences. For communication studies and philosophy, Vilém Flusser was one of the pioneer thinkers reflecting the rise of what he called "techno-images." The photograph is the paradigmatic case which condenses the whole crises of Western thought and culture on its grainy surface. It subverts the linear logic of writing.6 Flusser's central diagnosis detects a shift from the grand narratives associated with the literate mind and Modernity towards creating and envisioning the world through image based practices. Writing himself in a succinct, visually inspired style, Flusser contents that the web of meaning, spun throughout the centuries by our textual practices, has become transparent. We discover the void between the letters through which the world escapes: "We are alienated from the world of text, precisely because we see through them as our own product."7 As a consequence we are confronted with a world of dissociated facts and information bits. Flusser hails this fragmented world as the advent of the zero dimensional point-universe that will provide the new raw material for our image based practices. Techno-images are nothing else than a conscious, inherently artistic drawing-together of point-elements as exemplified first and foremost by the silver-grains of the photograph but also by more recent pixel based techno-images. Similar to blueprints, designs or statistical curves, the weather map for example visualizes and contracts a tremendous amount of meteorological data from widely distributed stations into a single image of high- and low pressure
areas, temperatures, wind speeds, etc. Techno-images "compute" in this sense the universe of unimaginable, abstract information-bits into a concrete surface – the image. Echoing Paul Feyerabend, these image practices have to be understood as creative practices where the truth of scientific models has been replaced by its aesthetic quality **Aesthetics, Writing, Networked Computers Jörg Muller Dissertation Submitted to the Division of Media and Communications of The European Graduate School in Candidacy for the Degree of Doctor of Philosophy**
Lothar Schäfer (1) and Sisir Roy (2) in Quantum Reality, the Importance of Consciousness in the Universe, the Discovery of a Non-Empirical Realm of Physical Reality, and the Convergence with Ancient Traditions of Indian and Western Philosophy investigate metaphysical aspects that include 1) the discovery of a non-empirical part of physical reality in a realm of potentiality; 2) the emanation of the empirical world out of a realm of non-material forms; 3) the discovery that the nature of physical reality is that of an indivisible Wholeness – the One; and 4) panpsychism: the possibility that the One is aware of its processes like a Cosmic Consciousness. The convergence of powerful traditions of seemingly disparate cultures is particularly important to point out in the present process of globalization, when a unifying view is needed to avoid controversy and conflict. To quote them in extenso : This is the nonlocality of the quantum world. Menas Kafatos and Robert Nadeau (1990) have drawn a remarkable conclusion from this phenomenon: If reality is nonlocal, the nature of the universe is that of an undivided wholeness. Because our consciousness has emerged from this wholeness and is part of it, it is possible to conclude that an element of consciousness is active in the universe: a Cosmic Consciousness. Pre-reflective self-consciousness is pre-reflective in the sense that (1) it is an awareness we have before we do any reflecting on our experience; (2) it is an implicit and first-order awareness rather than an explicit or higher-order form of self-consciousness. Indeed, an explicit reflective self-consciousness is possible only because there is a pre-reflective self-awareness that is an on-going and more primary self-consciousness. Although phenomenologists do not always agree on important questions about method, focus, or even whether there is an ego or self, they are in close to unanimous agreement about the idea that the experiential dimension always involves such an implicit pre-reflective self-awareness. In line with Edmund Husserl (1959, 189, 412), who maintains that consciousness always involves a self-appearance (Für-sich-selbst-erscheinens), and in agreement with Michel Henry (1963, 1965), who notes that experience is always self-manifesting, and with Maurice Merleau-Ponty who states that consciousness is always given to itself and that

the word 'consciousness' has no meaning independently of this self-givenness (Merleau-Ponty 1945, 488), Jean-Paul Sartre writes that pre-reflective self-consciousness is not simply a quality added to the experience, an accessory; rather, it constitutes the very mode of being of the experience: This self-consciousness we ought to consider not as a new consciousness, but as the only mode of existence which is possible for a consciousness of something (Sartre 1943, 20 [1956, liv]). The notion of pre-reflective self-awareness is related to the idea that experiences have a subjective 'feel' to them, a certain (phenomenal) quality of 'what it is like' or what it 'feels' like to have them. As it is usually expressed outside of phenomenological texts, to undergo a conscious experience necessarily means that there is something it is like for the subject to have that experience (Nagel 1974; Searle 1992). This is obviously true of bodily sensations like pain. But it is also the case for perceptual experiences, experiences of desiring, feeling, and thinking. There is something it is like to taste chocolate, and this is different from what it is like to remember what it is like to taste chocolate, or to smell vanilla, to run, to stand still, to feel envious, nervous, depressed or happy, or to entertain an abstract belief. Yet, at the same time, as I live through these differences, there is something experiential that is, in some sense, the same, namely, their distinct first-personal character. All the experiences are characterized by a quality of mineness or for-me-ness, the fact that it is I who am having these experiences. All the experiences are given (at least tacitly) as my experiences, as experiences I am undergoing or living through. All of this suggests that first-person experience presents me with an immediate and non-observational access to myself, and that consequently (phenomenal) consciousness consequently entails a (minimal) form of self-consciousness. To put it differently, unless a mental process is pre-reflectively self-conscious there will be nothing it is like to undergo the process, and it therefore cannot be a phenomenally conscious process. The mineness in question is not a quality like being scarlet, sour or soft. It doesn't refer to a specific experiential content, to a specific what; nor does it refer to the diachronic or synchronic sum of such content, or to some other relation that might obtain between the contents in question. Rather, it refers to the distinct givenness or the how it feels of experience. It refers to the first-personal presence or character of experience. It refers to the fact that the experiences I am living through are given differently (but not necessarily better) to me than to anybody else. It could consequently be claimed that anybody who denies the for-me-ness of experience simply fails to recognize an essential constitutive aspect of experience. Such a denial would be tantamount to a denial of the first-person perspective. It would entail the view that my own mind is either not given to me at all — I would be mind- or self-blind — or is presented to me in exactly the same way as the minds of others. There are also lines of argumentation in contemporary analytical philosophy of mind that is close to and consistent with the phenomenological conception of pre-reflective self-awareness. Alvin Goldman provides an example: [Consider] the case of thinking about x or attending to x. In the process of thinking about x there is already an implicit awareness that one is thinking about x. There is no need for reflection here, for taking a step back from thinking about x in order to examine it…When we are thinking about x, the mind is focused on x, not on our thinking of x. Nevertheless, the process of thinking about x carries with it a non-reflective self-awareness (Goldman 1970, 96). A similar view has been defended by Owen Flanagan, who not only argues that consciousness involves self-consciousness in the weak sense that there is something it is like for the subject to have the experience, but also speaks of the low-level self-consciousness involved in experiencing my experiences as mine (Flanagan 1992, 194). As Flanagan quite correctly points out, this primary type of self-consciousness should not be confused with the much stronger notion of self-consciousness that is in play when we are thinking about our own narrative self. The latter form of reflective self-consciousness presupposes both conceptual knowledge and narrative competence. It requires maturation and socialization, and the ability to access and issue reports about the states, traits, dispositions that make one the person one is. To claim that every kind of self-consciousness is conceptual is overly cognitive. Bermúdez (1998), to mention one further philosopher in the analytic tradition, argues that there are a variety of **nonconceptual forms of self-consciousness** that are "logically and ontogenetically more primitive than the higher forms of self-consciousness that are usually the focus of philosophical debate" (1998, 274; also see Poellner 2003). This growing consensus across philosophical studies supports the phenomenological view of pre-reflective self-consciousness. (References: Stanford Encyclopedia, Post Modersnist Theories, Internet Encyclopedia of Philosophy) Korzybski pointed out that there are

two ways to slice easily through life; to believe everything or to doubt everything. Both ways save us from thinking. So, from time to time you have to give yourself a Virtual Reality Check. The hallmark of postmodern philosophy has been disbelief or skepticism of all "metanarratives," or translations of reality. Postmodernism has even turned its profound skepticism on such important humanist concepts as "objective truth" and reason.Yet, for a deconstructionist postmodern society, individually we are still riddled with superstition and gullibility, and open to manipulation through our belief systems as any politician, philosopher, clergy, or salesperson will attest. Further, most people are painfully naive when it comes to even the simplest scientific understanding. Most of us don't have a clue about the fundamental nature of physical reality or our own psychological nature, and our ability to be fooled by our senses and mind, or methods of social and technological persuasion. We may think we know, but the penetration is deeper than we dare imagine. Nature rejects the naiveté that seeks absolute truth. We are beginning to realize, individually and culturally, that "realities" are all human constructions. The task becomes one of "catching ourselves in the act" of creating our own "reality" from the flow of events. Human truth is always an engagement of mind with experience.. We don't need to fear the collapse of our personalistic belief system (the "box" we live in), nor our belief in absolute truth. **A strong desire to engage in the "quest for uncertainty" complements our anxiety that perhaps there is no absolute, objective ground to reality**. The warrant of Truth is ever elusive when we deconstruct the foundational justifications of our convenient notions about the way the world works. It is easy to confuse what is actually the creation of beliefs with the "discovery of Truth," a common goal of science and theology. Science offers no ontological Scientific Picture of the World but substitutes a number of applied theories as meta-theory. It's close, powerful and useful: it 'works' and allows use to do work. But the still mysterious "true" metatheory may reveal the basis of consciousness. But who knows when we will lift that veil? Permeating the living reality of our culture are certain contagious notions, fads and trends that have the ability to influence the way we think about the nature of Reality and ourselves. Some of them are toxic and can consume you; others are just whims of pop culture infectious and benign as the common cold. **Jung** described the concept of psychic contagion by certain archetypal forces inherent in the human psyche, which manifest in our spiritual lives and belief systems. He spoke of both conscious and unconscious contamination. Notions like this range from simple superstitions to scientific concepts, to urban myths. Notions sweep through our culture and insinuate themselves within its fabric, as fads, whether they are "real" or not, they can be influential. They mood alter us, make us feel we belong. An analysis of these notions is useful in distinguishing a common human phenomenon from any potential "alien influence" which may or may not be exerted on us from an unknown source. **Strange Attractors: Transference, Holography, and an Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, and an Archetype (Doctoral dissertation, Pacifica Graduate Institute, 2003).** In consideration to the Nobel Proze for Chemistry this year(2014), a stability nalysis of the proble in to single molexues would be a contemporaneous question that has to be taken cognisace of and given importance to . Determination of molecular structure by geometry optimization became routine only after efficient methods for calculating the first derivatives of the energy with respect to all atomic coordinates became available. Evaluation of the related second derivatives allows the prediction of vibrational frequencies if harmonic motion is estimated. More importantly, it allows for the characterization of stationary points. The frequencies are related to the eigenvalues of the Hessian matrix, which contains second derivatives. If the eigenvalues are all positive, then the frequencies are all real and the stationary point is a local minimum. If one eigenvalue is negative (i.e., an imaginary frequency), then the stationary point is a transition structure. If more than one eigenvalue is negative, then the stationary point is a more complex one, and is usually of little interest. When one of these is found, it is necessary to move the search away from it if the experimenter is looking solely for local minima and transition structures. The total energy is determined by approximate solutions of the time-dependent Schrödinger equation, usually with no relativistic terms included, and by making use of the Born–Oppenheimer approximation, which allows for the separation of electronic and nuclear motions, thereby simplifying the Schrödinger equation. This leads to the evaluation of the total energy as a sum of the electronic energy at fixed nuclei positions and the repulsion energy of the nuclei. Notable exceptions are certain approaches called direct quantum chemistry, which treat electrons and nuclei on a common footing. Density

functional methods and semi-empirical methods are variants on the major theme. **Orlando Alvarez** discusses Polyakov's quantization of the string in the presence of a boundary allowing for an arbitrary topology for the world sheet. In addition to the dynamical conformal factor discovered by Polyakov, there are a finite number of new degrees of freedom if the surface is more complicated than a sphere or a disc. The quantization of the Liouville theory in an arbitrary topology is also discussed. A one-loop calculation shows that the model is renormalizable if one performs a mass renormalization and an additive field renormalization. The renormalization group equations have a perturbative infrared unstable fixed point in all topologies. Copyright © 1983 Published by Elsevier B.V. **Nuclear Physics B Volume 216, Issue 1, 25 April 1983, Pages 125–184 Theory of strings with boundaries: Fluctuations, topology and quantum geometry Orlando Alvarez DOI: 10.1016/0550-3213(83)90490-X** Quantum geometry (the modern loop quantum gravity involving graphs and spin-networks instead of the loops) provides microscopic degrees of freedom that account for black-hole entropy. However, the procedure for state counting used in the literature contains an error and the number of the relevant horizon states is underestimated. In our paper a correct method of counting is presented by **Marcin Domagala and Jerzy Lewandowski.** Results lead to a revision of the literature of the subject. It turns out that the contribution of spins greater than 1/2 to the entropy is not negligible. Hence, the value of the Barbero–Immirzi parameter involved in the spectra of all the geometric and physical operators in this theory is different than previously derived. Also, the conjectured relation between quantum geometry and the black-hole quasi-normal modes should be understood again. **Marcin Domagala and Jerzy Lewandowski 2004 Class. Quantum Grav 21 5233 doi:10.1088/0264-9381/21/22/014 Black-hole entropy from quantum geometry** Quantum geometry predicts that a universe evolves through an inflationary phase at small volume before exiting gracefully into a standard Friedmann phase. This does not require the introduction of additional matter fields with ad hoc potentials; rather, it occurs because of a quantum gravity modification of the kinetic part of ordinary matter Hamiltonians. Authors draw the cognizance of application of the same mechanism that can explain why the present day cosmological acceleration is so tiny. DOI: http://dx.doi.org/10.1103/PhysRevLett.89.261301 **Inflation from Quantum Geometry Phys. Rev. Lett 89, 261301 – Published 12 December 2002 Martin Bojowald** The loop quantum cosmology of the closed isotropic model is studied with special emphasis on a comparison with traditional results obtained in the Wheeler-DeWitt approach by **Martin Bojowald** .Investigation includes the relation of the dynamical initial conditions to boundary conditions such as the no-boundary or the tunneling proposal and a discussion of inflation from quantum cosmology. **DOI: http://dx.doi.org/10.1103/PhysRevD.67.124023 Loop quantum cosmology, boundary proposals, and inflation Phys. Rev. D 67, 124023 – Published 23 June 2003 Martin Bojowald and Kevin Vandersloot** In loop quantum cosmology, the universe avoids a big bang singularity and undergoes an early and short super-inflation phase. During super-inflation, non-perturbative quantum corrections to the dynamics drive an inflaton field up its potential hill, thus setting the initial conditions for standard inflation. **Shinji Tsujikawa et al** show that this effect can raise the inflaton high enough to achieve sufficient e-foldings in the standard inflation era. They also analyse the cosmological perturbations generated when slow-roll is **violated after super-inflation** and show that loop quantum effects can in principle leave an **indirect signature** on the largest scales in the CMB, with some loss of power and running of the spectral index. **Shinji Tsujikawa et al 2004 Class. Quantum Grav 21 5767 doi:10.1088/0264-9381/21/24/006 Loop quantum gravity effects on inflation and the CMB Quantum geometry from quantum information** is the study of **Florian Girelli and Etera R Livine .**Loop quantum gravity defines the quantum states of space geometry as spin networks and describes their evolution in time. We reformulate spin networks in terms of harmonic oscillators and show how the holographic degrees of freedom of the theory are described as matrix models. This allows us to make a link with non-commutative geometry and to look at the issue of the semi-classical limit of loop quantum gravity from a new perspective. This work is thought of as part of a bigger project of describing quantum geometry in quantum information terms. **Florian Girelli and Etera R Livine 2005 Class. Quantum Grav 22 3295 doi:10.1088/0264-9381/22/16/011 Reconstructing quantum geometry from quantum information: spin networks as harmonic oscillators.** Quantum geometric mechanics is the topic of study by **Dorje C. Brody Lane P. Hughston** establishing a clear relation between the quantum mechanics and quantum geometry

and also the entangled states. The manifold of pure quantum states can be regarded as a complex projective space endowed with the unitary-invariant Fubini–Study metric. According to the principles of geometric quantum mechanics, the physical characteristics of a given quantum system can be represented by geometrical features that are preferentially identified in this complex manifold. Here we construct a number of examples of such features as they arise in the state spaces for spin, spin 1, spin and spin 2 systems, and for pairs of spin systems. A study is then undertaken on the geometry of entangled states. A locally invariant measure is assigned to the degree of entanglement of a given state for a general multi-particle system, and the properties of this measure are analysed for the entangled states of a pair of spin particles. With the specification of a quantum Hamiltonian, the resulting Schrödinger trajectories induce an isometry of the Fubini–Study manifold, and hence also an isometry of each of the energy surfaces generated by level values of the expectation of the Hamiltonian. For a generic quantum evolution, the corresponding Killing trajectory is quasiergodic on a toroidal subspace of the energy surface through the initial state. When a dynamical trajectory is lifted orthogonally to Hilbert space, it induces a geometric phase shift on the wave function. The uncertainty of an observable in a given state is the length of the gradient vector of the level surface of the expectation of the observable in that state; a fact that allows us to calculate higher orders corrections to the Heisenberg relations. A general mixed state is determined by a probability density function on the state space, for which the associated first moment is the density matrix. The advantage of a general state is in its applicability in various attempts to go beyond the standard quantum theory, some of which admit a natural phase-space characterization. **Journal of Geometry and Physics Volume 38, Issue 1, April 2001, Pages 19–53 Geometric quantum mechanics Dorje C. Brody Lane P. Hughston DOI: 10.1016/S0393-0440(00)00052-8** Expansion (unmesa) of [consciousness] or the creative intuition {pratibha) is (=) [experienced] in the interval which divides two [moments] of differentiated perception (vikalpa) It is here that they arise and disappear. The Sastras and Agamas proclaim with reasoned argument that it is (=) free of thought-constructs {Nirvikalpa) and precedes (e) Path to Liberation all mental representations of any object. None can deny that a gap exists between perceptions insofar as two moments of thought are (=) invariably divided. This [gap] is (=) the undifferentiated unity of all the countless manifestations. Similarly, in the outer more objective sphere, where change consists (e) of the alterations in the configurations of manifest appearances (Abhasa\ the transition from one to (e&eb) another corresponds to (e&eb) a phase of pure luminosity that marks (eb) the beginning of one form and the end of another. The world of manifestation and differentiated perceptions (Yikalpa) thus extends from one Centre to (eb) the next. **What is meant again is the realisation or the evolution of individual consciousness or general ledger to be the cosmic general ledger is the sole criterion for the Satchidananda state where all dualities cease** Although it is never in fact divorced from (e) the subject who resides there, the ignorant fail (e) to grasp this fact and so, cut off from (e) the Centre, the world of objectivity becomes (=) for them the sphere of Maya. Bhagavatotpala quotes the Light of Consciousness amvitprakdsa): This ever pure experience (suddhanubhava) is variegated by (e) each form [revealed within it] Even so it remains unstained (nirmala) when moving to another. Just as a cloth which is naturally white, once dyed, cannot (e) change colour without [first] becoming white again, similarly the pure power of awareness, (citi) once coloured by (e) form, is pure [again] at the Centre where that form is (=) abandoned and from whence it proceeds to (e&eb) another. In his Essence of Vibration (Spandasarpdoha), K§Emaraja explains that the rise and fall of every individual perception in the field of awareness is (=) a specific pulsation of consciousness. This is exactly the individual general ledger transaction lifted to the transaction in cosmic general ledger. During The Initial Instant Of Perception, T Consciousness Is Manifestly Apparent And The Yogi, Participating In Its Plenitude, Observes The Outer World Without Being Attached To Any Particular Divine Body And Sacred Circle Of The Senses Singling It Out From Any Other, Like A Man Who Observes A City From A High Mountain Peak. He Sees The Outer World Reflected Within His Consciousness Free Of Thought-Constructs And So 'Stamps* The Outer On The Inner While Absorbing The Object And Means Of Knowledge In The Pure Subject Which Grasps Them As The Expansion Of His Own Nature. K§Emaraja Says: By Penetrating Into Bhairavimudra, The Yogi Observes The Vast Totality Of Beings Rising From, And Dissolving Into, The Sky [Of Conscious- Ness], Like A Series Of Reflections Appearing And Disappearing Inside A Mirror. Through The Practice Of Bhairavimudra The Yogi Realises That He Is The

Substratum Consciousness (Adhisfhatr) Which Both Underlies And Is The Essence Of All Things. He Discovers That Phenomena Have No Independent Existence Apart From Him And So Are, In This Sense, Void. At The Same Time, He Realises That Because All Things Are Consciousness, They Are Far From Unreal. He Views The Outer World Yet Sees It Not. Beyond Both Voidness And Non-Voidness He Penetrates Into The Supreme Abode (Pararn Padam) Of Siva's Consciousness. [The Powers Of The Senses] Endowed With The Attributes Of The Great Union [Between Subject And Object] Whose Form Is The Wakening Of Man's Spiritual Potential (Kuntfalinf), Fill [With Consciousness] The Outer Clatter Of Diversity (Bhedatfambara) Born Of Its Intense Power And Are Then Established In The Unobscured Abode Of The Void Of Consciousness To Shine [There] Eternally. Thus Residing Beyond Being And Non-Being, The Sole Protector Of The Unity Which Is Tranquil And Expanding [Consciousness], Whose Glory Is All-Embracing And Form Unobscured, Is Called Bhairavimudra. Through The Practice Of Bhairavimudra, The Yogi Unites The Universal Vibration Of T Consciousness With The Individual Pulsation Of Objectivised 'This' Consciousness. The Two Aspects Of Consciousness Are Now In A State Of Equilibrium Like The Two Pans Of An Evenly Weighed Balance And The Yogi Experiences The Pure Knowledge (Suddhavidya) That: 'I Alone Am All Things'. Thus Becoming The Master Of The Wheel Of Energies He Is Free, Like Siva, To Create And Destroy. When [The Yogi] Is Well Established, Without Wavering, Solely In The Integral Egoity Of His Authentic Nature, The Spanda Principle, And Is Absorbed In Contemplation (Samavisfa), He Becomes One With It The Doctrine Of Vibration (Tanmaya). Then . . . Dissolving And Creating The Universe By Means Of His Introverted And Extroverted Absorption, He Destroys And Creates All Things Out Of Sarikara, His Innate Nature. [Thus] He Assumes The State Of The Universal Experiencer And Having Absorbed All That Is To Be Experienced From [The Grossest Level] — Earth — To (The Subtlest) — Siva — He Reaches The State Of The Supreme Subject By Progressively Recognising [His Identity With Him]. Thus, Introverted And Extroverted Absorption Both Lead To The Recognition Of The Pulsation (Spanda) Of One's Own Consciousness. This Is Where Individual General Ledger Becomes Cosmic General Ledger. At The Level Of Consciousness Corresponding To Siva's Basic State (Saryibhavdvastha), The Alternation From Inner To Outer Is Instantaneously Resolved Into The Vibration Of His Nature. When The Yogi Finally Comes To Be Constantly Aware Of This Reality, His Enlightenment Is Full And Perfect. Freed Of All Means (Anupaya) And Delighting In The Power Of His Bliss (Anandasakti), He Knows And Does Whatever He Pleases. The Yogi Seeking Self-Realisation Must Acquire Mastery Over This Movement. Ksemaraja Stresses That The Doctrine Of Vibration Teaches That Liberation Can Only Be Achieved By First Withdrawing All Sense Activity In Introverted Contemplation (Nimilanasamadhi) To Then Experience The 'Great Expansion' (Mahdvikdsd) Of Consciousness While Recognising This To Be A Spontaneous Process Within It. This Is Done Through The Practice Of Kramamudra. A Passage From The Now Lost Kramasiitra Explains: Although The Adept's Attention [May Be] Outwardly Directed, He Enjoys Contemplative Absorption Through The Introverted Aspect Of Kramamudra. Initially He Turns Inward From The Outside World And [Then] From Within [Himself] He Exits Into The Outer World Under The Influence Of His Absorption. There Is No Necessity Of Any Glorification Or Mortification Here. It Is Just Like Joining A Job And Retiring And Realising All That Happened Is Just By His Own Effort And Nothing Else. Thus The Sequence (Krama) In This Attitude (Mudra) [Ranges Through] Both Inner And Outer. The Yogi Must Pervade The Surface Level Of Awareness (Yyutthana) With The Same Bliss He Experiences Plunging Into The Depths Of Con-Templative Absorption (Samadhi). Note Here That Samadhi Is A Death Like State And Still Being Matter And Information Stored You Experience Consciousness. Submerging Himself And Emerging Repeatedly From Samadhi, He Eventually Recognises That The Unity Of Consciousness Pervades Both States: The Best Of Yogi's, Who Has Achieved A State Of Complete Absorption Even When Risen From Meditation, [Inwardly] Vibrating Like A Drunkard In Blissful Inebriation From The After-Effects Of The Nectar Of Con- Templation, Sees All Things Dissolving In The Sky Of Consciousness Like A Cloud In The Autumn Sky. He Plunges Repeatedly Within Himself Divine Body And Sacred Circle Of The Senses And Becomes Aware Of His Identity With Consciousness By The Practice Of Introverted Contemplation. Thus Even When He Is Said To Have Risen From Absorption, He Is One With [His] Experience Of It. Ksemaraja Goes On To Explain That This Practice Is Called 'Mudra' Because It Both

Fills The Adept With Bliss (Mud) And Is Itself The Bliss Of Consciousness. Moreover, It Dissolves Away (Dra) All Bondage And 'Stamps' The Universe Of Experience With The Seal (Mudra) Of The Fourth State (Turiya) Of Enlightened Consciousness Beyond, And Including, The Three States Of Waking, Dreaming And Deep Sleep. It Is Called 'Krama' Because It Is The Root Source Of All Emanation And All Other Conscious Processes Which Succeed One Another In Ordered Sequence (Krama) And Is, At The Same Time, Their Successive (Krama) Appearance As Well. By The Practice Of Kramamudra The Opposites Fuse And Siva And Sakti Unite. They Yogi Comes To Experience The Simultaneous Pervasion Of All The Lower, Grosser Categories Of Existence By The Higher And The Presence Of The Lower In The Higher. Commencing His Practice In A Low Form Of Bhairavimudra, The Yogi Conjoins The Outer With The Inner; Then, In Kramamudra, He Fills Both The Outer With The Inner And The Inner With The Outer. When He Achieves Perfection In This Two-Fold Movement, He Attains To The Highest Form Of Bhairavimudra In Which The Two Merge Completely In The Experience Of The Absolute (Anuttara), Free Of All Differentiation And Polarities. If He Fails To Maintain Awareness Of This State, He Again Falls Into Kramamudra Until He Has Finally Completely Merged All The Highest States In The Lower And The Lower In The Higher. He Then No Longer Needs To Resort To Any Means (Anupaya) To Achieve Liberation. All He Says Or Does Anything He Perceives Or Thinks, Instantly Occasions In Him The Highest Level Of Consciousness. Thus The Fruit Of Bhairavimudra Is The Wonder (Camatkara) Or Amazement (Vismaya) That Overcomes The Yogi When He Reaches The Plane Of Union (Yogabhumika) Y Xsl Where All Opposites Merge In The Radiance Of The Great Light Of Consciousness. The Stanzas On Vibration Teach: How Can One Who, As If Astonished, Beholds His Own Nature As That Which Sustains [The Existence Of Everything] Be Subject To This Painful Round Of Transmigration (Kusrti)? The Yogi, Recognising His True Nature To Be The Supreme Subject, Is Astonished To Suddenly Discover That The Individual He Thought He Was, Caught Up In The Trammels Of Thought And Living In A World Enmeshed In The Web Of Time And Space, Does Not Really Exist At All. He Experiences A 'Turning About' (Paravrtti) In The Deepest Seat Of The Doctrine Of Vibration Consciousness As He Penetrates His True Nature. The Sudden Eruption Of This Intuition (Pratibha) Arouses In Him A Cry Of Amazement As He Transcends All Thought-Constructs And, Perfectly Absorbed In His Own Nature, Is Liberated. The Path To Liberation Essentially, Spanda Doctrine Is Concerned With Two Matters. The First Is To Impart To Those Who Are Fit To Receive The Teachings A Deeper Understanding Of The Ultimate Goal Of Life (Upeya). When We Have Understood What Truly Benefits Us And Is Worth Attaining And What, On The Contrary, Is Of No Real Value But Stands In The Way Of This Attainment, We Can Begin To Make Progress Towards Our Goal. This Is Spanda Doctrine's Second Concern, Namely, To Show The Way In Which We Can Develop Spiritually Through Siva's Grace And The Right Application Of The Means To Realisation That It Teaches. When Both These Aspects Of The Teaching Have Been Correctly Understood And Applied, The Spanda Yogi Achieves A Clear And Permanent Realisation Of His Goal And Is Liberated, Thus Fulfilling The Ultimate Aim Of The Teaching. The Doctrine Of Vibration Is Not Meant For The Spiritually Dull. It Is Not For The Worldly Whose Consciousness, Clouded By Ignorance, Is As If Dreaming, Even During The Waking State Of Daily Life, The Dream Of Its Own Thought-Constructs. The Teachings Are Meant For Those Who Are Awake (Prabuddha), Those Who, Full Of Faith And Reverence, Are Always Alert And Intent On Discerning The True Nature Of Ultimate Reality. This Reality Is Understood In Three Basic Ways. The First Is Purely Transcendental. The Stanzas Choose This Aspect As The One Which Formally Defines It Most Specifically. Ultimate Reality Transcends All The Opposites, Including Subject And Object. This Does Not Mean, However, That It Is An Unconscious Void, A Mere Absence Of All Existence. In Fact, This Negative Characterization Of Reality (Which Includes Also A Denial Of All That Is Unconscious) Implies A Positive Immanence In Which The Opposites Are United In The Oneness Of Pure Consciousness That Is Equally Siva And Spanda, His Universal Activity. These Two Seemingly Contrasting Aspects Are Reconciled In The Third, Namely, Reality Understood As The Essential Nature Of All Things. Although Universal And Everywhere The Same, It Is Understood To Be The Essential And Specific Nature Of Each Existent As Its 'Own Nature' (Svabhava). In The Case Of The Individual Soul It Is Even More Specific, More Personal As His Own 'Own Nature' (Svasvabhava). Note Here That Svabhava Is Not Svasbhava. In Fact We Made A Statement That Svabhava

Is The Very Characterstics Of Space Time Like, Crime Zones, Silent Zones, School Zones We Classify Space In To. Belonging To None Other Than Oneself It Is The Pure Subjectivity That Perceives Experiences, Enjoys, Reflects, Thinks And Senses As Well As Being The Conscious Agent Who Creates Every Possible Form Of Experience In All The States Of Consciousness. The Liberating Knowledge Of Reality Thus Corresponds To Our Regaining Possession Of Ourselves (Svatmagraha). We Must Lay Hold Of Ourselves And Abide In Our Authentic Nature. Reality Coincides With Our Own Most Fundamental State Of Being (Svasthiti), Free Of All Contrasts And Contradictions. Once We Have Overcome The Negative Forces That Arise From Our Ignorance And Prevent Us From Abiding In Ourselves, We Are Liberated. To Do This, We Must Penetrate Through The Pulsing Fluctuations Of Objectively Experienced States And Perceptions At The Surface Level Of Consciousness And Gain Insight Into The Timeless Rhythm Of Our Own Nature Manifest In The Universal Arising And Falling Away Of All Things. We Are Not Freed Of The Trammels Of Perpetual Change By Setting It Aside; On The Contrary, We Must Gain Insight Into The Recurrent Cycles Of Creation, Persistence And Destruction, Or Else Be Bound By Our Ignorance. This Spiritual Ignorance Consists Essentially Of Our Contracted State Of Consciousness And So Can Only Be Effectively Countered By Expanding It To Reveal Our Own Authentic Nature As This Expanded State Itself, Which Is The Universal Vibration (Samanyaspanda) Of Consciousness. Vibration Of Strings Which Gives Shape To Objects Is Itself A Information, A Sine Qua Non Of Individual Consciousness. The Spanda- Yogi Treads The Path Of Consciousness Expansion. The Movement From The Contracted To The Expanded State Marks The Transition From Ignorance To Understanding, From The Dispersion And Incompleteness Of A Form Of Consciousness Entirely Centred On An Objectively Perceived And Discursively Represented Reality To A Direct, Intuitive Awareness Of The Unity And Integral Wholeness Of Our Own Absolute Spanda Nature. Along The Way To This Supreme Realisation Consciousness Develops, As Veil After Veil Is Lifted, Until It Becomes Full And Perfect In The Absolute Which Encompasses Within Itself All Possible Formats Of Experience. As Abhinava Says: [This Realisation] Is The Supreme Limit Of Plenitude And As Such There Can Be No Higher Attainment. Any [Other] Attainment [We Can] Conceive Issues From A State That Falls Short Of [This] Perfection. Once* [This] Uncreated Fullness Has Been Attained, Pray Tell, What Other Fruit Can There Be [Beyond It]? The Fettered Soul's Contracted State Of Consciousness Binds Him Because He Is Deprived Thereby Of The Subtle, Intuitive Insight Into The Underlying Unity Of Existence And His Attention Is Focused Instead On Its Gross, Outer Diversity Easily Apparent To Everybody, However Restricted His Consciousness May Be. However, Although The Fettered Soul In This State Is Ignorant Of This Unity, This Does Not Mean That His Knowledge Of Diversity Is False. Ignorance Entails A Form Of Knowledge Which, Although Quite Correct, Is Binding. We Are Not Absolutely Ignorant Of Reality For If We Were We Would Be Totally Unconscious. Spiritual Ignorance Is Always Linked With Some Degree Of Consciousness. Those Subject To The Round Of Birth And Death Are Not Inert Clods Of Earth. This Also Leads To Conclusion That Shakti Responsible Is Impure Because Of Maya Imbibed In It. Thus, Although Ignorance Obscures Consciousness, It Is Wrong To Think Of It, As Dualist Saivites Do, In Terms Of A Defiling Impurity That Shrouds It Like A Cloth Covering Ajar. Spiritual Ignorance Can Be Nothing But Consciousness Itself, Albeit In A Limited State. Siva, Who Is Universal Consciousness, Is The Innate Nature Of Both Its Contracted And Expanded States, Both Of Which Are Forms Of Knowledge, Namely: 1) Supreme Knowledge (Parajnana) Defined As The Revelation Of One's Own Innate Nature As The One Reality Which Is The Being Of All Things. 2) Inferior Knowledge (Aparajndna) Which Jayaratha Explains Results From The Mental Activity (Yyapard) Of The Individual Subject Whose Consciousness Is Contracted. It Consists Of The Mental Representations (Vikalpa) He Forms Of Himself And His Object, Of The Type I Know This'. The Lower Knowledge Obscures The Higher And Binds The Soul By Breaking Up His Direct, Pervasive Awareness Of His Own Pure Consciousness Nature, Free Of Mental Representation. The Stanzas On Vibration Teach: Operating In The Field Of The Subtle Elements, The Arising Of Mental Representation Marks The Disappearance Of The Flavour Of The Supreme Nectar Of Immortality; Due To This [Man] Forfeits His Freedom. As We Have Already Seen, Three Factors Are Necessary For Perception And Thought To Be Possible, Namely, The Perceiving Subject, The Means Of Knowledge And The Object Perceived. Rajanaka Rama, In His Commentary On The Stanza Cited Above, Explains At Length That These Three Factors

Correspond To Three Major Divisions In The Lower Thirty-One Categories Of Existence, Namely: 1) The Object. This Consists Essentially Of The Five Primary Sensations Which Are The Subtle Elements {Tanmatras) Of Smell, Taste, Sight, Touch And Sound Along With The Five Gross Elements — Earth, Water, Fire, Air And Ether — Of Which These Sensations Are The Perceivable Qualities. 2) The Means Of Knowledge. This Consists Of The Senses And The Inner Mental Organ. 3) The Subject. At This Level, The Subject Is The Individual Soul (Purusa) Whose Consciousness Is Contracted By The Five Obscuring Coverings (Kancukas) Of Limited Knowledge And Action, Attachment, Natural Law And Time Along With Maya, Their Source. All These Categories Belong To The Impure Creation (Asuddhasrsti), Which Is The Sphere Of Maya Where The Lower Order Of Knowledge Operates And Subject And Object Are Divided. Above Them Are Five More Categories Which Belong To The Pure Creation (Suddhasrsfi) Where Subject And Object Are Still United. The Highest Of Those Categories Are Siva And Sakti. Combined They Represent The State Of Pure T Consciousness And Its Sentient Subjectivity (Upalabdhrta), Respectively. The Next Category Is Called Sadasiva. Here Faint Traces Of Objectivity Appear In The Pervasive, Undivided Consciousness Of Siva And Sakti. Consciousness, Now Full Of The Power Of Knowledge (Jnanasakti), Views The All In A State Of Withdrawal (Nimesa), Shining Within, And At One With Its Own Nature. T Consciousness Predominates Over 'This' Consciousness Which It Encompasses In The Awareness That: 'I Am This [Universe]' (Aham-Idam). Next Comes The Category 'Isvara' Corresponding To The Awareness: 'This (Universe) Is Me' (Idam-Aham). 'This' Consciousness Takes The Upperhand Over T Consciousness And Unfolds Externally Full Of The Creative Power Of Action (Kriya&Akti). The All Now Becomes More Clearly Manifest As An Independent Reality. It Is Still Experienced As One With Consciousness But Is No Longer Fully Merged Within It. Finally, When Both Subjective And Objective Aspects Share An Equal Status In The Two-Fold Awareness That: 'I Am This (Universe) And This (Universe) Is Me (Ahamidam-Idamaham)\ Pure Knowledge (Fuddhavidya), The Last Of These Categories, Emerges. The Pure Categories Are The Experience Of The Impure Categories When They Are Recognised To Be One With Consciousness. They Are Experienced Within The Domain Of The Pure Universal Subject The Enlightened Yogi Realises Himself To Be. Mental Representations (Yikalpa) Emerge From This Pure Awareness And Subside Into It In Consonance With The Rhythm Of The Emanation And Withdrawal Of The Lower Categories. Impelled By The Universal Will, This Movement Is Spontaneous And Free. Free Of All Hopes And Fears The Enlightened Yogi Sees All Things As Part Of This Eternal Cosmic Game, Played In Harmony With The Blissful Rhythm Of His Own Sportive Nature At One With All Things. The Stanzas On Vibration Teach: Everything Arises [Out Of) The Individual Soul And He Is All Things. Being Aware Of Them, He Perceives His Identity [With Them]. Therefore There Is No State In The Thoughts Of Words Or [Their] Meanings That Is Not Siva. It Is The Enjoyer Alone Who Always And Everywhere Abides As The Object Of Enjoyment. Or, Constantly Attentive, And Perceiving The Entire Universe As Play, He Who Has This Awareness (Sawvitti) Is Undoubtedly Liberated In This Very Life. According To The Doctrine Of Vibration, Only Liberation In This Life (Jivanmukti) Is Authentic Liberation. Liberation After Death (Yideha- Mukti) In Some Form Of Disembodied State Free Of All Perceptions And Notions Of The World Of Diversity Is Not The Ultimate Goal. Ksemaraja Stresses That Liberation Is Only Possible By Realising One's Own Identity With The Whole Universe, However Difficult This May Be. Similarly, He Maintains That The Suspension Of All Mental And Sensory Activity, Which Takes Place In The Introverted Absorption Of Contemplation With The Eyes Closed (Nimilanasamadhi) That Leads To Identification With Transcendent Consciousness Is Complemented And Fulfilled By The Cosmic Vision Had Through The Expansion Of Consciousness That Takes Place In Contemplative Absorption With The Eyes Open (Unmilanasamadhi). Consequently, Ksemaraja Explains That The First Of The Three Sections, Into Which He Divides The Stanzas, Deals With The Former Mode Of Contemplation And The Second Section With The Latter. Significantly, The Last Stanza Of The Second Section Ends With The Declaration That 'This Is The Initiation That Bestows Siva's True Nature'. In Other Words, This Realisation, Attained Through The Expanding Consciousness Of Contemplation With The Eyes Open, Initiates The Yogi Into The Liberated State, Which Is Identification With Siva Whose Body Is The Universe. In Order To Attain This Expanded State Of Liberated Consciousness, The Yogi Must Find A Spiritual Guide Because The Master (Guru) Is The Means To Realisation. The Master Is For His Disciple

Siva Himself For It Is He Who Through His Initiation, Teaching And Grace, Reveals The Secret Power Of Spiritual Discipline. Instructing In The Purport Of Scripture He Does More Than Simply Explain Its Meaning: He Transmits The Realisation It Can Bestow. The Master Is At One With Siva's Divine Power Through Which He Enlightens His Disciple. It Is This Power That Matters And Makes The Master A True Spiritual Guide, Just As It Was This Same Power That Led The Disciple To Him In His Quest For The Path That Leads To The Tranquility That Can Only Be Found 'In The Abode Beyond Mind'. The Master Is The Ferry That Transports The Disciple Over The Ocean Of Thought — If, That Is, The Disciple Is Ready. The Disciple Must Be 'Awake' (Prabuddha) 2% Attending Carefully To The Pulse Of Consciousness. This Alert State Of Wakefulness Is At Once The Keen Sensitivity Of Insight As Well As The Receptivity Of One Who Has No Other Goal To Pursue Except Enlightenment. The Highest, Most Perfect Relationship The Disciple Can Have With His Master Is Such As It Is With Siva Himself: One Of Identity. The Exchange That Takes Place Between Them Is An Internal Dialogue Within Universal Consciousness, Their Common Identity (Svabhava). Limiting Itself To A Point Source (Anu) And Obscured By The Thought-Constructs Born Of Doubt And Ignorance, Consciousness Assumes The Guise Of The Disciple Who Seeks To Attain The Expanded Fullness Of His Master's Consciousness. The Master, On The Other Hand, Embodies The Aspect Of Consciousness Which Responds To The Inquiring Consciousness Of His Disciple. Free Of The Notions Of 'Self And 'Other', When The Disciple Is Liberated By His Grace, It Is The Master Who In Reality Liberates Himself. Although Ksemaraja Assures Us That The Master Can By Himself Enlighten His Disciple By The Initiation He Imparts To Him And The Other Means (Yukti) N He Adopts, Even So, He Is Not The Only Guide On The Path. Apart From The Master There Is Scripture And, Above All, One's Own Personal Experience, Because, As Abhinava Says: The Knowledge [Acquired] By Gradually [Coming To Understand The Meaning Of] The Scriptures And Following The Master [Who Knows Them] Leads, [When] Confirmed For Oneself, To The Realisation Of One's Own Identity With Bhairava. It Is Important To Know The Scriptures. God Reveals Himself Through Them; They Are One Of The Forms In Which He Is Directly Apparent In This World. They Teach Man What Is Worth Attaining And What Should Be Avoided And So Like A Boat Convey Him Across The Ocean Of Profane Existence (Samsara) To The Other Shore Where God's True Nature Is Revealed To Him. However, The Study Of The Scriptures Is Of Value Only If Accompanied By The Spiritual Knowledge That Results From Personal Experience. Mahesvarananda Writes: Being Well Versed In The Nature Of Deity Is One Thing, But Being Well Versed In The Sacred Scripture Is Another, Just As The Peace Of That Abode Is One Thing And What Worldly People Experience Is Another. Vasugupta, Who Found The Sivasutra, Knew The Means To Realisation (Yukti) As Well As The Scriptures And Had Fully Experienced The One Ultimate Reality. Therefore, Ksemaraja Declares Him To Be Amongst The Best Of Teachers. The Stanzas On Vibration (That Ksemaraja Attributes To Vasugupta) Accordingly Transmit The Secrets Of The Sivasutra In Accord With Scripture, Sound Reasoning And Personal Experience. The Latter Is Particularly Important For The Spanda Yogi; He Is Not Interested In Wasting His Time In Useless Discussion About The Experience Of Consciousness Expansion And Its Fruits, For That Can Only Be Known For Oneself. The Yogi Can Achieve This Experience Either Through Faith In The Master Or Personal Insight (Svapratyayatah) Acquired By Unswerving Devotion To God. Ksemaraja Accordingly Quotes A Passage From The Bhagavadgita Where Krsna Says: Those I Deem To Be The Best Yogis Who Fix Their Thoughts On Me And Serve Me, Ever Integrated [In Themselves], Filled With The Highest Faith. But While The Yogi's Development Depends On Faith And Personal Experience Of The Higher States Of Consciousness, He Can, And Must, Strengthen His Conviction In The Light Of Reason .When Reason (Upapatti) And Direct Insight (Upalabdhi) Work Together, They Serve As A Means To Liberation. Reason Alone Cannot Help Us, But When It Is Based On An Intuitive Insight Of Fundamental Principles Along With A Direct Experience Of Reality, Error Is Eradicated And The Yogi Is Freed. In This Way The Awakened Yogi Realizes His Inherent Spiritual Power (Svabala) With Which He Exerts Himself To Distinguish Between The **Motions Of Individualized Consciousness And The Universal Vibration (Samanyaspanda)** Of The Collective Consciousness That Is Their Ultimate Ground And Firm Foundation. Note The Information Of Individual, Collective And Cosmic General Ledger Is Alone Responsible For Creation And Also Destruction. Thus, Although The Doctrine Taught In The Stanzas On Vibration Accords With Scripture, It Is

Supported By Reason And Above All By Personal Experience. Thus, For Example, The Seventeenth Stanza Describes The Difference In The Manner In Which The Well Awakened And The Unawakened Experience Their Own Nature (Atmopalambha While The Eighteenth Describes The Experience Of The Well Awakened In The Three States Of Waking, Dreaming And Deep Sleep. Indeed, Rajanaka Rama, One Of The Commentators, Explains That The First Sixteen Stanzas Establish On The Basis Of Personal Experience (Svanubhava) That One's Own True Nature Is Independent Of The Body. Similarly, The Remaining Stanzas Also Discuss The Direct Experience Of One's Own Nature, But This Time As The Unity Of All Things. This Direct Experience, In Its Diverse Aspects, Is Both The Means By Which The Yogi Develops His Consciousness As Well As His Ultimate Goal. **Doctrine Of Vibration: Mark S.G.Dyczkowski Models See Next Paper** From the point of view of the object, the expansion (unmesa) of this pulse is represented(eb) by the initial desire to perceive (didrksa) a particular object, while the contracted (nimesha) phase is (=) the withdrawal of attention from the object previously perceived. From the point of view of the perceiving subjectivity, the phases are (=) reversed, so that the initial desire to perceive marks (eb) the contraction (nimesha) of subjective consciousness while the falling away of the previous perception is (=) its expansion (unmesa). At the higher level, where these two phases are experienced within (eb) consciousness, they represent (eb) the state of the categories of Isvara ('this universe is me') and Sadasiva ('I am this universe'). Utpaladeva says: Expansion (unmesa\ which is in (eb) the external manifestation [of objectivity], is (=) Kvaratattva while contraction (Niemba), which is in the internal manifestation [of subjectivity], is Sadasiva. **There is no glorification or mortification here. It just like joining the job and retiring the involution and evolution phases**.(italics mine)The Doctrine of Vibration At this level all the powers of consciousness fuse (e&eb) and both phases are (=) manifest as part of one reality. This unity is in fact apparent (eb) to everybody at each moment. However, within the domain of Maya, which is the sphere of differentiated perceptions (Yikalpa), it is clearly manifest (eb) only at the juncture (Madhya) between two cognitions. In this Centre resides (eb) the void (kha) of consciousness (free of (e) thought-constructs) which divested of (e) diversity, digests into (eb) itself all the psycho-physical processes that give (eb) life to the multiplicity of perceptions. **Here it is important to note that the when the cosmic general ledger zero reflects upon itself and it is projected on the individual consciousness you see a film an illusory animations wherein creation itself becomes subjective wherein the individual consciousness is both the perceiver the perceived being the image on the screen on the screen of individual consciousness**. The yogi moves from the particular vibrations of consciousness at its periphery to (e&eb) the universal throb of the Heart in the Centre. As Abhinava explains: The self-reflective awareness in (eb) the Heart of pure consciousness, present (eb) at the beginning and end of each perception, within (eb) which the entire universe is (=) dissolved away without residue, is (=) called in the scriptures, the universal vibration of consciousness (samanyaspanda) and is (=) the outpouring (uccalana) [of awareness] within one's own nature. All the categories of existence (tattvas) are united in (eb) the Heart of the Centre where the life-giving elixir of Siva's consciousness floods (e&eb) one's own inner nature. To reside in the Centre is to abide by (e) the law of totality (gramadharma) in a state which transcends (e) the workings of the mind (unmana).Consciousness (jnana) with Light as its support, residing in (eb) the Centre between being and non-being is (=) known as the act of abiding in (eb) one's own abode as (=) the perceiving subjectivity (dratfrtva) free of (e) all obscuration. That which has been purified by (e) pure awareness **(suddhavijnana)** is called the transcendent (viviktavastu), said to be (=) the mode of being (v/7//) of the law of totality (gramadharma) through (e&eb) which everything is (=) easily attainable. The power in the Centre (madhyasakti) is (=) the eternal Present Beyond time it is the source of both past and future. To be established there is to abide without a break in Rama, the supreme enjoyer, in every action of one's life. Rama is Siva, the supreme cause Who pervades the fourteen aspects which embrace the entire universe of experience, namely, moving, standing, dreaming, waking, the opening and closing of the eyes, running, jumping, exertion, knowledge [born] of the power of the senses, the [three] aspects of the mind, living beings, names and all kinds of actions. This came as a natural development in Spanda doctrine not only for **this reason but also because the universal ego is experienced as the inner dynamics of absolute consciousness. To conclude our summarial exposition of the Divine Means, which is centred on the direct experience of this pure ego (and hence on Spanda in this form), we turn now to a brief description of its inner,**

cyclic activity. We shall do this by examining Abhinava's esoteric exegesis of the symbolic significance of the word 'A HAM', which in Sanskrit means T, and symbolises by its form the ego's dynamic nature. “Dasein Does Not Fill Up A Track Or Stretch ‘Of Life’ — One Which Is Somehow Present-At-Hand — With The Phases Of Its Momentary Actualities. It Stretches Itself Along In Such A Way That Its Own Being Is Constituted In Advance As A Stretching-Along. The ‘Between’ Which Relates To Birth And Death Already Lies In The Being Of Dasein … It Is By No Means The Case That Dasein ‘Is’ Actual In A Point Of Time, And That, Apart From This, It Is ‘Surrounded’ By The Non-Actuality Of Its Birth And Death. Understood Existentially, Birth Is Not … Something Past In The Sense Of Something No Longer Present-At-Hand; And Death Is Just As Far From Having The Kind Of Being Of Something … Not Yet Present-At-Hand But Coming Along … Factical Dasein Exists As Born; And, As Born, It Is Already Dying, In The Sense Of Being-Towards-Death. As Long As Dasein Factically Exists, Both The ‘Ends’ And Their ‘Between’ *Are*, And They Are In The Only Way Possible On The Basis Of Dasein’s Being As *Care* … As Care, Dasein Is The ‘Between’.” **— Martin Heidegger (Thomas Piel - Traducteur), Being And Time** Path to Liberation On without knowledge that everything is manifest within consciousness is illusory or unreal in that sense alone. 78 Things are more real or more tangibly experienced according to their own essential nature (svabhava) to the degree in which we recognise that they are appearances (abhasa) within absolute consciousness As Jayaratha says: Just as images manifest in a mirror, for example, are essentially mere appearances, so too are [phenomena] manifest within conscious- ness. Thus, beause they are external, [phenomena] have no being (sattva) of their own. The Lord says this [not with the intention of saying anything about the nature of things] but in order to raise the level of consciousness of those people who are attached to outer things; thus everything in this sense is essentially a mere appearance. [Knowing this], in order to quell the delusion of duality, one should not be attached to anything external. 79 The ultimate experience is the realisation that everything is contained within consciousness. We can discover this in two ways. Either we merge the external world into the inner subject, or we look upon the outer as a gross form of the inner. In these two ways we come to recognise that all things reside within our own consciousness just as consciousness resides within them. This all-embracing inwardness is only possible if there is an essential identity between the universe and consciousness. The events which constitute the universe are always internal events happening within consciousness because their essential nature is consciousness itself. 80 We can only account for the fact that things appear if there is an essential identity between consciousness and the object perceived. If a physical object were really totally material, that is, part of a reality independent of, and external to, consciousness, it could never be experienced. Abhinava says: The existence or non-existence of phenomena within the domain of the empirical (iha) cannot be established unless they rest within consciousness. In fact, phenomena which rest within consciousness are apparent (prakasamana). And the fact of their appearing is itself their oneness {abheda) with consciousness because consciousness is nothing but the fact of appearing {prakasa). If one were to say that they were separate from the light of [that consciousness] and that they appeared [it would be tantamount to saying that] 'blue' is separate from its own nature. However, [insofar as it appears and is known as such] one says: 'this is blue 1 . Thus, in this sense, [phenomena] rest in conscious- ness; they are not separate from consciousness. The Doctrine of Vibration The universe and consciousness are two aspects of the whole, just as quality and substance constitute two aspects of a single entity. The universe is an attribute (dharma) of consciousness which bears (dharmiri) it as its substance. It is said that 'substance' is that resting in which this entire group of categories manifests and is made effective. Now, if you don't get angry [we insist that] this entire class of worlds, entities, elements and categories (tattva) rests in consciousness and [resting in it] is as it is. Thus consciousness contains everything in the sense that it is the ground or basis (adhard) of all things, their very being (satta) and substance from which they are made. But, unlike the Brahman of the Advaita Vedanta, it is not the real basis (adhisthand) of an unreal projection or illusion. Consciousness and its contents are essentially identical and equally real. They are two forms of the same reality. Consciousness is both the substratum and what it supports: The perceiving awareness and its object. In this respect, the Kashmiri Saiva is frankly and without reserve an idealist. Although he does not deny the reality of the object, his position is at odds with most commonly accepted forms of realism. The realist maintains that the content perceived is independent of the act of perception. The content is only accidentally an

object of perception and undergoes no change in the process of being perceived. His contention, however, is essentially unverifiable; to verify it, we would have to know an object without perceiving it. This, from the Kashmiri Saiva point of view, is not possible. Objects of which we have no knowledge may indeed exist, but they are knowable as objects only if they are related to subjects who perceive them. In this sense, if there were no subjects, there could be no objects. 86 The subject, however, as opposed to the object is, in terms of the phenomenology of perception, apparent to himself. He is self-luminous (svaprakdsa). Thus, conscious- ness (the essence of subjectivity) is one s own awareness by virtue of which all things exist. The realist maintains that consciousness clearly differs from its object insofar as their properties are contrary to each other. The Saivite idealist. However, says that the object is a form of awareness (vijnanakara)™ The objective status of the object is cognition itself. Perception manifests its object and renders it immediately apparent (sphuta) to those who perceive it. It does not appear at any other time. If 'blue' were to exist apart from the cognition of k blue\ two things would appear: 'blue' and its cognition, which is not the case. It is the perception of the object which constitutes its manifest nature. An entity becomes an Integral Monism of Kashmiri Saivism object of knowledge not by virtue of the entity itself but by our knowledge of it. If objects had the property of making other objects appear, it would be possible for one object to make another appear in its own likeness. 'Blue' is perceived to be 'blue' because it is manifest as such to the perceiver. As Abhinava points out: The [nature of an] object of knowledge could not be established through a means of knowledge totally unrelated to it — a crow does not become white because a swan [sitting next to it] is white. Perception, on the other hand, is immediately apparent to conscious- ness. It is self-luminous in the sense that it is directly known without need of being known by any ulterior acts of perception and makes its object known at the same time. Adopting the Buddhist Yogacara doctrine that things necessarily perceived together are the same (sahopa- lambhaniyamavada), the Saivite affirms that because the perceived is never found apart from perception, they are in fact identical. Reality (satya) is the point where the intelligible and the sensible meet in the common unity of being; it cannot be said to exist in itself outside, and apart from, knowledge or vision. Bhagavatotpala in his commentary on the Stanzas on Vibration quotes: Once the object is reduced to its authentic nature, one knows [the true nature of] consciousness. What then [remains of] objectivity? What [indeed could be] higher than consciousness? Consciousness is essentially active. Full of the vibration of its own energy engaged in the act of perception, it manifests itself externally as its own object. When the act of perception is over, consciousness reabsorbs the object and turns in on itself to resume its undifferentiated inner nature. Knowledge (jnana) manifests internally and externally as each individual entity.... Once knowledge has assumed that form it falls back [into itself]. The Yogacara Buddhist similarly maintains that consciousness creates its own forms. But, according to him, because the perceived and perception are identical; there is no perceived object at all. The so-called outer world is merely a flux of cognitions, it is not real. He is firmly committed to a doctrine of illusion. The reality of consciousness from The Doctrine of Vibration his point of view is established by proving the unreality of the universe. "All this consists of the act of consciousness alone", says Vasubandhu, "because unreal entities appear, just as a man with defective vision sees unreal hair or a moon, etc."He points to dreams as examples of purely subjective constructs which appear to be objective realities. The apparent reality dreams possess is not derived from any concrete, objective world, but merely from the idea of objectivity. While the Yogacara does not say that an idea has, for example, spatial attributes, it does have a form manifesting them. While he agrees with the Saiva idealist that appearances have no independent existence apart from their appearing to consciousness, he maintains that for this reason they are unreal. The creativity of consciousness consists in its diversification in many modes having apparent externality; it is not a creation of objects. While the Kashmiri Saivite agrees that the world is pure conscious- ness alone, he maintains that it is such because it is a real creation of consciousness. The effect is essentially identical with the cause and shares in its reality. Matter and the entire universe are absolutely real, as 'congealed' (sty ana) or 'contracted' (samkucita) forms of consciousness. "This God of consciousness", writes Ksemaraja, "generates the universe and its form is a condensation of His own essence (rasa)" m By boiling sugarcane juice it condenses to form treacle, brown sugar and candy which retains its sweetness. Similarly, consciousness abides unchanged even though it assumes the concrete material form of the five gross elements. The same reality thus abides equally in

gross and subtle forms. Consequently no object is totally insentient. Even stones bear a trace (vasana) of consciousness, although it is not clearly apparent because it is not associated with the vital breath (prana) and other components of a psycho-physical organism. Somananda goes so far as to affirm that physical objects, far from being insentient, can only exist insofar as they are aware of themselves as existing. The jar performs its function because it knows itself to be its agent. Indeed, all things are pervaded by consciousness and at one with it and hence share in its omniscience. Thus, Siva, Who perceives Himself in the form of physical objects, is the one ultimate reality. "The jar knows because it is of my nature", writes Somananda, "and I know it because I am of the jar's nature. I know because I am of Sadasiva's nature and He knows because He is of my nature; Yajnadatta [knows] because he is of Siva's nature and Siva [knows] because He is of Yajnadatta's nature". Integral Monism of Kashmiri Saivism Everything in this sense is directly perceived by absolute conscious- ness, and this direct perception (pratyaksa) unifies the knowable into a single, undivided whole. This is the central concept behind a doctrine originally expounded by Narasimha called 'the non-dualism of direct perception' (pratyaksadvaita). This states that consciousness is essentially perceptive and that its perception of all things operates throughout the universe. Insofar as phenomena are clearly evident (sphufa) to us, everything is directly perceived by absolute consciousness, with which our individual consciousness is identical. This direct perception unfolds everywhere; the one true reality, it is alone and without companion or rival (nihsapatna). Even though it remains one, it can, by its very nature, perceive distinctions (bheda) between one entity and another, without this engendering any division within it. We distinguish between two entities in empirical terms on the basis of their mutual exclusion (anyonyabhava). The relative distinction {bheda) between them is essentially the perceived difference between their respective characteristics. Despite this difference they are united within the purview of a single cognition insofar as they are equally both manifest appearances. This cognition is the undivided essence (rasa) or 'own nature' (svabhava) of both. Encompassed by the 'fire of consciousness', there is no essential difference between them. Just as when an emerald and ruby reflect each other's light, the ruby is reddish-green and the emerald greenish-red, similarly everything is connected with everything else as part of the single variegated (vicitra) cognition of absolute consciousness. Mahesvarananda writes: The Supreme Lord's unique state of emotivity (asadharanabhava) is the outpouring of pure Being (mahasatta). It is manifest as the brilliance (sphuratta) of the universe which, if we ponder deeply, [is realized to be] the single flavour (ekarasa) of the essence of Beauty which is the vibration of the bliss of one's own nature. In this way all things are in reality one although divided from the one another sharing as they do the 'single flavour' (ekarasa) of the pure vibration of consciousness. Kashmiri Saiva Realism Kashmiri Saivism as a whole has been variously called a form of 'realistic idealism', 'monistic idealism', ‘idealistic monism' and 'concrete monism'. It is easy to understand why Kashmiri Saivism is The Doctrine of Vibration said to be 'idealistic' and 'monistic', but in what sense is it also 'realistic*? The answer to this question is of no small importance in trying to under- stand the central idea behind its metaphysics and the fundamental importance of the concept of Spanda, in this seemingly impossible marriage between monistic idealism and pluralistic realism. The Kashmiri Saiva approach understands the world to be a symbol of the absolute, that is, as the manner in which it presents itself to us. Again we can contrast this view with that of the Advaita Vedanta. The Advaita Vedanta understands the world to be an expression of the absolute insofar as it exists by virtue of the absolute's Being. Being is understood to be the real unity which underlies empirically manifest separateness and as such is never empirically manifest. It is only transcendentally actual as 'being-in-itself. The Kashmiri Saiva position represents, in a sense, a reversal of this point of view. The nature of the absolute, and also that of Being, is onceived as an eternal becoming (satatodita), a dynamic flux or Spanda, 'the agency of the act of being'. It is dentified with the concrete actuality of the fact of appearing, not passive unmanifest Being. Appearance (abhasa) alone is real.ppearing (prakasamanatva) is equivalent to the fact of being (astitva). Ksemaraja writes in his commentary on the Stanzas on Vibration: Indeed, all things are manifest because they are nothing but manifestation. The point being that nothing is manifest apart from manifestation The absolutely unmanifest, from this point of view, can have as little existence as the space in a lattice window of a sky-palace. Nay, even less, because even that space can appear as an imagined mage manifest within consciousness. Everything is real according to the manner in which it appears. Even an illusion is in this sense real, insofar as it

appears and is known in the manner in which it appears. The empirical and the real are identical categories of thought. As Abhinava says: Thus this is the supreme doctrine (upanisatf), namely that, when- ever and in whatever form [an entity] appears, that then is its particular nature. Perhaps at this stage a brief comparison with Heidegger's ideas might prove to be enlightening and not altogether out of place. According to Heidegger's phenomenology of Being, reality is intelligible in a two-fold Integral Monism of Kashmiri Saivism manner as 'phenomenon' and 'logos'. Heidegger defines what he means by 'phenomenon' as: "that-which-shows-itself. The manifest . . . phenomena are then the collection of that which lies open in broad daylight or can be brought to the light of day — what the Greeks at times implicitly identified as 'ta onta' (the things-which-are)". 127 In his later writings Heidegger drops the term 'phenomenon' in preference for the verbal form 'phainesthai' in order to emphasize even more the actuality or presentational property of Being. Explaining this new form of the term he writes: "Being disclosed itself to the ancient Greeks as 'physis'. The etymological roots 'phy-' and 'pha-' designate the same thing: 'phyein', the rising-up or upsurge which resides within itself as 'phainesthai', lighting-up, self-showing, coming-out, appearing-forth." Heidegger contrasted his notion of phenomenon with semblance (Schein) and with appearing (Erscheinung). In the case of semblance a thing can show itself as that which it is not, as when fool's gold shows itself to be gold. The ancients always allied semblance with non-being. Heidegger points out, however, that semblances are grounded in showings, and so does Abhinava. Both Heidegger and Abhinava consequently maintain that all semblances have a real basis and are to be treated as instances of phenomena along with the so-called real showing or manifestation of non-deceptive objects. So Heidegger states that: 'how- ever much seeming, just that much being'. Thus self-showing or appearing defines Being as phenomenon, but this definition of Being is as yet incomplete. Being is not only self-showing but 'logos' which Heidegger explains means 'discourse' (Rede) in the sense of 'apophansis': 'letting-be-seen'. Phenomenology, which according to Heidegger is the only correct study of Being, means 'letting-be-seen-that-which- shows-itself. This is true of Saiva Paramadvaita as well. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real." A thousand proofs could not make 'blue' other than the colour blue. The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. Although the nature of the absolute is discovered at a higher level of consciousness, The Doctrine of Vibration nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. All things are known to be just as they present themselves. The concrete actuality of being known (pramiti), irrespective of content, is itself the vibrant (spanda) actuality of the absolute. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. "The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. Within the sphere of Maya, every entity's 'own nature' (svabhdva) corresponds to its specific manifest form. Accordingly it is defined as that which distinguishes it from all else and from which it never deviates. Above the sphere of Maya, that is, above the level of objectivity, is the domain of the subject. At this level,

everything is realised to be part of the fullness of the experience r and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto. In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri SaivismWe have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. This is the level of consciousness in which the absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature. The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things] and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. Atomic appearings can combine in any number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. Insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific The Doctrine of Vibration awareness of an object of the form: 'here this is such.' While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. The form of our experience is thus 'I now see this here'. But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. The form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its

constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'. The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Integral Monism of Kashmiri Saivism Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; it arises from a distinction between appearances and the individual subject. So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their functions; when they are merged within the subject and at one with him, they cannot do so. Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness. Together these individual pulsations constitute the universal pulse (samanyaspandd) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. Light and Awareness: The Two Aspects of Consciousness Absolute consciousness understood as the unchanging ontological ground of all appearing is termed 'Prakasa'. As the creative awareness of its own Being, the absolute is called ' Vimarsa '. Prakasa and Vimarsa — the Divine Light of consciousness and the reflective awareness this Light has of its own nature — together constitute the all-embracing fullness (purnata) of consciousness. The Recognition (pratyabhijfia) school of Kashmiri Saivism develops this concept of the absolute which finds its fullest expression in Utpaladeva's Stanzas on the Recognition of God. Even though neither of these two key terms appear in the Stanzas on Vibration or the Aphorisms of Siva, they recur frequently in their commentaries. Thus, although the original formulation of the Doctrine of Vibration differs from the theology of Recognition in this respect, it was extended in the course of its development to accommodate this concept of the absolute as well. This was possible, and quite justified, insofar as the absolute understood in Pratyabhijfia terms does not, as we shall see, differ essentially from that of the Spanda school. We can, as Kashmiri Saivites themselves have done, explain one in terms of the other. The Doctrine of Vibration Prakasa: The Light of Consciousness Prakasa is the pure luminosity' (bhdna) or 'self-showing' that constitutes the essence and ultimate identity (atman) of phenomena. That things appear at all is due to the light of consciousness, and their appearing (avabhasana) is itself this Light which bestows on all things their evident, manifest nature. Established in the light of consciousness everything appears there according to its own specific nature (svabhava). Anything that supposedly does not rest in this Light is as unreal as a sky-flower. 3 Thus, according to Rajanaka Rama, unlike the light of the sun, or any other light, this Light not only makes all things apparent, it is also their ultimate source. 4 Full of its divine vibration the Light makes all things manifest and withdraws them into itself. This supra- temporal activity characterises it most specifically; devoid of it, it would be no better than an inert physical phenomenon. At the same time, this light is the conjunction (slesa) or oneness (aikdtmya) of its countless manifest forms, 6 and the collective whole (sampina'ana) of all the categories of existence. The universe is nothing but the shining of the Light within itself. It is the radiant vibration (sphuratta) of this Light, the state (avast hand) in which consciousness becomes manifest.Although the Light shines as all things at all times and hence also makes their diversity manifest, 9 penetrating each object individually as well as collectively, it is not totally 'merged' (magna) or identified with the object so as to suffer any division within itself. Our experience of any object is of the form: T see this': it is not itself an object, but the manifest form the object assumes as a luminous principle of

experience. The Light is ever revealed and can never be obscured; objectivity can never cast a shadow on the light of consciousness. The Stanzas on Vibration declare: That in which all this creation is established and from whence it arises is nowhere obstructed because it is unconditioned by [its very] nature. This Light is the highest reality (paramartha). It is the 'Ancient Light' (puranaprakasa) that makes all things new and fresh every moment. It is 'always new and secret, ancient and known to all'. It is the form of the Present (vartamanarupa), the Eternal Now. Time and space are relations between the contents of consciousness; they cannot impinge on the integrity of the absolute itself. 16 Neither space nor time can divide it, for they are one with the Light that illumines them Light and Awareness: Two Aspects of Consciousness and makes them known as elements of experience. But this Light is the shining of the absolute; it is not an impersonal principle. It is the living Light of God, indeed it is God Himself, the Master Who instructs the entire universe. 18 Siva is this 'auspicious lamp', Who illumines all things. He is the Light of consciousness that reveals the presence of both the real and the unreal, of light' and 'darkness'. 20 Abhinavagupta writes: Thus Bhairava, the Light, is self-evident (svatahsiddha); without beginning, He is the first and last of all things, the Eternal Present. And so what else can be said of Him? The unfolding of the categories of existence (tattva) and creation, which are the expansion of His own Self, He illumines, luminous with His own Light, in identity with Himself, and because He illumines Himself, so too He reflects on His own nature, without His wonder (camatkara) being in any way diminished. Since Zwicky's early observations, similar data from other galaxy clusters has yielded the same result – we consistently see that clusters of galaxies appear to By developing an awareness of the Centre, the yogi experiences the bliss of consciousness. 117 Through this gap he plunges into introverted absorption (nimilanasamadhi) and then emerges again to pervade the field of awareness between Centres and so experience the Cosmic Bliss (jagadananda) of the universal vibration of consciousness. 1 i8 He then recognises that this state pervades every aspect of experience. In this way the yogi's consciousness is no longer afflicted by the power which obscures it, hemming the Centre in on both sides with thought-constructs that seemingly deprive it of its fullness. As he realises directly his pure conscious nature as the universal ego free of all mental representations, it expands out to embrace all things within itself. Thus the realisation the Divine Means leads to, and is directly based upon, is that this pure ego is in all things just as all things are within it. In the Spanda tradition, as recorded in the Stanzas on Vibration, no such ego is recognised. ' I9 Man's authentic nature is, however, understood in personal terms as every individual's own 'own nature* (svasvabhava) which is Siva, the universal vibration of pure subjectivity (upalabdhfta). It is not surprising, therefore, that later commentators found these two conceptions to be essentially the same and accordingly identified one's own inner nature with the pure ego. This came as a natural development in Spanda doctrine not only for **this reason but also because the universal ego is experienced as the inner dynamics of absolute consciousness. To conclude our summarial exposition of the Divine Means, which is centred on the direct experience of this pure ego (and hence on Spanda in this form), we turn now to a brief description of its inner, cyclic activity. We shall do this** by examining Abhinava's esoteric exegesis of the symbolic significance of the word 'A HAM', which in Sanskrit means T, and symbolises by its form the ego's dynamic nature. The objective world of perceptions is, as we have seen, essentially a chain of thought-constructs (prapanca) closely linked to one another and woven into the fabric of diversity (vicitrata). This thought (vikalpa) is a form of speech (vac) uttered internally by the mind (citta), which is itself an outpouring of consciousness. Consciousness also, in its turn, resounds with the silent, supreme form of speech {para vac) which is the reflective awareness through which it expresses itself to itself. Consequently, the fifty letters of the Sanskrit alphabet, which are the smallest phonemic units into which speech can be analysed, are symbolic of the principal elements of the activity of consciousness. Letters come together to generate words and words go on to form sentences. In the same way the fifty phases in the cycle of consciousness represent, in the realms of denoted meaning (vacya)the sum total of its universal activity (kriya) corresponding to the principal forces (kala) which come together to form the metaphysical categories of 186 The Doctrine of Vibration experience, which in their turn appear in the grossest, most explicitly 'articulate' form as the one hundred and eighteen world-systems (bhuvana). 'A', the first letter of both AHAM and the Sanskrit alphabet, is the point of departure or initial emergence of all the other letters and hence denotes Anuttara — the absolute. 'Ha', is the final letter of the alphabet and represents the point of completion when

ail the letters have emerged. It represents the state in which all the elements of experience, in the domains of both inner consciousness and outer unconsciousness, are fully displayed. It is also the generative, emission (visarga) which, like the breath, casts the inner into the outer, and draws what is outside inward. The two letters 'A' and 'Ha' thus represent Siva, the transcendental source and Sakti, His cosmic outpouring that flows back into Him. The combined 'A-Ha' contains within itself all the letters of the alphabet — every phase of consciousness, both transcendental and universal. (For a graphic representation of this analysis, see figure 1.) M, the final letter of AHAM, is written as a dot placed above the letter which precedes it. It comes at the end of the vowel series and before the consonants and so is called 'anus vara '(lit. 'that which follows the vowels') and also 'bindu ' (lit. 'dot,' 'drop,' 'point' or 'zero'). While the consonant 'M' symbolises the individual soul (purusa), 'bindu 1 rep resents the subtle vibration of T, which is the life force (jivakala) and essence of the soul's subjectivity manifest at the transcendental, supra-mental level (unmana). 12 ° It is the zero-point in the centre between the series of negative numbers, in this case the vowels which represent the processes happening internally within Siva, and the series of positive numbers — the consonants which symbolise the processes happening externally within Sakti. Bindu, as a point without area, symbolises the non-finite nature of the pure awareness (pramitibhava) of AHAM. It is the pivot around which the cycle of energies from 'A' to 'Ha' rotates, the Void in the centre from which all the powers emanate and into which they collapse. As such, it is the supreme power of action which holds subject, object and means of knowledge together in a potential state in the one Light that shines as all three 121 containing them in its repose 122 (visrdnti). Bindu is the 'knower' (jndtr), who is essentially consciousness that, though omniscient does not manifest its intelligence, like a man who knows the scriptures but having no occasion to explain them to others silently bears this knowledge within himself. As such, it symbolises the union of Siva and Sakti (sivasakti- mithunapina'a) 123 in a state of heightened potency in which they have not yet divided to generate the world of diversity. It stands, in other words, at the threshold of differentiation in the stream of emanation still contained within Siva. Expansion Commences Bindu — The Individual Soul Withdrawal Commences Figure 1 188 The Doctrine of Vibration Then, to the degree in which that which is to be accomplished by the power of action residing within it [as a potential] penetrates into the absolute, it appears initially as bindu, which is the light of pure consciousness. 124 When outer objectivity is reabsorbed into its transcendent source, bindu is the point into which all the manifest powers of consciousness are gathered and fused together. The universal potency of all the letters is thus contained in bindu which, as the reflective awareness of supreme T consciousness, 125 gives them all life. Thus bindu also marks the beginning of Siva's internal movement back to the undifferentiated absolute and so stands at the threshold of both emission and absorption without being involved in either. The three aspects of AH AM together constitute a movement from the undifferentiated source of transcendental consciousness — 'A' — through the expansion or emission of its power — 4 Ha' — to the subject — 'M 1 — which contains and makes manifest the entire universe of experience. The reverse of this movement, which of withdrawal (samhara), is represented by M-Ha-A. AHAM and M-Ha-A alternate in the rotation (ghurnana) of the reflective awareness of T consciousness as immanent Sakti emerges from transcendental Siva to then merge back into Him. As Abhinava says: The universe rests within Sakti and She on the plane of the absolute (anutiara) and this again within Sakti ... for the universe shines within consciousness and [consciousness shines] there [within the universe by the power of] consciousness. These three poles, forming a couple and merging, make up the one supreme nature of Bhairava Whose essence is AHAM- 126 At the microcosmic level, fc A' represents the initial moment when the subject begins to rise out of himself to view the object. The movement from 'A to 'Ha' marks the emergence of sensation within the field of awareness, which is represented by the fifty letters of the alphabet symbolic of the fifty aspects of the flux of consciousness leading to objectified perception. 'Nl' is tne subject who, resting content within himself when he has perceived his object, merges through the inner flow of awareness into k A\ the absolute. Then from the absolute (A) its emission (Ha) flows back into the pure subject (M) set to perceive his object. Thus all the cycles of creation and destruction are contained within AHAM through which they are experienced simultaneously as the spontaneous play of the absolute. The yogi who recognises this recurrent pulse of awareness to be the movement of his own consciousness merges his limited ego with the Path to Liberation universal ego. **This is natures general ledger** Thus he realises

that its power to create, sustain and destroy all things is his own inner strength (svabaia) that he exerts effortlessly in the same state of mystical absorption (turlya) in universal consciousness that the absolute itself enjoys. In this way he shares in the three-fold awareness Siva Himself has of His own nature which Abhinava describes as follows: 4 I make the universe manifest within myself in the Sky of Consciousness. I, who am the universe, am its creator! ' — this awareness is the way in which one becomes Bhairava. 'AH of manifest creation (sadadhvari) is reflected within me, I cause it to persist 1 — this awareness is the way in which one becomes the universe. The universe dissolves within me. I who am the flame of the [one] great and eternal fire of consciousness' — seeing thus one achieves peace. 127 The experience of the liberated thus coincides with the realisation of their own divine nature which, through its power, rules and guides the cosmic order. Thus this attainment (siddhi), which is liberation itself, is in the DoctrLie of Vibration technically called 'Mastery over the Wheel of Energies' (cakresvaratvasiddhi) because the liberated soul, identified with Siva, now governs, as does Siva, the cycle of the powers that bring about the creation and destruction of all things, Here we are talking of individual consciousness becoming cosmic general ledger and the liberationist realises all destruction and creation is by him. This clearly means that he is in his own world of illusion. Creation is subjective quintessentially The Empowered Means (Saktopaya) All the practices taught in the Stanzas on Vibration are internal. Whenever ritual is mentioned, it is invariably interpreted in terms of the dynamics of the inner processes the yogi experiences and implements in the course of his yogic practice. The Doctrine of Vibration, Ksemaraja affirms, 129 is concerned entirely with these inner disciplines centred, as it is, in one way or another, on consciousness or, at least, on the inner activity of the mind. Thus the Empowered Means which, like the other categories we have discussed, is entirely internal, includes an important part of Spanda practice. Spanda practice belonging to the Divine Means centres on one's own inherent nature (svasvabhava) as Siva, the universal perceiver and agent, that belonging to the Empowered Means on His power. Instead of arriving directly at the all-embracing emptiness of subjective conscious- ness, the yogi practising the Empowered Means realises his true nature through the fullness of its energy. Practising the Divine Means, the yogi plunges, as it were, straight into the fire of consciousness; practising the 190 The Doctrine of Vibration Empowered Means he merges with its rays. Either way the yogi is centred equally on ultimate reality. The power of consciousness is no less absolute than its possessor. To make this point Abhinava quotes the Matanga- tantra: This reality consists of the rays of [Siva's] power and is variously said to be the abode of the Lord's manifestation . . . That same [power] illumined [by Siva] is itself also luminous, unshaken and unmoving. That very [power] is the supreme state, subtle, omnipresent, the nectar of immortality, free of obscuration, peaceful, yearning for pure Being alone {vastumatra) and devoid of beginning and end. Perfectly pure, it is said to be the body [of ultimate reality]. 110 The yogi concentrates on the powers operating in all of life's activities as particular pulsations (visesaspanda) in the universal rhythm (samanyaspanda) of the power of consciousness. In this way he rises progressively from the particular to the universal until he reaches pure Being (satta), the greatest of all universals (mahasamanyd) and the highest form of Siva's power. Thus the creative power of Maya, manifest through countless lesser powers, no longer causes the yogi to stray from Siva's consciousness but becomes the means through which it can be realised 13 1 in the illuminating brilliance (sphuratta) which is Siva's pure Being. Thus by discovering the true nature of Sakti, the yogi realises himself to be Siva, its possessor Who consists of all its countless powers. Thus practise belonging to this Means leads to the same pure consciousness free of thought-constructs realised through the Divine Means. Although the ultimate realisation is instantaneous, the yogi rises to it gradually by freeing his consciousness of the limitations imposed upon it by thought. Abhinava explains: The same occurs in the Empowered Means [as does in the Divine]. At the discursive level of consciousness {yaikalpikibhumi) [where the Empowered Means functions] knowledge and action, although evident, are, for the reasons explained previously, contracted. A blazing energy [is revealed within] the one who dedicates himself to removing the burden of this contraction. [This energy eventually] brings about the inner manifestation {antarabhasa) of pure consciousness he seeks. 132 Consciousness is individualized and its power of knowledge and action contracted by the thought-constructs born of ignorance. The arising of these mental representations, as the Stanzas on Vibration say, deprives the soul of its freedom and immortal life. 133 The practise of the Path to Liberation 191 Empowered Means is meant to free the fettered soul of this constriction on his

consciousness. It operates within the mental sphere (cetasy™ and is designed to purify thought (vikalpasamskara) in order to. reveal the pure consciousness which is its ground and ultimate source. Thus, the Empowered Means is concerned with the second instant of perception, during which the subject forms mental representations of his object. Thought functions on the basis of an awareness of relative distinctions between specific particulars, distinguishing them from one another and thus seemingly fragmenting the essential unity of reality. 135 The vibrant vitality of consciousness, universally manifest, is clouded like a mirror by a child's breath 136 and the soul is deprived of the liberating intuition of the one reality free of thought-constructs (nirvikalpa). Abhinava writes: The [fettered soul] is like a dancing girl who although wishing to leave the dancehall is collared by the doorkeeper of thought and thrown back onto the stage of Maya. 137 All thought is centred on objectivity and hence dislodges awareness from the plenitude of pure subjective consciousness. Thus, to regain the original state of rest (visranti) consciousness enjoys, the yogi must rid himself of thought. As thought-forms decrease, pure, thought-free awareness is strengthened 138 until the yogi is fully established in a state in which the relative distinctions (bheda) conceived between entities dissolve away. Everything appears to him as pure Being (sattdmdtra) l}9 and the entire universe shines before him pervaded by Siva's radiance. 140 His intuitive faculty (mati) thus purified, the yogi gains both the perfections (siddhi) of yogic practice and liberation (mukti). His consciousness is now like a well-polished mirror which reflects everything he desires and grants it to him. 141 Abhinava writes: Just as a man who has been ill for a long time forgets his past pain completely when he regains his health, absorbed as he is in the ease of his present condition, so too those who are grounded in pure awareness free of thought-constructs are no longer conscious of their previous [fettered] state. Consciousness, the sole truly existent reality, free of thought- constructs is made fully and evidently manifest by eliminating thesedifferentiated perceptions. The wise man should therefore exert himself to attend closely to this [state of awareness]. I42 The thought-constructs generated within consciousness do not in reality affect it at all. They can neither break up nor add anything to the Light which shines as all things. 143 They are in fact nothing but 192 The Doctrine of Vibration consciousness itself 144 which perceives, through its power of reflective awareness (yimarsa), the multitude of objects in diverse ways, and so assumes this form. 145 Although thought-constructs are mental representations of objects once seen or present, they are products of the power of consciousness and not of the objects they represent. 146 Thought is both analytic and synthetic; 147 it serves the useful purpose of separating individual elements of experience from others and linking together those that appear to be distinct from one another so that they can be better understood. 148 It does not consist merely of false mental constructs projected onto reality that need to be wholly rejected. Thought obscures consciousness and distracts it only when it appears in the form of doubt, vacillating between alternatives. 149 Once this conflicting duality (dvaitddhivasa) 150 is eliminated, thought is purified and rests in itself as the 'thought-less thought' of pure consciousness. 151 By gradually eliminating the multitude of conflicting notions that agitate him, the yogi ultimately achieves the certainty (niscaya) corresponding to a direct awareness of his own divine nature. 152 Abhinava explains: Thought is in reality none other than pure consciousness. Even so, it serves as a means to liberation for the individual soul (anu) only when it takes the form of certainty (niscaya). 153 The yogi must eliminate every doubt and misguided notion that leads him to believe himself to be other than Siva. By developing the thought: 4 1 am Siva', it ultimately affirms itself directly as a pure awareness beyond thought without any intervening mental representations. Abhinava says: Just as the man who thinks intensely that he is a sinner becomes such, just so one who thinks himself to be Siva, and none other than He, becomes Siva. This certainty (ddrtfhya), which penetrates and affirms itself in our thoughts, coincides with an awareness free of thought- constructs engendered by a series of differentiated mental representa- tions, the object of which is our identity with Siva. 154 As thought is gradually purified, it becomes progressively clearer until its object becomes maximally apparent (sphufatama). 155 The stream of perceptive consciousness (pramana) progressively reveals each aspect of its object which, thus affirming itself with increasing clarity, reveals its ultimate nature. The yogi reflects repeatedly upon it as the object of his realisation and loving devotion, for all that is perceptible and need be known (j fie yd) is Siva alone. As Abhinava says: Path to Liberation 193 What should we say of those who before they are satisfied have to see their beloved again and again, caress her and think about her for a long time? 156 The yogi practising the

Empowered Means is initiated into the Great Sacrificial Rite (mahayaga), eternally enacted at the interface between the inner and outer aspects of consciousness, by a direct infusion of awareness from his master who is the embodiment and outer symbol of the yogi's enlightened identity. ,57 The rite begins with ritual bathing (snana) which is in this case the immersion of the body of thought in the white ashes of the cosmic fuel of duality, burnt in the fire of consciousness. 158 The yogi then goes on to worship (puja) by uniting all that is pleasing to the senses in the oneness of consciousness. 159 The ritual formula (mantra) he recites is the eternal resonance of the awareness which is the pulsation of the Heart of his own consciousness. 160 Repetition (Japa) of the formula is every activity, perception, breath or thought which arises within him while plunged in the universal awareness of his true nature. 161 The mental image he visualises meditating (dhyana) on the Deity in the course of the rite, is whatever the yogi spontaneously imagines and contemplates as the outpouring of the universal creativity of consciousness. 162 Ritual gesture (mudra) is whatever bodily posture the yogi may assume when, fully absorbed in consciousness, he moves, staggering about (ghumita) as it were, drunk with the wine of self-realisation. 163 Oblation is performed by offering with devotion and awareness all the sensations which flow in through the channels of the senses to the fire of his subjectivity, which is thus inflamed (uddipita) and makes all things one with itself. 164 The outer ritual which commences in the sphere of the Individual Means thus leads naturally to the inner rite of the Empowered Means. When the yogi's practise (abhyasa) reaches fruition, the rite merges with the spontaneous activity of consciousness. This is fullness (purnata), the completion and reunification of the forces within consciousness which, through the power of ignorance, were formerly dispersed and divided. "Just as a horse driven here and there", writes Abhinava, "over plains, hills and dales follows the will of its rider, so also consciousness, driven by various expedients (bharigi), quiescent or terrific, abandoning duality, becomes Bhairava. Just as by looking repeatedly at one's own face in a mirror one comes to know that it is the same [as the image reflected], so also, [one sees] in the mirror of mental representations of meditation {dhyana), ritual (puja) and worship (area) one's own Self as Bhairava and so quickly identifies with Him. This identification is the realisation that takes place in the absolute (anuttara)." 165 194 The Doctrine of Vibration By ridding himself of the relative distinctions engendered by thought, the yogi practising the Empowered Means, illumined by the power of self- awareness of Pure Knowledge {suddhavidya), transcends the distinction between right and wrong, purity and impurity. He is led to the conviction that the pure consciousness, which is his true nature, is unaffected by whatever action he may do, whether conventionally accepted as good or bad. Abhinava quotes the Malinivijaya as saying: All here is enjoined and all prohibited. This alone, O Lord of the gods, is here prescribed as obligatory, namely that the mind be firmly applied to the true reality. It matters little how this is achieved. He whose mind is firmly established in [this] reality, even if he eats poison, is as little affected by it as are lotus petals by water. 166 Impurity is a state of seeming separation from consciousness. 167 The yogi who has freed himself of all false notions comes to realise that the true nature of consciousness can never be sullied or limited by any object appearing within it. 168 This is the realisation the ancient sages achieved through a direct intuition of reality free of intruding thought-constructs {avikalpabhava), but kept secret in order not to confuse the worldly. 169 Similarly, in reality nobody is ever bound. It is ignorance to believe bondage exists and to contrast it with a conceived state of liberation. 170 If the Self is one with Siva, how can it be either bound or released? 171 Nothing essentially distinguishes those who are bound from those who are free. 172 The difference between their states is merely conceptual. 173 Pure consciousness abides free of all such distinctions. Thus Bhagavatotpala, <n his commentary on the Stanzas, repeatedly stresses that thought- constructs obscure consciousness and misguide the individual soul. 174 Those who are bound are convinced that they are dull witted, conditioned by Karma, sullied by their sin and helplessly impelled to action by some power beyond their control. He who manages to counter this conviction with its opposite achieves freedom. 175 He who considers himself to be free is free indeed, while he who thinks himself bound remains so. Thus at the highest level of realisation, as Abhinava says: Nothing new is achieved nor is that which in reality is unmanifest, revealed- [only] the idea is eradicated that the luminous being shines not. 176 Nothing is impure, all is perfect, including Maya and the diversity it engenders. To say that illusion exists and that ignorance must be Path to Liberation 195 eradicated implies that it has a separate existence apart from conscious- ness. If this is so, it has as little reality as the shadow of a shadow, but if not,then it must be

consciousness itself. Thus, as Kallata says, bondage, the binder and the bound are in fact one. 177 It is Siva Himself Who freely obscures His own nature. Siva binds Himself by Himself. 178 Concealing and revealing Himself, Siva plays His timeless game. At the Divine (sambhava) level of pure Siva-consciousness, the Spanda yogi directly lays hold of the power inherent in his own conscious nature (svabala) which gives life to the psycho-physical organism and impels the senses and mind to action. 179 In this way every thought- construct, and with it the ego, is instantly annulled in the immediacy of the pure subjectivity that remains unaltered throughout every perception and state of consciousness. The same takes place at the Empowered level b> attending to the recurrent activity — Spanda — of the subject, that is, the flux of awareness through the cyclic movement of the powers of consciousness. 180 By attending (avadhana) to this movement the thought- constructs that emerge and subside in the course of perception are seen to be part of this universal process, and, in this way purified, are no longer binding. Thus, Ksemaraja says that the Spanda teachings are concerned most directly with the Empowered Means. m The yogi who is always alert to discern the pulse of Spanda quickly realises his own authentic state of being (nijarfi bhavam). 1 * 2 He is then truly awake, not only literally, but also in the deeper sense that he is awake to his authentic nature, its power and activity. When attention (avadhana) slackens, this movement takes place unconsciously and so the thought-constructs and perceptions generated through it appear to take on an autonomous existence of their own, just as happens when we dream. 183 The spontaneity of the movement that travels between subject and object and holds them together in the pure awareness of the universal subject's identity with his cosmic object devolves into the creative activity of waking and dreaming. Man, in other words, becomes a victim of his states of consciousness and the contents that they, by their very nature, generate within themselves. 184 The Spanda teachings are not only concerned with the structure of thought and its functions, but also with the powers and properties of its vehicle, namely, speech. Speech issues out of consciousness, develops into thought to then become articulated sound. A focal point of Spanda doctrine is thus the role speech plays in the formation of thought- constructs and their purification. Although this takes place at all levels of practise below the Divine (sambhava), the Spanda teachings, meant as they are for advanced yogis, ignore the outer forms of spiritual discipline to concentrate on practise in the Empowered (sakta) psychic sphere (cetas) and what lies beyond it where speech is the pure inner awareness (yimarsa) **Although Abhinava does not differentiate between individual consciousness and cosmic general ledger, one tends to comprehend that right interpretation is the transfor mentioned the individual consciousness to cosmic consciousness responsible for the creation and destruction which the sadhaka sees as his own power. Every one is rama and every one is Shiva. only thing that is essential is striving and concerted efforts and sustained struggle to reach.** The Doctrine of Vibration of the light of consciousness. The Doctrine of Vibration identifies this, the highest level of speech (para vac), with the universal pulse of consciousness that resounds spontaneously within it as the inner flow of its own undifferentiated awareness. 185 Beyond the realms of language, it is the transcendental consciousness in which all language is rooted and pervades all that language denotes as its essential being. Utpaladeva writes: The Supreme Voice is consciousness. It is self-awareness spontaneously arisen, the highest freedom and sovereignty of the Supreme Lord. That pulsing radiance (sphuratta) is pure Being, unqualified by time and space. As the essence [of all things] it is said to be the Heart of the Supreme Lord. 186 When the intention arises within consciousness to discern its own brilliance manifest in the world of denotations and denoted meanings, speech turns from the supreme transcendental level to that of immanence and assumes the form of a pure intuitive awareness (pratibha) which perceives and comprehends its universal manifestation. This is the voice of intuition (pasyanti), which grasps the meaning inherent inwardly in all words and externally in all that they denote. Analogous to the non- discursive, instinctual knowledge animals possess, it is a pure generic perception not yet formed into language in which the act of denotation, its object and that which denotes it are indistinguishable. Illumined by the voice of intuition birds migrate in their due seasons, the cock crows at dawn and young mammals suck at the breast. 187 Infants similarly reflect and respond instinctively to their environment by virtue of this intuitive sense 188 and through it come to grasp the link between words and the objects they denote. As they learn to speak, they begin to form concepts and so the next two levels of speech develop. One is the outer corporeal speech (yaikhari) and the other the subtler, inner discourse (antah- saryijalpa) of

thought that forms at the intermediate level (madhyama) where the ratiocinating mind stands between the higher levels of intuition and its outer verbal expression. In this way the development of speech in infancy reflects its progressive actualisation in every spoken word. A hymn to the Goddess quoted by Bhagavatotpala describes this process well: Therefore, O Supreme Goddess, the highest form of speech should be worshipped as the [universal] cause that establishes the existence of all things by insight {niscaya) into their nature (artha) brought about by their manifestation through the superimposition [of verbal designations]. O Mother, insight into the true nature [of things] is nothing but the Path to Liberation 197 act of intent of that [same speech], apart from which [speech itself and all that it expresses] could not attain to its own nature. Again, in that state [speech] is said to be the light of one's own nature. Free of division and succession it is attainable [only] by the yogi. Then from the state of intent, O Siva, speech [assumes] the nature of thought as the radiant pulse (sphuraria) of desire to speak of that which is in the domain of words. Then consisting of words, it bears a clearly expressed meaning, for if [speech] were not such, meaning could not be understood. 189 personal experience clearly proves that thought is invariably associated with speech. 190 Thought is a function of language. Through it we communicate to ourselves a mental image of the world about us and can construct complex ideas about ourselves. Language is the fabric from which our world of ideas is woven. Mental representation which orders the influx of sensation and presents us with a meaningful, picture of the world, memory, the elaboration of ideas and the shifting tides of emotion are all intimately connected with language and through it to the consciousness which underlies them. To think of language as nothing more than a system of denotation based on a commonly accepted convention (sanketd) fails to fully account forks inherent power to convey meaning (vacakasakti). In order to learn the convention we must be born with an innate ability to grasp meaning, and this ability is not itself learned nor found anywhere within the domain of convention. Lacking this ability we would be caught in an infinitely expanding system of denotation in which each element pointed to some other within it, without ever coming to rest anywhere. Unless we can couple the word 'jar' with the object it denotes, explaining that the word 'pot' is a synonym of the word 'jar' would leave us none the wiser. 191 The connection between word and meaning is only explicable if we postulate that it is an inherent property of the power of awareness to link one with the other. Language must be grounded in the pure cognitive awareness (prama) of consciousness which stands beyond, and yet illumines, the sphere of experience we define and understand through the medium of language. As Abhinava says: Someone may hear another person speak, but if his awareness (prama) is obscured, he is unable to rise, unconscious as he is, to the level of the experiencing subject [who understands] what has been said. He only grasps the outer successive (sound) of what the other person says and thus can only repeat it as would a parrot. An understanding of its meaning presupposes that he has caught hold of his own power of awareness (prama) by attaining the autonomy [of the conscious, universal subject]. **Note that we have defined sachitananada as one which understand Truth and Untruth, Bliss and Unbliss, and Knowledge and ignorance where all these are equal. Truth=Untruth, Bliss=Unbliss and Ignorance=Knowledge** The Doctrine of Vibration Outer, articulate speech consists of a series of ordered phonemic elements produced and combined by the vocal organs to form meaningful words. In order for this to be possible, these elements must also be grounded in consciousness (prama). The articulated phonemes are merely outer, gross manifestations of the phonemic energies (yarnagrama) held in a potential state within consciousness. This 'mass of sounds* (sabdarasi) is the light of consciousness (prakdsd) which makes the universe manifest and contains all things within itself. In other words, it is the totality of consciousness expressed as the collective awareness symbolised by all the letters corresponding to the introverted subjectivity of Siva Himself. The power through which this potential actualises itself into speech and the world of denotation is technically called * Mdtrkd*. It is the reflective awareness (vimarsa) and radiance (sphuratta) of the supreme subject — the 'mass of sounds' (Jabdarasi) — and the undivided wonder Siva experiences when He contemplates the universe He gathers up into Himself in the form of countless words (vdcaka) and their meanings (vdcya). 193 Mdtfkdsakti is manifest in the second movement of consciousness after the primal vibration of the pure luminosity of the 'mass of sounds', as the state of pure potency which arises when its unsullied subjectivity begins to turn away from itself and is associated with faint traces of objectivity (dmrsya- cchdyd). 194 Mdtrkd contains within itself the various aspects of objectivity that, although not yet manifest, are ready to issue forth. Thus this power, at

one with Siva, is called 'Mdtrkd' because she is the mother (mdtrkd) of the universe that she contains within herself as does a pregnant woman her child. 195 The circle of the powers of Mdtrkd (mdtrkdcakra) consists of the phonemic energies contained in AH AM, the universal ego. 196 When grasped in its entirety at its source, these energies elevate the consciousness of the enlightened, but when split up and dispersed give rise to the obscuring forces (kald) which lead the ignorant away from realisation. The fettered soul is ignorant of the pure egoity that is the source of speech and so it generates, through its powers, the many thought-constructs that deprive him of the awareness of unity and obscure Siva's universal activity. 197 The Stanzas on Vibration declare: He who is deprived of his power by the forces of obscuration {kala) and a victim of the powers arising from the mass of sounds (sabdardsi) is called the fettered soul. 198 The powers [of speech] are always ready to obscure his true nature as no mental representation can arise that is not penetrated by speech. 199 The rays of phonemic energies emanate from the light of Siva, the Path to Liberation 199 'mass of sounds' (sabdarasi) in eight groups. They constitute the powers of the inner mental organ and the five senses, figuratively arranged in a circle around the sacred shrine {pi(ha) of Matrka&akti who manifests externally as the body. 200 The eight classes and the names of the deities presiding over them are as follows: 201 Gutturals Brahman! Intellect (buddhi) Palatals Mahegvari Ego (ahankara) Cerebrals Kaumarl Mind (manas) Dentals Narayani Hearing Labials Varahi Touch Semivowels Aindri Sight Sibilants Camuntfa Taste Vowels Mahalak§ml Smell The yogi who grasps the true nature of the power of Matfka and its phonemic forces is liberated 202 by recognising that the activity of the senses and the discursive representations of the mind are in fact emanations of universal consciousness. Conversely, when ignorant, he is affected by its power in its multiple negative aspects known as 'Mahdghora' ('greatly terrible') and, unable to rest within himself free of the sense of diversity, he is constantly disturbed by the flux of extroverted perceptions. Abhinava explains: When the [phonemic energies] are not known to be [emanations of the Lord] they conceal the wonder (camatkara) of consciousness which is the one essential non-discursive awareness [present throughout perception] and even in discursive thought. They obscure it with thought-constructs constituted by the diverse configurations of phonemes and syllables which [although also] a form of the deity [are no longer benevolent but] most terrible. Inducing doubt and fear, they engender the fettered soul's state, bound by the shackles of transmigra tion. But once their true nature is understood correctly in this way, they bestow freedom in this very life This knowledge of their intimate being [at one with the absolute] consists of this, namely, that even in the midst of all these fluctuations, free at their inception of discursive representations, thought-constructs do not conjoin [individualised consciousness] with the wheel of energies consisting of the totality of phonemes, even though [these constructs] are coloured by the many diverse words generated by the aggregate of phonemes. Language has a powerful effect on us. A few words we may hear or read can inspire us with joy, fear or sadness, and the constant inner 200 The Doctrine of Vibration dialogue of thought arouses intense feelings within us. This power hidden in language, which binds us through the thought-constructs it generates, can also be used to free us of them by channeling it through Mantra. Mantric practice begins at the Individual (driava) level where Mantras are recited in consonance with the rising and falling away of the breath. In this way they are charged with the vibration (spandd) of consciousness and, in their turn, make consciousness vibration (Doctrine of Vibration S.G.Dyczkowski) On without knowledge that everything is manifest within consciousness is illusory or unreal in that sense alone. 78 Things are more real or more tangibly experienced according to their own essential nature (svabhava) to the degree in which we recognise that they are appearances (abhasa) within absolute consciousness As Jayaratha says: Just as images manifest in a mirror, for example, are essentially mere appearances, so too are [phenomena] manifest within conscious- ness. Thus, beause they are external, [phenomena] have no being (sattva) of their own. The Lord says this [not with the intention of saying anything about the nature of things] but in order to raise the level of consciousness of those people who are attached to outer things; thus everything in this sense is essentially a mere appearance. [Knowing this], in order to quell the delusion of duality, one should not be attached to anything external. 79 The ultimate experience is the realisation that everything is contained within consciousness. We can discover this in two ways. Either we merge the external world into the inner subject, or we look upon the outer as a gross form of the inner. In these two ways we come to recognise that all things reside within our own consciousness just as consciousness resides within them. This all-

embracing inwardness is only possible if there is an essential identity between the universe and consciousness. The events which constitute the universe are always internal events happening within consciousness because their essential nature is consciousness itself. 80 We can only account for the fact that things appear if there is an essential identity between consciousness and the object perceived. If a physical object were really totally material, that is, part of a reality independent of, and external to, consciousness, it could never be experienced. Abhinava says: The existence or non-existence of phenomena within the domain of the empirical (iha) cannot be established unless they rest within consciousness. In fact, phenomena which rest within consciousness are apparent (prakasamana). And the fact of their appearing is itself their oneness {abheda) with consciousness because consciousness is nothing but the fact of appearing {prakasa). If one were to say that they were separate from the light of [that consciousness] and that they appeared [it would be tantamount to saying that] 'blue' is separate from its own nature. However, [insofar as it appears and is known as such] one says: 'this is blue 1 . Thus, in this sense, [phenomena] rest in conscious- ness; they are not separate from consciousness. The Doctrine of Vibration The universe and consciousness are two aspects of the whole, just as quality and substance constitute two aspects of a single entity. The universe is an attribute (dharma) of consciousness which bears (dharmiri) it as its substance. It is said that 'substance' is that resting in which this entire group of categories manifests and is made effective. Now, if you don't get angry [we insist that] this entire class of worlds, entities, elements and categories (tattva) rests in consciousness and [resting in it] is as it is. Thus consciousness contains everything in the sense that it is the ground or basis (adhard) of all things, their very being (satta) and substance from which they are made. But, unlike the Brahman of the Advaita Vedanta, it is not the real basis (adhisthand) of an unreal projection or illusion. Consciousness and its contents are essentially identical and equally real. They are two forms of the same reality. Consciousness is both the substratum and what it supports: The perceiving awareness and its object. In this respect, the Kashmiri Saiva is frankly and without reserve an idealist. Although he does not deny the reality of the object, his position is at odds with most commonly accepted forms of realism. The realist maintains that the content perceived is independent of the act of perception. The content is only accidentally an object of perception and undergoes no change in the process of being perceived. His contention, however, is essentially unverifiable; to verify it, we would have to know an object without perceiving it. This, from the Kashmiri Saiva point of view, is not possible. Objects of which we have no knowledge may indeed exist, but they are knowable as objects only if they are related to subjects who perceive them. In this sense, if there were no subjects, there could be no objects. 86 The subject, however, as opposed to the object is, in terms of the phenomenology of perception, apparent to himself. He is self-luminous (svaprakdsa). Thus, conscious- ness (the essence of subjectivity) is one s own awareness by virtue of which all things exist. The realist maintains that consciousness clearly differs from its object insofar as their properties are contrary to each other. The Saivite idealist. However, says that the object is a form of awareness (vijnanakara)™ The objective status of the object is cognition itself. Perception manifests its object and renders it immediately apparent (sphuta) to those who perceive it. It does not appear at any other time. If 'blue' were to exist apart from the cognition of k blue\ two things would appear: 'blue' and its cognition, which is not the case. It is the perception of the object which constitutes its manifest nature. An entity becomes an Integral Monism of Kashmiri Saivism object of knowledge not by virtue of the entity itself but by our knowledge of it. If objects had the property of making other objects appear, it would be possible for one object to make another appear in its own likeness. 'Blue' is perceived to be 'blue' because it is manifest as such to the perceiver. As Abhinava points out: The [nature of an] object of knowledge could not be established through a means of knowledge totally unrelated to it — a crow does not become white because a swan [sitting next to it] is white. Perception, on the other hand, is immediately apparent to conscious- ness. It is self-luminous in the sense that it is directly known without need of being known by any ulterior acts of perception and makes its object known at the same time. Adopting the Buddhist Yogacara doctrine that things necessarily perceived together are the same (sahopa- lambhaniyamavada), the Saivite affirms that because the perceived is never found apart from perception, they are in fact identical. Reality (satya) is the point where the intelligible and the sensible meet in the common unity of being; it cannot be said to exist in itself outside, and apart from, knowledge or vision. Bhagavatotpala in his commentary on the Stanzas on Vibration quotes: Once the object is reduced to its authentic nature, one knows [the

true nature of] consciousness. What then [remains of] objectivity? What [indeed could be] higher than consciousness? Consciousness is essentially active. Full of the vibration of its own energy engaged in the act of perception, it manifests itself externally as its own object. When the act of perception is over, consciousness reabsorbs the object and turns in on itself to resume its undifferentiated inner nature. Knowledge (jnana) manifests internally and externally as each individual entity.... Once knowledge has assumed that form it falls back [into itself]. The Yogacara Buddhist similarly maintains that consciousness creates its own forms. But, according to him, because the perceived and perception are identical; there is no perceived object at all. The so-called outer world is merely a flux of cognitions, it is not real. He is firmly committed to a doctrine of illusion. The reality of consciousness from The Doctrine of Vibration his point of view is established by proving the unreality of the universe. "All this consists of the act of consciousness alone", says Vasubandhu, "because unreal entities appear, just as a man with defective vision sees unreal hair or a moon, etc."He points to dreams as examples of purely subjective constructs which appear to be objective realities. The apparent reality dreams possess is not derived from any concrete, objective world, but merely from the idea of objectivity. While the Yogacara does not say that an idea has, for example, spatial attributes, it does have a form manifesting them. While he agrees with the Saiva idealist that appearances have no independent existence apart from their appearing to consciousness, he maintains that for this reason they are unreal. The creativity of consciousness consists in its diversification in many modes having apparent externality; it is not a creation of objects. While the Kashmiri Saivite agrees that the world is pure conscious- ness alone, he maintains that it is such because it is a real creation of consciousness. The effect is essentially identical with the cause and shares in its reality. Matter and the entire universe are absolutely real, as 'congealed' (sty ana) or 'contracted' (samkucita) forms of consciousness. "This God of consciousness", writes Ksemaraja, "generates the universe and its form is a condensation of His own essence (rasa)" m By boiling sugarcane juice it condenses to form treacle, brown sugar and candy which retains its sweetness. Similarly, consciousness abides unchanged even though it assumes the concrete material form of the five gross elements. The same reality thus abides equally in gross and subtle forms. Consequently no object is totally insentient. Even stones bear a trace (vasana) of consciousness, although it is not clearly apparent because it is not associated with the vital breath (prana) and other components of a psycho-physical organism. Somananda goes so far as to affirm that physical objects, far from being insentient, can only exist insofar as they are aware of themselves as existing. The jar performs its function because it knows itself to be its agent. Indeed, all things are pervaded by consciousness and at one with it and hence share in its omniscience. Thus, Siva, Who perceives Himself in the form of physical objects, is the one ultimate reality. "The jar knows because it is of my nature", writes Somananda, "and I know it because I am of the jar's nature. I know because I am of Sadasiva's nature and He knows because He is of my nature; Yajnadatta [knows] because he is of Siva's nature and Siva [knows] because He is of Yajnadatta's nature". Integral Monism of Kashmiri Saivism Everything in this sense is directly perceived by absolute conscious- ness, and this direct perception (pratyaksa) unifies the knowable into a single, undivided whole. This is the central concept behind a doctrine originally expounded by Narasimha called 'the non-dualism of direct perception' (pratyaksadvaita). This states that consciousness is essentially perceptive and that its perception of all things operates throughout the universe. Insofar as phenomena are clearly evident (sphufa) to us, everything is directly perceived by absolute consciousness, with which our individual consciousness is identical. This direct perception unfolds everywhere; the one true reality, it is alone and without companion or rival (nihsapatna). Even though it remains one, it can, by its very nature, perceive distinctions (bheda) between one entity and another, without this engendering any division within it. We distinguish between two entities in empirical terms on the basis of their mutual exclusion (anyonyabhava). The relative distinction {bheda) between them is essentially the perceived difference between their respective characteristics. Despite this difference they are united within the purview of a single cognition insofar as they are equally both manifest appearances. This cognition is the undivided essence (rasa) or 'own nature' (svabhava) of both. Encompassed by the 'fire of consciousness', there is no essential difference between them. Just as when an emerald and ruby reflect each other's light, the ruby is reddish-green and the emerald greenish-red, similarly everything is connected with everything else as part of the single variegated (vicitra) cognition of absolute consciousness.

Mahesvarananda writes: The Supreme Lord's unique state of emotivity (asadharanabhava) is the outpouring of pure Being (mahasatta). It is manifest as the brilliance (sphuratta) of the universe which, if we ponder deeply, [is realized to be] the single flavour (ekarasa) of the essence of Beauty which is the vibration of the bliss of one's own nature. In this way all things are in reality one although divided from the one another sharing as they do the 'single flavour' (ekarasa) of the pure vibration of consciousness. Kashmiri Saiva Realism Kashmiri Saivism as a whole has been variously called a form of 'realistic idealism', 'monistic idealism', 'idealistic monism' and 'concrete monism'. It is easy to understand why Kashmiri Saivism is The Doctrine of Vibration said to be 'idealistic' and 'monistic', but in what sense is it also 'realistic*? The answer to this question is of no small importance in trying to under- stand the central idea behind its metaphysics and the fundamental importance of the concept of Spanda, in this seemingly impossible marriage between monistic idealism and pluralistic realism. The Kashmiri Saiva approach understands the world to be a symbol of the absolute, that is, as the manner in which it presents itself to us. Again we can contrast this view with that of the Advaita Vedanta. The Advaita Vedanta understands the world to be an expression of the absolute insofar as it exists by virtue of the absolute's Being. Being is understood to be the real unity which underlies empirically manifest separateness and as such is never empirically manifest. It is only transcendentally actual as 'being-in-itself. The Kashmiri Saiva position represents, in a sense, a reversal of this point of view. The nature of the absolute, and also that of Being, is onceived as an eternal becoming (satatodita), a dynamic flux or Spanda, 'the agency of the act of being'. It is dentified with the concrete actuality of the fact of appearing, not passive unmanifest Being. Appearance (abhasa) alone is real.ppearing (prakasamanatva) is equivalent to the fact of being (astitva). Ksemaraja writes in his commentary on the Stanzas on Vibration: Indeed, all things are manifest because they are nothing but manifestation. The point being that nothing is manifest apart from manifestation The absolutely unmanifest, from this point of view, can have as little existence as the space in a lattice window of a sky-palace. Nay, even less, because even that space can appear as an imagined mage manifest within consciousness. Everything is real according to the manner in which it appears. Even an illusion is in this sense real, insofar as it appears and is known in the manner in which it appears. The empirical and the real are identical categories of thought. As Abhinava says: Thus this is the supreme doctrine (upanisatf), namely that, when- ever and in whatever form [an entity] appears, that then is its particular nature. Perhaps at this stage a brief comparison with Heidegger's ideas might prove to be enlightening and not altogether out of place. According to Heidegger's phenomenology of Being, reality is intelligible in a two-fold Integral Monism of Kashmiri Saivism manner as 'phenomenon' and 'logos'. Heidegger defines what he means by 'phenomenon' as: "that-which-shows-itself. The manifest . . . phenomena are then the collection of that which lies open in broad daylight or can be brought to the light of day — what the Greeks at times implicitly identified as 'ta onta' (the things-which-are)". 127 In his later writings Heidegger drops the term 'phenomenon' in preference for the verbal form 'phainesthai' in order to emphasize even more the actuality or presentational property of Being. Explaining this new form of the term he writes: "Being disclosed itself to the ancient Greeks as 'physis'. The etymological roots 'phy-' and 'pha-' designate the same thing: 'phyein', the rising-up or upsurge which resides within itself as 'phainesthai', lighting-up, self-showing, coming-out, appearing-forth." Heidegger contrasted his notion of phenomenon with semblance (Schein) and with appearing (Erscheinung). In the case of semblance a thing can show itself as that which it is not, as when fool's gold shows itself to be gold. The ancients always allied semblance with non-being. Heidegger points out, however, that semblances are grounded in showings, and so does Abhinava. Both Heidegger and Abhinava consequently maintain that all semblances have a real basis and are to be treated as instances of phenomena along with the so-called real showing or manifestation of non-deceptive objects. So Heidegger states that: 'how- ever much seeming, just that much being'. Thus self-showing or appearing defines Being as phenomenon, but this definition of Being is as yet incomplete. Being is not only self-showing but 'logos' which Heidegger explains means 'discourse' (Rede) in the sense of 'apophansis': 'letting-be-seen'. Phenomenology, which according to Heidegger is the only correct study of Being, means 'letting-be-seen-that-which- shows-itself. This is true of Saiva Paramadvaita as well. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left

which could be said to be real" A thousand proofs could not make 'blue' other than the colour blue. The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. Although the nature of the absolute is discovered at a higher level of consciousness, The Doctrine of Vibration nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. All things are known to be just as they present themselves. The concrete actuality of being known (pramiti), irrespective of content, is itself the vibrant (spanda) actuality of the absolute. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. "The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. Within the sphere of Maya, every entity's 'own nature' (svabhdva) corresponds to its specific manifest form. Accordingly it is defined as that which distinguishes it from all else and from which it never deviates. Above the sphere of Maya, that is, above the level of objectivity, is the domain of the subject. At this level, everything is realised to be part of the fullness of the experience r and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto. In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri SaivismWe have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. This is the level of consciousness in which the absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature. The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things] and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. Atomic appearings can combine in any

number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. Insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific The Doctrine of Vibration awareness of an object of the form: 'here this is such.' While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. The form of our experience is thus 'I now see this here'. But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. The form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'. The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Integral Monism of Kashmiri Saivism Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; it arises from a distinction between appearances and the individual subject. So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their functions; when they are merged within the subject and at one with him, they cannot do so. Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness. Together these individual pulsations constitute the universal pulse (samanyaspandd) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. Light and Awareness: The Two Aspects of Consciousness Absolute consciousness understood as the unchanging ontological ground of all appearing is termed 'Prakasa'. As the creative awareness of its own Being, the absolute is called ' Vimarsa '. Prakasa and Vimarsa — the Divine Light of consciousness and the reflective awareness this Light has of its own nature — together constitute the

all-embracing fullness (purnata) of consciousness. The Recognition (pratyabhijfia) school of Kashmiri Saivism develops this concept of the absolute which finds its fullest expression in Utpaladeva's Stanzas on the Recognition of God. Even though neither of these two key terms appear in the Stanzas on Vibration or the Aphorisms of Siva, they recur frequently in their commentaries. Thus, although the original formulation of the Doctrine of Vibration differs from the theology of Recognition in this respect, it was extended in the course of its development to accommodate this concept of the absolute as well. This was possible, and quite justified, insofar as the absolute understood in Pratyabhijfia terms does not, as we shall see, differ essentially from that of the Spanda school. We can, as Kashmiri Saivites themselves have done, explain one in terms of the other. The Doctrine of Vibration Prakasa: The Light of Consciousness Prakasa is the pure luminosity' (bhdna) or 'self-showing' that constitutes the essence and ultimate identity (atman) of phenomena. That things appear at all is due to the light of consciousness, and their appearing (avabhasana) is itself this Light which bestows on all things their evident, manifest nature. Established in the light of consciousness everything appears there according to its own specific nature (svabhava). Anything that supposedly does not rest in this Light is as unreal as a sky-flower. 3 Thus, according to Rajanaka Rama, unlike the light of the sun, or any other light, this Light not only makes all things apparent, it is also their ultimate source. 4 Full of its divine vibration the Light makes all things manifest and withdraws them into itself. This supra- temporal activity characterises it most specifically; devoid of it, it would be no better than an inert physical phenomenon. At the same time, this light is the conjunction (slesa) or oneness (aikdtmya) of its countless manifest forms, 6 and the collective whole (sampina'ana) of all the categories of existence. The universe is nothing but the shining of the Light within itself. It is the radiant vibration (sphuratta) of this Light, the state (avast hand) in which consciousness becomes manifest.Although the Light shines as all things at all times and hence also makes their diversity manifest, 9 penetrating each object individually as well as collectively, it is not totally 'merged' (magna) or identified with the object so as to suffer any division within itself. Our experience of any object is of the form: T see this': it is not itself an object, but the manifest form the object assumes as a luminous principle of experience. The Light is ever revealed and can never be obscured; objectivity can never cast a shadow on the light of consciousness. The Stanzas on Vibration declare: That in which all this creation is established and from whence it arises is nowhere obstructed because it is unconditioned by [its very] nature. This Light is the highest reality (paramartha). It is the 'Ancient Light' (puranaprakasa) that makes all things new and fresh every moment. It is 'always new and secret, ancient and known to all'. It is the form of the Present (vartamanarupa), the Eternal Now. Time and space are relations between the contents of consciousness; they cannot impinge on the integrity of the absolute itself. 16 Neither space nor time can divide it, for they are one with the Light that illumines them Light and Awareness: Two Aspects of Consciousness and makes them known as elements of experience. But this Light is the shining of the absolute; it is not an impersonal principle. It is the living Light of God, indeed it is God Himself, the Master Who instructs the entire universe. 18 Siva is this 'auspicious lamp', Who illumines all things. He is the Light of consciousness that reveals the presence of both the real and the unreal, of light' and 'darkness'. 20 Abhinavagupta writes: Thus Bhairava, the Light, is self-evident (svatahsiddha); without beginning, He is the first and last of all things, the Eternal Present. And so what else can be said of Him? The unfolding of the categories of existence (tattva) and creation, which are the expansion of His own Self, He illumines, luminous with His own Light, in identity with Himself, and because He illumines Himself, so too He reflects on His own nature, without His wonder (camatkara) being in any way diminished. Although It Is Possible To Catch Glimpses Of The Highest Reality In Advanced States Of Contemplation Before Attaining Perfect Enlightenment, These States, However Long They Last, Are Transitory (Kadacitka) And When They End The Vision Of The Absolute Ceases With Them. The Highest Realisation, However, Persists In All States Of Consciousness. It Happens Once And Need Never Occur Again. A Passage From A Lost Tantra Declares: "The Self Shines Forth But Once, It Is Full [Of All Things] And Can Nowhere Be Unmanifest." All Spiritual Discipline Culminates In This Moment Of Realisation. Accordingly, Abhinava Stresses That The Goal Of All The Means To Realisation, Even The Individual Means, Is This Absolute Consciousness. Finally, It Is Worth Noting That Although Abhinava Affirms That The Teachings Concerning Anupaya Are Found In The Siddhayogesvarimata And The Malinxvijaya, Both Of Which, According To Abhinava,

Are Major Tantras Of The Trika School, It Is In The Theology Of The School Of Recognition That It Is Best Exemplified. Abhinava Himself Refers To Somananda, The Founder Of This School, As Teaching It And Alludes To The Following Passage In The Vision Of Siva To Support His Own Exposition: When Siva, Who Is Everywhere Present, Is Known Just Once Through The Firm Insight Born Of Right Knowledge (Pramana), The Scripture And The Master's Words, No Means [To Realisation] Serves Any Purpose And Even Contemplation {Bhavana) [Is Of No Further Use]. Anupaya Is Therefore, According To Abhinavagupta, The Recognition Of One's Own Authentic Siva-Nature, Which All The Higher Tantric Traditions Teach Is The Ultimate Realisation. This Is Also True Of The Doctrine Of Vibration Whose Precedents Are Clearly Traceable To These Same Traditions. Thus, Although The Stanzas Themselves Never Refer Directly To Enlighten- Ment As An Experience Of Recognition, There Can Be Little Doubt That Spanda Practice Leads To This Same Realisation. Accordingly, Commentators Stress That We Realise The Vibration Of Consciousness By Recognising Its Activity And That Liberation Depends On The Recognition Of This As One's Own Nature. Ksemaraja Describes What Happens In This Moment Of Recognition According To The Doctrine Of Vibration Thus: At The End Of Countless Rebirths, The Yogi's [Psycho-Physical] Activity [Which Issues From Ignorance] Is Suddenly Interrupted By The Recognition Of His Own Transcendent Nature, Full Of A Novel And Supreme Bliss. He Is Like One Struck With Awe. And In This Attitude Of Astonishment (Vismaya- Mudra) Achieves The Great Expansion [Of Consciousness] {Mahavikasa). Thus He, The Best Of Yogis, Whose True Nature Has Been Revealed [To Him] Is Well Established [At The Highest Level Of Consciousness], Which He Grasps Firmly And His Hold Upon It Never Slackens. Thus He Is No Longer Subject To Profane Existence (Pravftti), The Abhorrent And Continuing Round Of Birth And Death, Which Inspires Fear In All Living Beings, Because Its Cause, His Own Impurity, No Longer Exists. The Divine Means (Sambhavopava) In Anupaya The Yogi Does Not Need To Deal With The World Of Diversity At All; Only Paramasiva Exists There. Beyond Both Immanence And Transcendence, He Has Nothing To Do With The World Of Practice And Realisation. Anupaya Is The Experience Of The Undefinable (Qnakhya) Light Of Consciousness, Which Is The Pure Bliss Beyond Even The Supreme State (Paratlta) Of Sivatattva. At A Slightly Lower Level, Corresponding To The Divine Means, A Subtle Distinction Emerges Between The Goal And The Path. The Yogi Now Practises Within The Domain Of The Outpouring Of The Power Of Consciousness. From This Level He Penetrates Directly Into The Universal Egoity Of Pure Consciousness By The Subtle Exertion (Udyama) Of Its Freedom (Svatantrya) And Reflective Awareness. The Yogi Who Practises The Divine Means Is Not Concerned With Any Partial Aspect Of Reality But Centers His Attention Directly On Its Abounding Plenitude. Hence This Means Is Based On Siva's Own State (Sambhavavastha) In Which Only The Power Of Freedom Operates As The Pure Being (Satta) Or Essence Of All The Other Powers. This State Is The Light Of Consciousness Which, Free Of All Thought- Forms, Is The Basis Of All Practice." The Yogi Who Recognises That Pure Consciousness, Free Of Thought-Constructs (Nirvikalpa), Is His Basic State, Can Practice In Any Way He Chooses; Even The Most Common Mantra Will Lead Him Directly To The Highest State. Thus The Forms Of Contemplative Absorption, Empowered (Sakta) And Individual (Anava), Which Are The Fruits Of The Other Means To Realisation Both, Attain Maturity In This Same Undifferentiated Awareness. This Awareness Is The Pure Ego Manifest At The Initial Moment Of Perception (Prathamikalocana), When The Power Of The Will To Perceive Is Activated. It Is The Subtle State Of Consciousness That Reveals The Presence And Nature Of Its Object Directly: That Which Shines And Is Directly Grasped In The First Moment Of Perception While It Is Still Free Of Differentiated Representations And Reflects Upon Itself Is [The Basis Of The Divine Means] Said To Be The Will. Just As An Object Appears Directly To One Whose Eyes Are Open Without The Intervention Of Any Mental Cogitation (Anusaipdhana), So, For Some, Does Siva's Nature. The Movement Of Awareness At This Level Of Practice Attains Its Goal Quickly. While Consciousness Is Heightened Progressively In The Other Means, Here It Expands Freely To The Higher Levels, Unconfined By Any Intruding Thought-Constructs. The Divine Means Is A 'Thoughtless Thought', A 'Processless Process', That Occurs At The Juncture Between Being And Becoming. Abhinava Explains: When The Heart [Of Consciousness] Is Pure And [Free Of Thought- Constructs], It Harbours The Light Which Illumines The Radiant, Primordial Plane Ipragrabhumi) Together With All The Categories Of Existence. [The Yogi] Then Realises Through It His Identity With Siva Who

Is Pure Consciousness The Yogi Must Catch The Initial Moment Of Awareness (Adiparamarsa) Just When Perception Begins^ He Must Not Move On From The First Pure Sensation Of The Object But Return To Its Original Source In His Own T Consciousness. Observing In This Way The Objective Field Of Consciousness Without Laboring To Distinguish Particulars, The Yogi Penetrates Into His Own Subjectivity Which, Vacuous And Divested Of All Outer Supports (Niralamba), Is Not Directed Anywhere Outside Itself (Ananyamukha- Preksin). Here He Can Hold Of The Power Inherent In His Own Consciousness Through Which He Discerns The True Nature Of Whatever Appears Before Him. Thus The Stanzas On Vibration Teach: Just As An Object, Which Is Not Seen Clearly At First Even When The Mind Attends To It Carefully, Becomes Later Fully Evident When Observed With The Effort Exerted Through One's Own [Inherent] Strength (Svabala), In The Same Way, When [The Yogi] Lays Hold Of That Same Power, Then Whatever [He Perceives Manifests To Him] Quickly According To Its True Nature, Whatever Be Its Form, Locus, Time Or State. Thus, Although The Practice Of This Divine Means Starts By Catching Hold Of The Will In The First Moment Of Awareness, It Also Concerns The Second And Third Moments In Which The Means Of Knowledge And The Object Are Made Manifest. When Practice At This Level Proceeds Smoothly And Without Interruption, The Three Powers Of Will, Knowledge And Action Fuse Into The Trident (Trisula) Of Power, Which Is The Subject Free Of All Obscuration (Niranjana), M At One With The Power Of Action In Its Most Powerful And Evident Form. The Kaula Schools Call This State The Stainless (Niranjanatattva). Equated In The Spanda Tradition With The Dawning Of The Vibration Of Consciousness {Spandodaya), It Is The Enlightenment The Spanda Yogi Seeks. Spanda Practice Is Based On The Experience Of Spanda Which, As We Have Seen, Is Defined As The Intent (Aunmukhya) Of Consciousness, Unrestricted To Any Specific Object And Hence Free Of Thought-Constructs. Spanda Can Therefore Be Experienced Directly When A Powerful Intention Develops Within Consciousness, Whatever Is Its Ultimate Goal Or Cause. We Have Already Noted That Intense Anger, Joy, Grief Or Confusion Is Such Occasions. Similarly, The Yogi Can Make Contact With The Omnipotent Will, Which He As Siva Possesses, Through Intense Prayer. Directing His Entire Attention To Siva, The Benefactor Of The World, Entreating Him Fervently And Without Break, His Will Merges With Siva's Universal Will, Which Is The Source Of Every Impulse And Perception. As He Looks About Him, The Yogi Realises That It Is Siva Himself, The Universal Consciousness And The Yogi's Authentic Identity, Who Ordains His Every Action, Thought And Perception. Thus The Yogi's Cognitive Intent On His Object Coincides With The Universal Will To Make That Object Known To Him, Whether The Yogi Is Awake Or Dreaming. He Is Thus No Longer Like The Worldly Man Who Cannot Dream As He Wishes And Is Forced To Experience Whatever Spontaneously Happens In These States Of Consciousness. Ultimately The Yogi Manages, By Siva's Grace, To Maintain A Constant Awareness Of His Own Pure Perceptive Consciousness (Upalabdhrta) Divested Of All Obscuring Thought-Constructs In Deep Sleep As Well As In The Contemplative State (Turiya) Beyond It. When He Rises To The Higher Levels Of Contemplation In Which The Breath Is Suspended And All Sensory And Mental Activity Ceases, The Yogi Who Manages To Sustain This Pure, Undifferentiated Awareness Does Not Succumb To Sleep As Do Less Developed Yogis. Perfection In The Practice Of The Divine Means Thus Coincides With The Goal Of Spanda Practice, Namely, A Constant, Alert Attention To The Perceiving Subjectivity Which Persists Unchanged In Every State Of Consciousness Both As The Perceiver And Agent Of All That It Experiences. Another Important Spanda Practice Belonging To This Means Is Centering. The Spanda Yogi Seeks To Find The Centre (Madhya) Between One Cognition And The Next; For It Is There That He Discovers The Expansion (Unmesd) Of Consciousness Free Of Thought-Constructs From Whence All Differentiated Perceptions (Yikalpa) Emerge. 107 Abhinava Explains That This Pure Awareness Is Called: . . . The Expansion (Unme$A) Of [Consciousness] Or The Creative Intuition {Pratibha) [Experienced] In The Interval Which Divides Two [Moments] Of Differentiated Perception (Vikalpa). It Is Here That They Arise And Disappear. The Sastras And Agamas Proclaim With Reasoned Argument That It Is Free Of Thought-Constructs {Nirvikalpa) And Precedes All Mental Representations Of Any Object. None Can Deny That A Gap Exists Between Perceptions Insofar As Two Moments Of Thought Are Invariably Divided. This [Gap] Is The Undifferentiated Unity Of All The Countless Manifestations. Similarly, In The Outer More Objective Sphere, Where Change Consists Of The Alterations In The Configurations Of Manifest Appearances (Abhasa\ The

Transition From One To Another Corresponds To A Phase Of Pure Luminosity That Marks The Beginning Of One Form And The End Of Another. The World Of Manifestation And Differentiated Perceptions (Yikalpa) Thus Extends From One Centre To The Next. Although It Is Never In Fact Divorced From The Subject Who Resides There, The Ignorant Fail To Grasp This Fact And So, Cut Off From The Centre, The World Of Objectivity Becomes For Them The Sphere Of Maya. Bhagavatotpala Quotes The Light Of Consciousness (Samvitprakdsa): This Ever Pure Experience (Suddhanubhava) Is Variegated By Each Form [Revealed Within It]; Even So It Remains Unstained (Nirmala) When Moving To Another. Just As A Cloth Which Is Naturally White, Once Dyed, Cannot Change Colour Without [First] Becoming White Again, Similarly The Pure Power Of Awareness, (Citi) Once Coloured By Form, Is Pure [Again] At The Centre Where That Form Is Abandoned And From Whence It Proceeds To Another. In His Essence Of Vibration (Spandasarpdoha), K§Emaraja Explains That The Rise And Fall Of Every Individual Perception In The Field Of Awareness Is A Specific Pulsation Of Consciousness. From The Point Of View Of The Object, The Expansion (Unmesa) Of This Pulse Is Represented By The Initial Desire To Perceive (Didrksa) A Particular Object, While The Contracted (Nimesa) Phase Is The Withdrawal Of Attention From The Object Previously Perceived. From The Point Of View Of The Perceiving Subjectivity, The Phases Are Reversed, So That The Initial Desire To Perceive Marks The Contraction (Nimesa) Of Subjective Consciousness While The Falling Away Of The Previous Perception Is Its Expansion (Unmesa). At The Higher Level, Where These Two Phases Are Experienced Within Consciousness, They Represent The State Of The Categories Of Isvara ('This Universe Is Me') And Sadasiva ('I Am This Universe'). Utpaladeva Says: Expansion (Unme$A\ Which Is In The External Manifestation [Of Objectivity], Is Kvaratattva While Contraction (Nime$A), Which Is In The Internal Manifestation [Of Subjectivity], Is Sadasiva. At This Level All The Powers Of Consciousness Fuse And Both Phases Are Manifest As Part Of One Reality. This Unity Is In Fact Apparent To Everybody At Each Moment. However, Within The Domain Of Maya, Which Is The Sphere Of Differentiated Perceptions (Yikalpa), It Is Clearly Manifest Only At The Juncture (Madhya) Between Two Cognitions. In This Centre Resides The Void (Kha) Of Consciousness (Free Of Thought-Constructs) Which, Divested Of Diversity, Digests Into Itself All The Psycho-Physical Processes That Give Life To The Multiplicity Of Perceptions. The Yogi Moves From The Particular Vibrations Of Consciousness At Its Periphery To The Universal Throb Of The Heart In The Centre. As Abhinava Explains: The Self-Reflective Awareness In The Heart Of Pure Consciousness, Present At The Beginning And End Of Each Perception, Within Which The Entire Universe Is Dissolved Away Without Residue, Is Called In The Scriptures, The Universal Vibration Of Consciousness (Samanyaspanda) And Is The Outpouring (Uccalana) [Of Awareness] Within One's Own Nature. All The Categories Of Existence (Tattvas) Are United In The Heart Of The Centre Where The Life-Giving Elixir Of Siva's Consciousness Floods One's Own Inner Nature. To Reside In The Centre Is To Abide By The Law Of Totality (Gramadharma) In A State Which Transcends The Workings Of The Mind (Unmana). Consciousness (Jnana) With Light As Its Support, Residing In The Centre Between Being And Non-Being Is Known As The Act Of Abiding In One's Own Abode As The Perceiving Subjectivity (Dratfrtva) Free Of All Obscuration. That Which Has Been Purified By Pure Awareness (Suddhavijnana) Is Called The Transcendent (Viviktavastu), Said To Be The Mode Of Being (V/7//) Of The Law Of Totality (Gramadharma) Through Which Everything Is Easily Attainable. The Power In The Centre (Madhyasakti) Is The Eternal Present. Beyond Time It Is The Source Of Both Past And Future. To Be Established There Is To Abide Without A Break In Rama, The Supreme Enjoyer, In Every Action Of One's Life. Rama Is Siva, The Supreme Cause Who Pervades The Fourteen Aspects Which Embrace The Entire Universe Of Experience, Namely, Moving, Standing, Dreaming, Waking, The Opening And Closing Of The Eyes, Running, Jumping, Exertion, Knowledge [Born] Of The Power Of The Senses, The [Three] Aspects Of The Mind, Living Beings, Names And All Kinds Of Actions. By Developing An Awareness Of The Centre, The Yogi Experiences The Bliss Of Consciousness. Through This Gap He Plunges Into Introverted Absorption (Nimilanasamadhi) And Then Emerges Again To Pervade The Field Of Awareness Between Centres And So Experience The Cosmic Bliss (Jagadananda) Of The Universal Vibration Of Consciousness. 1 I8 He Then Recognises That This State Pervades Every Aspect Of Experience. In This Way The Yogi's Consciousness Is No Longer Afflicted By The Power Which Obscures It, Hemming The Centre In On Both

Sides With Thought-Constructs That Seemingly Deprive It Of Its Fullness. As He Realises Directly His Pure Conscious Nature As The Universal Ego Free Of All Mental Representations, It Expands Out To Embrace All Things Within Itself. Thus The Realisation The Divine Means Leads To, And Is Directly Based Upon, Is That This Pure Ego Is In All Things Just As All Things Are Within It. In The Spanda Tradition, As Recorded In The Stanzas On Vibration, No Such Ego Is Recognised. ' I9 Man's Authentic Nature Is, However, Understood In Personal Terms As Every Individual's Own 'Own Nature* (Svasvabhava) Which Is Siva, The Universal Vibration Of Pure Subjectivity (Upalabdhfta). It Is Not Surprising, Therefore, That Later Commentators Found These Two Conceptions To Be Essentially The Same And Accordingly Identified One's Own Inner Nature With The Pure Ego. This Came As A Natural Development In Spanda Doctrine Not Only For This Reason But Also Because The Universal Ego Is Experienced As The Inner Dynamics Of Absolute Consciousness. To Conclude Our Summarial Exposition Of The Divine Means, Which Is Centred On The Direct Experience Of This Pure Ego (And Hence On Spanda In This Form), We Turn Now To A Brief Description Of Its Inner, Cyclic Activity. We Shall Do This By Examining Abhinava' S Esoteric Exegesis Of The Symbolic Significance Of The Word 'A HAM', Which In Sanskrit Means T, And Symbolises By Its Form The Ego's Dynamic Nature. The Objective World Of Perceptions Is, As We Have Seen, Essentially A Chain Of Thought-Constructs (Prapanca) Closely Linked To One Another And Woven Into The Fabric Of Diversity (Vicitrata). This Thought (Vikalpa) Is A Form Of Speech (Vac) Uttered Internally By The Mind (Citta), Which Is Itself An Outpouring Of Consciousness. Consciousness Also, In Its Turn, Resounds With The Silent, Supreme Form Of Speech {Para Vac) Which Is The Reflective Awareness Through Which It Expresses Itself To Itself. Consequently, The Fifty Letters Of The Sanskrit Alphabet, Which Are The Smallest Phonemic Units Into Which Speech Can Be Analysed, Are Symbolic Of The Principal Elements Of The Activity Of Consciousness. Letters Come Together To Generate Words And Words Go On To Form Sentences. In The Same Way The Fifty Phases In The Cycle Of Consciousness Represent, In The Realms Of Denoted Meaning (Vacya), The Sum Total Of Its Universal Activity (Kriya) Corresponding To The Principal Forces (Kala) Which Come Together To Form The Metaphysical Categories Of Experience, Which In Their Turn Appear In The Grossest, Most Explicitly 'Articulate' Form As The One Hundred And Eighteen World-Systems (Bhuvana). 'A', The First Letter Of Both AHAM And The Sanskrit Alphabet, Is The Point Of Departure Or Initial Emergence Of All The Other Letters And Hence Denotes Anuttara — The Absolute. 'Ha', Is The Final Letter Of The Alphabet And Represents The Point Of Completion When Ail The Letters Have Emerged. It Represents The State In Which All The Elements Of Experience, In The Domains Of Both Inner Consciousness And Outer Unconsciousness, Are Fully Displayed. It Is Also The Generative, Emission (Visarga) Which, Like The Breath, Casts The Inner Into The Outer, And Draws What Is Outside Inward. The Two Letters 'A' And 'Ha' Thus Represent Siva, The Transcendental Source And Sakti, His Cosmic Outpouring That Flows Back Into Him. The Combined 'A-Ha' Contains Within Itself All The Letters Of The Alphabet — Every Phase Of Consciousness, Both Transcendental And Universal. (For A Graphic Representation Of This Analysis, See Figure 1.) M, The Final Letter Of AHAM, Is Written As A Dot Placed Above The Letter Which Precedes It. It Comes At The End Of The Vowel Series And Before The Consonants And So Is Called 'Anus Vara '(Lit. 'That Which Follows The Vowels') And Also 'Bindu ' (Lit. 'Dot,' 'Drop,' 'Point' Or 'Zero'). While The Consonant 'M' Symbolises The Individual Soul (Purusa), 'Bindu 1 Rep Resents The Subtle Vibration Of T, Which Is The Life Force (Jivakala) And Essence Of The Soul's Subjectivity Manifest At The Transcendental, Supra-Mental Level (Unmana). 12 ° It Is The Zero-Point In The Centre Between The Series Of Negative Numbers, In This Case The Vowels Which Represent The Processes Happening Internally Within Siva, And The Series Of Positive Numbers — The Consonants Which Symbolise The Processes Happening Externally Within Sakti. Bindu, As A Point Without Area, Symbolises The Non-Finite Nature Of The Pure Awareness (Pramitibhava) Of AHAM. It Is The Pivot Around Which The Cycle Of Energies From 'A' To 'Ha' Rotates, The Void In The Centre From Which All The Powers Emanate And Into Which They Collapse. As Such, It Is The Supreme Power Of Action Which Holds Subject, Object And Means Of Knowledge Together In A Potential State In The One Light That Shines As All Three Containing Them In Its Repose (Visrdnti). Bindu Is The 'Knower' (Jndtr), Who Is Essentially Consciousness That, Though Omniscient, Does Not Manifest Its Intelligence, Like A Man Who Knows The

Scriptures But Having No Occasion To Explain Them To Others Silently Bears This Knowledge Within Himself. As Such, It Symbolises The Union Of Siva And Sakti (Sivasakti- Mithunapina'a) In A State Of Heightened Potency In Which They Have Not Yet Divided To Generate The World Of Diversity. It Stands, In Other Words, At The Threshold Of Differentiation In The Stream Of Emanation Still Contained Within Siva. The Absolute. Expansion Commences Bindu — The Individual Soul Withdrawal Commences Then, To The Degree In Which That Which Is To Be Accomplished By The Power Of Action Residing Within It [As A Potential] Penetrates Into The Absolute, It Appears Initially As Bindu, Which Is The Light Of Pure Consciousness. When Outer Objectivity Is Reabsorbed Into Its Transcendent Source, Bindu Is The Point Into Which All The Manifest Powers Of Consciousness Are Gathered And Fused Together. The Universal Potency Of All The Letters Is Thus Contained In Bindu Which, As The Reflective Awareness Of Supreme T Consciousness, Gives Them All Life. Thus Bindu Also Marks The Beginning Of Siva's Internal Movement Back To The Undifferentiated Absolute And So Stands At The Threshold Of Both Emission And Absorption Without Being Involved In Either. The Three Aspects Of AH AM Together Constitute A Movement From The Undifferentiated Source Of Transcendental Consciousness — 'A' — Through The Expansion Or Emission Of Its Power — 4 Ha' — To The Subject — 'M 1 —Which Contains And Makes Manifest The Entire Universe Of Experience. The Reverse Of This Movement, That Of Withdrawal (Samhara), Is Represented By M-Ha-A. AHAM And M-Ha-A Alternate In The Rotation (Ghurnana) Of The Reflective Awareness Of T Consciousness As Immanent Sakti Emerges From Transcendental Siva To Then Merge Back Into Him. As Abhinava Says: The Universe Rests Within Sakti And She On The Plane Of The Absolute (Anutiara) And This Again Within Sakti ... For The Universe Shines Within Consciousness And [Consciousness Shines] There [Within The Universe By The Power Of] Consciousness. These Three Poles, Forming A Couple And Merging, Make Up The One Supreme Nature Of Bhairava Whose Essence Is AHAM- At The Microcosmic Level, Fc A' Represents The Initial Moment When The Subject Begins To Rise Out Of Himself To View The Object. The Movement From 'A To 'Ha' Marks The Emergence Of Sensation Within The Field Of Awareness, Which Is Represented By The Fifty Letters Of The Alphabet Symbolic Of The Fifty Aspects Of The Flux Of Consciousness Leading To Objectified Perception. 'Nl' Is Tne Subject Who, Resting Content Within Himself When He Has Perceived His Object Merges Through The Inner Flow Of Awareness Into K A\ The Absolute. Then From The Absolute (A) Its Emission (Ha) Flows Back Into The Pure Subject (M) Set To Perceive His Object. Thus All The Cycles Of Creation And Destruction Are Contained Within AHAM Through Which They Are Experienced Simultaneously As The Spontaneous Play Of The Absolute. The Yogi Who Recognises This Recurrent Pulse Of Awareness To Be The Movement Of His Own Consciousness Merges His Limited Ego With The Universal Ego. Thus He Realises That Its Power To Create, Sustain And Destroy All Things Is His Own Inner Strength (Svabaia) That He Exerts Effortlessly In The Same State Of Mystical Absorption (Turlya) In Universal Consciousness That The Absolute Itself Enjoys. In This Way He Shares In The Three-Fold Awareness Siva Himself Has Of His Own Nature Which Abhinava Describes As Follows: I Make The Universe Manifest Within Myself In The Sky Of Consciousness. I, Who Am The Universe, Am Its Creator! ' — This Awareness Is The Way In Which One Becomes Bhairava. 'AH Of Manifest Creation(Sadadhvari) Is Reflected Within Me, I Cause It To Persist 1 — This Awareness Is The Way In Which One Becomes The Universe. The Universe Dissolves Within Me. I Who Am The Flame Of The [One] Great And Eternal Fire Of Consciousness' — Seeing Thus One Achieves Peace. The Experience Of The Liberated Thus Coincides With The Realisation Of Their Own Divine Nature Which, Through Its Power, Rules And Guides The Cosmic Order. Thus This Attainment (Siddhi), Which Is Liberation Itself, Is In The Doctrlie Of Vibration Technically Called 'Mastery Over The Wheel Of Energies' (Cakresvaratvasiddhi) Because The Liberated Soul, Identified With Siva, Now Governs, As Does Siva, The Cycle Of The Powers That Bring About The Creation And Destruction Of All Things, The Empowered Means (Saktopaya) All The Practices Taught In The Stanzas On Vibration Are Internal. Whenever Ritual Is Mentioned, It Is Invariably Interpreted In Terms Of The Dynamics Of The Inner Processes The Yogi Experiences And Implements In The Course Of His Yogic Practice. The Doctrine Of Vibration, Ksemaraja Affirms, Is Concerned Entirely With These Inner Disciplines Centred, As It Is, In One Way Or Another, On Consciousness Or, At Least, On The Inner Activity Of The Mind. Thus The Empowered Means Which, Like

The Other Categories We Have Discussed, Is Entirely Internal Includes An Important Part Of Spanda Practice. Spanda Practice Belonging To The Divine Means Centres On One's Own Inherent Nature (Svasvabhava) As Siva, The Universal Perceiver And Agent, That Belonging To The Empowered Means On His Power Instead Of Arriving Directly At The All-Embracing Emptiness Of Subjective Conscious- Ness, The Yogi Practising The Empowered Means Realises His True Nature Through The Fullness Of Its Energy. Practising The Divine Means, The Yogi Plunges, As It Were, Straight Into The Fire Of Consciousness; Practising The Empowered Means He Merges With Its Rays. Either Way The Yogi Is Centred Equally On Ultimate Reality. The Power Of Consciousness Is No Less Absolute Than Its Possessor. To Make This Point Abhinava Quotes The Matanga- Tantra: This Reality Consists Of The Rays Of [Siva's] Power And Is Variously Said To Be The Abode Of The Lord's Manifestation . . . That Same [Power] Illumined [By Siva] Is Itself Also Luminous, Unshaken And Unmoving. That Very [Power] Is The Supreme State, Subtle, Omnipresent, The Nectar Of Immortality, Free Of Obscuration, Peaceful, Yearning For Pure Being Alone {Vastumatra) And Devoid Of Beginning And End. Perfectly Pure, It Is Said To Be The Body [Of Ultimate Reality]. The Yogi Concentrates On The Powers Operating In All Of Life's Activities As Particular Pulsations (Visesaspanda) In The Universal Rhythm (Samanyaspanda) Of The Power Of Consciousness. In This Way He Rises Progressively From The Particular To The Universal Until He Reaches Pure Being (Satta), The Greatest Of All Universals (Mahasamanyd) And The Highest Form Of Siva's Power. Thus The Creative Power Of Maya, Manifest Through Countless Lesser Powers, No Longer Causes The Yogi To Stray From Siva's Consciousness But Becomes The Means Through Which It Can Be Realised In The Illuminating Brilliance (Sphuratta) Which Is Siva's Pure Being. Thus By Discovering The True Nature Of Sakti, The Yogi Realises Himself To Be Siva, Its Possessor Who Consists Of All Its Countless Powers. Thus Practise Belonging To This Means Leads To The Same Pure Consciousness Free Of Thought-Constructs Realised Through The Divine Means. Although The Ultimate Realisation Is Instantaneous, The Yogi Rises To It Gradually By Freeing His Consciousness Of The Limitations Imposed Upon It By Thought. Abhinava Explains: The Same Occurs In The Empowered Means [As Does In The Divine]. At The Discursive Level Of Consciousness {Yaikalpikibhumi) [Where The Empowered Means Functions] Knowledge And Action, Although Evident, Are, For The Reasons Explained Previously, Contracted. A Blazing Energy [Is Revealed Within] The One Who Dedicates Himself To Removing The Burden Of This Contraction. [This Energy Eventually] Brings About The Inner Manifestation {Antarabhasa) Of Pure Consciousness He Seeks. Consciousness Is Individualized And Its Power Of Knowledge And Action Contracted By The Thought-Constructs Born Of Ignorance. The Arising Of These Mental Representations, As The Stanzas On Vibration Say, Deprives The Soul Of Its Freedom And Immortal Life. The Practise Of The Empowered Means Is Meant To Free The Fettered Soul Of This Constriction On His Consciousness. It Operates Within The Mental Sphere (Cetasy™ And Is Designed To Purify Thought (Vikalpasamskara) In Order To. Reveal The Pure Consciousness Which Is Its Ground And Ultimate Source. Thus, The Empowered Means Is Concerned With The Second Instant Of Perception, During Which The Subject Forms Mental Representations Of His Object. Thought Functions On The Basis Of An Awareness Of Relative Distinctions Between Specific Particulars, Distinguishing Them From One Another And Thus Seemingly Fragmenting The Essential Unity Of Reality. The Vibrant Vitality Of Consciousness, Universally Manifest, Is Clouded Like A Mirror By A Child's Breath And The Soul Is Deprived Of The Liberating Intuition Of The One Reality Free Of Thought-Constructs (Nirvikalpa). Abhinava Writes: The [Fettered Soul] Is Like A Dancing Girl Who Although Wishing To Leave The Dancehall Is Collared By The Doorkeeper Of Thought And Thrown Back Onto The Stage Of Maya All Thought Is Centred On Objectivity And Hence Dislodges Awareness From The Plenitude Of Pure Subjective Consciousness. Thus, To Regain The Original State Of Rest (Visranti) Consciousness Enjoys, The Yogi Must Rid Himself Of Thought. As Thought-Forms Decrease, Pure, Thought-Free Awareness Is Strengthened Until The Yogi Is Fully Established In A State In Which The Relative Distinctions (Bheda) Conceived Between Entities Dissolve Away. Everything Appears To Him As Pure Being (Sattdmdtra) L} And The Entire Universe Shines Before Him Pervaded By Siva's Radiance. His Intuitive Faculty (Mati) Thus Purified, The Yogi Gains Both The Perfections (Siddhi) Of Yogic Practice And Liberation (Mukti). His Consciousness Is Now Like A Well-Polished Mirror Which Reflects Everything He Desires

And Grants It To Him. Abhinava Writes: Just As A Man Who Has Been Ill For A Long Time Forgets His Past Pain Completely When He Regains His Health, Absorbed As He Is In The Ease Of His Present Condition, So Too Those Who Are Grounded In Pure Awareness Free Of Thought-Constructs Are No Longer Conscious Of Their Previous [Fettered] State. Consciousness, The Sole Truly Existent Reality, Free Of Thought- Constructs Is Made Fully And Evidently Manifest By Eliminating These Differentiated Perceptions. The Wise Man Should Therefore Exert Himself To Attend Closely To This [State Of Awareness]. The Thought-Constructs Generated Within Consciousness Do Not In Reality Affect It At All. They Can Neither Break Up Nor Add Anything To The Light Which Shines As All Things. They Are In Fact Nothing But Consciousness Itself Which Perceives, Through Its Power Of Reflective Awareness (Yimarsa), The Multitude Of Objects In Diverse Ways, And So Assumes This Form. Although Thought-Constructs Are Mental Representations Of Objects Once Seen Or Present, They Are Products Of The Power Of Consciousness And Not Of The Objects They Represent. Thought Is Both Analytic And Synthetic; It Serves The Useful Purpose Of Separating Individual Elements Of Experience From Others And Linking Together Those That Appear To Be Distinct From One Another So That They Can Be Better Understood. It Does Not Consist Merely Of False Mental Constructs Projected Onto Reality That Need To Be Wholly Rejected. Thought Obscures Consciousness And Distracts It Only When It Appears In The Form Of Doubt, Vacillating Between Alternatives. Once This Conflicting Duality (Dvaitddhivasa) Is Eliminated, Thought Is Purified And Rests In Itself As The 'Thought-Less Thought' Of Pure Consciousness. By Gradually Eliminating The Multitude Of Conflicting Notions That Agitate Him, The Yogi Ultimately Achieves The Certainty (Niscaya) Corresponding To A Direct Awareness Of His Own Divine Nature. Abhinava Explains: Thought Is In Reality None Other Than Pure Consciousness. Even So, It Serves As A Means To Liberation For The Individual Soul (Anu) Only When It Takes The Form Of Certainty (Niscaya). This surprising result--that information capacity depends on surface area—has (e) a natural explanation if the holographic principle (proposed in 1993 by Nobelist Gerard't Hooft of the University of Utrecht in the Netherlands and elaborated by Susskind) is true**.** In the everyday world, a hologram is a special kind of photograph that generates a full three-dimensional image when it is illuminated in the right manner. When it com,es to structures,one cannot bypass Deleuze and his work. Levi Strauss Has Also Noticed That Signs Always Offer An Excess. The System Of Language, 'The Order Of The Known' Exceeds Actual Speech, Even Attempts At Totalization (48). Laws Pre-Exist Actual Cases. [So We Were Getting Close To A Role For Social Life, But Then It Gets Metaphysical Again]. As LS Put It, The Universe Signified Long Before Human Beings Knew What It Was Signifying. By Contrast, The Domination Of Nature Proceeds Partially And Progressively, Step By Step Unlike Social Life Where All Its Goals And Possibilities Given At Once. We're Back With Two Series, This Time Conceived As Rhythms, Social And Natural. Both Technocrats And Dictators Attempt A False Synthesis Of These Two Rhythms. Levi Strauss Referred To 'The Floating Signifier' As A Creative Force And Deleuze Wants To Say It's 'The Promise Of All Revolutions' (49). There Are Also 'Floated Signifieds', Which Seem To Be Possibilities Which Have Not Yet Been Realized. These Can Fill The Gap Between Signifier And Signified [And Are Found In Common Sense Expressions Like 'Gadgets' Or 'Whatnot' – Maybe Connected To The Idea Of A Bricoleur?]. It Implies A Symbolic Content, But Does Not Attempt To Fill It With Specifics. Together, These Possibilities Constitute A Structure, Two Heterogeneous Series, One Signifying, One Signified, Interdependent, And Including Particular Events, Singularities, Emitted By A Differentiator. The Singularities Belong To Neither Series Exclusively And Thus Have No Coherent Identity –Each Is An Excess In One Series And A Lack In The Other. The Singularities Can React Back On The Series, So Structures And Events Are Interdependent [So Structures Need A Dynamic Element, And, D Argues, An Excess, An Empty Square Instead Of Total Systematic Closure. Addresses The Old Issue Of The Static Nature Of Structuralism]. The Signifying Series Contains A Series Of Ideal Events, An Internal History. Differentiators Articulate Series And This Produces A 'Tangled Tale' Overall (51). **Sense Can Be Found In Either Series. It Is Not Just Signification But The Relation That Produces Signifier And Signified**, [The Operation Of The Whole Structure In This Expanded Sense]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press Attributable And Ascribable To Constraints On Space

Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules.All the information describing the 3-D scene is encoded into the pattern of light and dark areas on the two-dimensional piece of film, ready to be regenerated. The holographic principle contends that an analogue of this visual magic applies to the full physical description of any system occupying a 3-D region It proposes that another physical theory defined only on the 2-D boundary of the region completely describes the 3-D physics. If a 3-D system can be fully described by a physical theory operating solely on its 2-D boundary, one would expect the information content of the system not to exceed that of the description on the boundary. Can we apply the holographic principle to the universe at large? The real universe is a 4-D system It has volume and extends in time. If the physics of our universe is holographic, there would be an alternative set of physical laws, operating on a 3-D boundary of spacetime somewhere, which would be equivalent to our known 4-D physics. We do not yet know of any such 3-D theory that works in that way. Indeed, what surface should we use as the boundary of the universe? One step toward realizing these ideas is to study models that are simpler than our real universe. A class of concrete examples of the holographic principle at work involves so-called anti-de Sitter spacetimes **.** The original de Sitter spacetime is a model universe first obtained by Dutch astronomer Willem de Sitter in 1917 as a solution of Einstein's equations, including the repulsive force known as the cosmological constant. De Sitter's spacetime is empty, expands at an accelerating rate and is very highly symmetrical. In 1997 astronomers studying distant supernova explosions concluded that our universe now expands in an accelerated fashion and will probably become increasingly like a de Sitter spacetime in the future. Now, if the repulsion in Einstein's equations is changed to attraction, de Sitter's solution turns into the anti-de Sitter spacetime, which has equally as much symmetry. More important for the holographic concept, it possesses a boundary, which is located "at infinity" and is a lot like our everyday spacetime. Using anti-de Sitter spacetime, theorists have devised a concrete example of the holographic principle at work: a universe described by superstring theory functioning in an anti-de Sitter spacetime is completely equivalent to a quantum field theory operating on the boundary of that spacetime Thus, the full majesty of superstring theory in an anti-de Sitter universe is painted on the boundary of the universe. Juan Maldacena, then at Harvard University, first conjectured such a relation in 1997 for the 5-D anti-de Sitter case, and it was later confirmed for many situations by Edward Witten of the Institute for Advanced Study in Princeton, N.J., and Steven S. Gubser, Igor R. Klebanov and Alexander M. Polyakov of Princeton University. Examples of this holographic correspondence are now known for spacetimes with a variety of dimensions. This result means that two ostensibly very different theories--not even acting in spaces of the same dimension--are equivalent. Creatures living in one of these universes would be incapable of determining if they inhabited a 5-D universe described by string theory or a 4-D one described by a quantum field theory of point particles. (Of course, the structures of their brains might give them an overwhelming "commonsense" prejudice in favor of one description or another **Holographic Space-Time (Crystal Links, Tom Hanks and Wikipedia Francisco Di Biase** propose a quantum-informational holographic model of brain-consciousness-universe interactions based in the holonomic neural networks of Karl Pribram, in the holographic quantum theory developed by David Bohm, and in the non-locality property of the quantum field described by Hiroomi Umezawa. He considers this model an extension of the interactive dualism of Sir John Eccles, of an interconnection between brain and spirit by means of quantum microsites named dendrons and psychons. **Francisco Di Biase** proposes a dynamic concept of consciousness seen as a holoinformational flux interconnecting the holonomic informational quantum brain dynamics, with the quantum informational holographic nature of the universe. This self-organizing flux is generated by the holographic mode of treatment of neuronal information and can be optimized through practices of deep meditation, prayer, and others states of higher consciousness that underlie the coherence of cerebral waves. In brain mapping studies performed during the occurrence of these harmonic states we can see the spectral array of brain waves highly synchronized and perfectly ordered like a unique harmonic wave, as if all frequencies of all neurons from all cerebral centers played the same symphony. This highly coherent brain state generates the non-local holographic informational cortical field of consciousness that interconnect the human brain and the holographic cosmos. The comprehension of this holonomic quantum informational nature of brain-consciousness-

universe interconnectedness allows us to solve the old mind-matter Cartesian hard problem, unifying science, philosophy, and spiritual traditions in a more transdisciplinary, holistic, integrated paradigm. In this new arrangement cosmovision, consciousness and transpersonal phenomena becomes part of Science and of the very holoinformational nature of the Holographic Conscious Multiverse. **DOI: 10.14704/nq.2009.7.4.259 Quantum-Holographic Informational Consciousness Francisco Di Biase NeuroQuantology** Holography suggests a considerable reduction of degrees of freedom in theories with gravity. However it seems to be difficult to understand how holography could be realized in a closed re-contracting universe. In this Letter we claim that a scenario which achieves that goal will eliminate all spatial degrees of freedom. This would require a different concept of quantum mechanics and would imply an intriguing increase of power for the natural laws. **Physics Letters B Volume 451, Issues 1–2, 1 April 1999, Pages 19–26 Richard Dawid doi:10.1016/S0370-2693(99)00205-1Vijay Balasubramanian, Per Kraus, Albion Lawrence, and Sandip P. Trivedi** describe probes of anti–de Sitter spacetimes in terms of conformal field theories on the AdS boundary. Basic tool is a formula that relates bulk and boundary states—classical bulk field configurations are dual to expectation values of operators on the boundary. At the quantum level they relate the operator expansions of bulk and boundary fields. Using our methods, we discuss the CFT description of local bulk probes including normalizable wave packets, fundamental and D-strings, and D-instantons. Radial motions of probes in the bulk spacetime are related to motions in scale on the boundary, demonstrating a scale-radius duality. They also discuss the implications of these results for the holographic description of black hole horizons in the boundary field theory.DOI: http://dx.doi.org/10.1103/PhysRevD.59.104021 **Holographic probes of anti–de Sitter spacetimes Phys. Rev. D 59, 104021 – Published 26 April 1999 Vijay Balasubramanian, Per Kraus, Albion Lawrence, and Sandip P. Trivedi** The existence of a fundamental scale, a lower bound to any output of a position measurement, seems to be a model-independent feature of quantum gravity. In fact, different approaches to this theory lead to this result. The key ingredients for the appearance of this minimum length are quantum mechanics, special relativity and general relativity. As a consequence, classical notions such as causality or distance between events cannot be expected to be applicable at this scale. They must be replaced by some other, yet unknown, structure. **LUIS J. GARAY, Int. J. Mod. Phys A, 10, 145 (1995) DOI: 10.1142/S0217751X95000085 QUANTUM GRAVITY AND MINIMUM LENGTH** A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored. Copyright © 1997 Society for Industrial and Applied Mathematics **SIAM J. Comput., 26(5), 1484–1509. (26 pages) Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer ISSN (print): 0097-5397 ISSN (online): 1095-7111 Publisher: Society for Industrial and Applied Mathematics Peter W. Shor DOI: 10.1137/S0097539795293172** The classical dynamics of M-dimensional extended objects arising from stationary points of the world volume swept out in space time is discussed from various points of view. A introduction to the Hamiltonian mechanics of bosonic compact M (em) branes is given, emphasing the diversity of the different formulations and gauge choices. For moving hypersurfaces, a graph description—including its nonlinear realization of Lorentz invariance—and hydrodynamic formulations (in light-cone coordinates as well as when choosing the time coordinate of a Lorentz observer as the dependent variable) are presented. A matrix regularization for M = 2 (existing for all topologies) is explained in detail for the 2-sphere, as well as multilinear formulations for M > 2. The recently found dynamical symmetry that exists for all M and related reconstruction algebras are covered, just as some explicit solutions of the level-set equations. **Jens Hoppe 2013 J. Phys A: Math. Theor. 46 023001 doi:10.1088/1751-8113/46/2/023001 Relativistic membranes N Bodendorfer et al** rederive the results of our companion paper, for matching space–time and internal signature, by applying in detail the Dirac algorithm to the Palatini action. While the constraint set

of the Palatini action contains second class constraints, by an appeal to the method of gauge unfixing, we map the second class system to an equivalent first class system which turns out to be identical to the first class constraint system obtained via the extension of the ADM phase space performed in our companion paper. Central to analysis is again the appropriate treatment of the simplicity constraint. Remarkably, the simplicity constraint invariant extension of the Hamiltonian constraint, that is a necessary step in the gauge unfixing procedure, involves a correction term which is precisely the one found in the companion paper and which makes sure that the Hamiltonian constraint derived from the Palatini Lagrangian coincides with the ADM Hamiltonian constraint when Gauß and simplicity constraints are satisfied. **N Bodendorfer et al** therefore have rederived new connection formulation of general relativity from an independent starting point, thus confirming the consistency of this framework. **N Bodendorfer et al 2013 Class. Quantum Grav 30 045002 doi:10.1088/0264-9381/30/4/045002 new variables for classical and quantum gravity in all dimensions: II. Lagrangian analysis** As K§emaraja points out, none of the practices taught in the Stanzas on Vibration belong to the Individual Means and so it does not, strictly speaking, concern Spanda doctrine, if that is, we consider the Stanzas to be the basic text of the Spanda school. From K§emaraja's point of view, however, the third section of the Aphorisms of Siva (Sivasutra), which is both the last and most extensive, is largely an exposition of this category of practice. The Stanzas and Aphorisms have been traditionally linked together and so, even though we feel that they should be distinguished sofar as the Stanzas rather than the Aphorisms teach the Doctrine of vibration as such, we are nonetheless justified in referring to the Aphorisms as its major source. Our exposition of the Individual Means will therefore be largely based on K§emaraja's interpretation of the third section of Aphorisms and we will present it, as he does, as an exposition of a possible mystical journey of individualised (anava) consciousness to realisation. We follow K§emaraja because he understood the practise taught in the Aphorisms in these terms, thereby not only illustrating for us how it fits into this scheme but also how he understood the basic categories of practice and their relationship to one another. According to K§emaraja, the first Aphorism of each section of the Sivasutra characterises the condition and nature of the Self at the corresponding three levels of practice. In other words, they indicate the yogi's basic state at each level in terms of his self-identification. This identification corresponds to his existential condition as a degree of self- realisation in the process leading to the authentic self-awareness of the liberated. The very first Aphorism starts directly with this, the highest state, by declaring that the Self is pure, dynamic and universal consciousness (caitanya).This is true for the yogi who has awakened to his authentic nature at the Divine (sambhava) level of being. At the Individual (anava) level, however, the situation has changed. In this sphere of consciousness the intermediate processes of discernment, analysis and classification of perceptions, which bridge the gap in the flow of awareness from the universal subject to a specific object of knowledge, appear to take over the status of the perceiving subjectivity which underlies them. The universal Self recedes into the background as a pure, undefinable awareness, and the individual ego, consisting of the perceptions, thoughts and emotions generated by the contact between the universal perceiver and the perceived, emerges in the juncture between them. Thus at this level, as the Aphorisms say, the Self is the mind. This is the Self which moves (atati) from one state of being to another, from one body to the next carrying with it subtle traces left behind by its sensory and mental activity. Together these are said to constitute, and be caused by, the subtle body technically called the 'City of Eight' (puryasfaka) with which consciousness is identified and due to which it is subject to the constant alterations of pleasure, pain and inertia. The Stanzas teach: [The soul] is bound by the City of Eight (puryaffaka) that resides in the mind, intellect and ego and consists of the arising of the [five] subtle elements [of sensory perception]. He helplessly suffers worldly pleasure and pain (bhoga) which consists of the arising of mental representations born of that [City of Eight] and so its existence subjects him to transmigration.**Whereas consciousness itself is the subject who practises the Divine Means (Sambhavopayd), the subject who practises the Individual Means is the mind. Unlike the Empowered Means, however, the mind is not directed inwards onto itself. At the Empowered level, enlivened by the direct intuition (pratibha) consciousness has of its own nature, mind ceases to function merely in the paradigmatic, formative manner which gives rise to mental representations, but operates instead as the subtle introverted activity of reflective awareness (vimarsa), the power of consciousness (sakti).** This activity, as we have seen, is

the essence of Mantra which, independent of the senses, is no longer restricted in any way. At the Individual lew, however, the creative powers of consciousness reflected through the eAtroverted mind are greatly attenuated. All that remains is the power to form thought-constructs and make determined resolutions (sankalpa) which go on to issue through the body into outer action to make the private creations of the mind apparent to others. The Individual Means, therefore, deals with the objectively perceived contents of consciousness and hence with the individual subject as a composite aggregate of objective elements, ranging from the subtle life force (prdrta) to the physical body and its outer environment. The practices belonging to this Means are thus of two types. One is concerned with the individual subject who resides in, and as, the psycho-physical organism; the other with external reality. What this implies essentially is that practice at this level is not concerned as much with the will or cognitive consciousness **(refer hiedigger who proposes the same philosophy: italics mine**) as are the other two Means, but with the power of action applied, in the context of the practice taught in the Aphorisms, to the spiritual activity of Yoga. According to Ksemaraja, the Individual Means culminates in the Empowered state and hence leads to the levels of practice beyond it. 251 This is possible because, despite their differences, there is an essential similarity between them. The aim of both the Individual and Empowered Means is to purify the discursive representa- tions of differentiated perceptions (vikalpasamskara) 252 and so lead the yogi to the expanded (yikasita) consciousness of the Divine (sambhava) state. The other levels of practice therefore both sustain and complement it. The activity of individual consciousness can be fully perfected only when it operates through the flow of the conative and cognitive powers which together constitute the pure activity of universal consciousness beyond all means (anupaya). In fact, according to Ksemaraja, all three soteriological types function together in various ways, their corresponding states representing dimensions of the same experience. For example, the upsurge of consciousness (udyama) which is the supreme, **illuminating intuition (parapralibha)** of the Divine state (sambhavavastha) is concomitant with the gathering together of all the powers of consciousness in the Empowered state. The Divine Means, in other words, leads to the experience of Power (sakti) which in its turn, when fully affirmed, marks the attainment of a permanent contemplative consciousness (turiyatita) at the Divine level which persists unaltered in every state of consciousness. Consequently, Ksemaraja concludes his exposition of the first section of the Aphorisms which exemplifies, according to him, the Divine Means, by saying: Thus we have explained the first expansion which starts with [the Aphorism] 'the Self is pure dynamic consciousness' **(note this happens in evolutionary pocess and in normal human being it remains as witness consciousness italics mine)** and expounds the nature of the realisation (prathana) attained through the Divine Means. It is the intuitive insight (samapatti) of Bhairava's nature which is, as we have said, the upsurge of consciousness that quells all bondage, namely, the ignorance of that freedom which makes it manifest. Transforming all things into the nectar of one's own innate bliss, it bestows every yogic accomplishment (siddhi) including mystic absorption in the vitality of Mantra, the highest of them all. Accordingly, we have, in the course of this exposition, explained the nature of Sakti in order to show that the Divine nature (Sambhavarupa) possesses [every] power. Another way in which the Means are related to one another is illustrated by the recurrence of the same Aphorism in different sections of the Sivasutra which indicates, according to Ksemaraja, that the same practice belongs to more than one Means. Both times this happens, the Aphorism appears first in the section dealing with the Divine Means and then recurs in that concerned with the Individual Means. In one case, Ksemaraja tells us this is because practice at the Divine level requires no effort whereas at the Individual level, the yogi must exert himself to achieve the same state that at the Divine level dawns spontaneously. At the Empowered level also, as the Sivasutra says, 'effort achieves the goal'. Here, however, because as the Empowered Means is, according to Ksemaraja, predominantly concerned with the contemplation (anusamdhi) of the vitality of Mantra, the effort exerted is that required to bring the practice of Mantra to fulfillment. It is, as Ksemaraja says, 'the spontaneous effort exerted to grasp the initial expansion of intention to apply oneself to the contemplation [of Mantra]. It is this exertion which wins the favour of the gods of Mantra and identifies the adept with them.' The second case of the same practice being taught in different sections of the Aphorisms concerns the realisation of the Fourth State of contemplative consciousness (turiya) in the other three states of waking, dreaming and deep sleep. At the Divine level this takes place by **'violently digesting' (hafhapaka)** the three states in the Fourth. At the

Individual level the Fourth state is first experienced at the junctures between the other three states and then induced gradually to spread out from these Centres to pervade the other states like oil extending slowly through a piece of cloth. The difference in this case between the levels of practice is not only that at the Divine level it reaches fulfillment spontaneously, but it is also sudden and complete, leading directly to the liberated state of consciousness Beyond the Fourth (turiyatita) At the Individual level, however, practice is gradual and even when the yogi manages to rise to states of contemplation, he must take care not to fall to lower levels of consciousness. Indeed, until the yogi attains the sudden and direct realisation of perfect enlightenment, whatever be his state of consciousness or level of practice, he is bound to rise and fall because his contemplative state is necessarily transitory (kadacitka) however long it may last. The yogi is more prone to these ups and downs the lower his basic state of consciousness. Consequently, the last section of the Sivasutra repeatedly instructs the yogi not only how to rise to higher levels of consciousness and maintain them, but also in what way he is liable to fall from them and how to regain them. Ksemaraja stresses that the rise from one level of consciousness to another is marked by the transition from a lower Means to a higher. Conversely, a fall from the higher level to the lower entails practice of a lower Means. The measure of the yogi's level of consciousness and that which sustains him in it allowing him to progress further, is his attentiveness (avadhana) to the higher realities he experiences in the more elevated states. Thus the last Aphorism of the second section of the Sivasutra warns the yogi that if his pure awareness (suddhavidya) of his oneness with all things slackens, he will fall from his awakened state to dream the dream of thought-constructs. From Ksemaraja's point of view this means that the negligent yogi must now resort to the Individual Means described in the next section to return to his former, higher Empowered practice in which he experiences this oneness. Ksemaraja expounds practice at the Individual level, as he sees it in the Aphorisms, as extending from one Means to the next. For example, practice at the Individual level diverts the flow of the vital breath (prana) from its more usual course and induces it to enter the Central Channel (susumna) along which it rises as a pure conscious energy (technically called 'kuritfalini'). This leads the yogi to the Empowered state in which he enjoys the pure awareness of unity. If he manages to make it truly his own and it becomes his basic state of being, he enters the Divine plane (sambhavapadd) of identity with Siva. The Individual Means is both a point of departure to higher levels of practice and the level to which the yogi returns if he falls. Thus although the practices taught in the last section of the Aphorisms may belong to any one of the three Means, they are collectively treated as part of the Individual Means because they start from it and because it is the yogi's abiding standby if he falls. Let us turn now to the basic practice at the Individual level, as Ksemaraja understands it. This is essentially Yoga. According to the Classical Yoga system taught by Patanjali in the Aphorisms of Yoga (Yogasutra), Yoga is defined as 'the quelling of the fluctuations of the mind' (cittavrttinirodha). The aim is to sever the spiritual essence of the Person (purusa) from the defiling materiality of Nature (prakfti), even though the word 'Yoga' means to 'unite' or 'yoke together.' Here, however, Yoga combines both union and cessation. It is the act (kriya) of removing the latent traces (yasana) of differentiated perceptions (yikalpa) born of the impurities (mala) which contract consciousness **(like pravrutti and Nivrutti; italcis mine)**. This is achieved by uniting all the elements of experience (tattva) together in the wholeness of the activity of consciousness. As Jayaratha explains: The [wise] consider Yoga to be the union of one thing with another/ thus, in accord with this dictum, Yoga is the [act] of uniting [all] the metaphysical principles together within consciousness . . . , Ksemaraja seeks initially to establish the best form of Yoga for the yogi to practice at the Individual level. His sources are two Tantras he knew well and considered to be amongst the most important, namely, The Tantra of (Siva's Third) Eye (Netratantra) and The Tantra of the Liberated Bhairava (Svacchandabhairavatantra). The basic model is that of the Eight-limbed Yoga (asfariga) taught by Patanjali which consists of: 1) The five restraints (yama\ namely, abstention from violence (ahirrisa), falsehood (satya), dishonesty (asteya), sexual intercourse (brahmacarya) and desire for more than the essential (aparigrahd) 2) The five disciplines (niyama), namely, cleanliness (sauca), contentment (santo$a), austerity (tapas\ study (svadhyaya) and reverence for God (Uvarapranidhand) 3) Posturing of the body (asana) in a manner conducive to the practice of meditation and physical health. 4) Regulation of the breath (pranayama) 5) Withdrawal of the senses from their objects (pratyahara) 6) Focusing of attention (dharana) 7) Meditation (dhyana), that is, steady, uninterrupted concentration. 8)

Contemplation (samadhi). Ksemaraja **rejects** Patanjali's system because he believes it to be a form of Yoga that can, at best, lead only to limited yogic attainments (mitasiddhi). In the Netratantra, however, Siva teaches a different, higher form of the Eight Limbs of Yoga which lead to perfect penetration into the supreme, transcendental principle 280 of which the Netratantra says: Speech cannot express, nor the eye see, the ears hear, or the nose smell, the tongue taste, the skin touch or the mind conceive that which is eternal. Free of all colour and flavour, endowed with all colours and flavours, it is beyond the senses and cannot be objectively perceived. O goddess, those yogis who attain it become immortal gods! By great practice and supreme dispassion . . . one attains Siva, the supreme imperishable, eternal and unchanging reality. A necessary preliminary of all Tantric Yoga is a process technically called the 'purification of the elements' (bhutasuddhi), through which the body is homologized with the macrocosm and so made a fit vessel for the pure, conscious presence of the Deity within it. Ksemaraja equates this with the meditation (dhyana) which, according to the Mdlinivijayatantra, characterises the Individual Means. In order to practice this meditation the yogi must visualise the dissolving away of all the forces in the body. There are two ways in which this can be done. The first is called 'the contemplation of dissolution' (layabhdvana). Through it the progressive differentiation of consciousness from its causal, pre-cosmic form to its phenomenal manifestation is reversed As the Vijnanabhairava teaches: "One should meditate on the All in the form of the Paths of the world-orders etc. considered in their gross, subtle and supreme forms until, at the end, the mind dissolves away." Mediated by consciousness, the macrocosm rests in the microcosm which is emitted along with it successively in the emptiness of the individual subject, vital breaths, mind, psychic nerves (ndtfi), senses and external body. The yogi reproduces this process by visualising the totality of reality including the world-systems, metaphysical principles and cosmic forces along with the Mantras, letters and syllables which represent them, as arising successively throughout the psycho-physical body so as to constitute it. Deployed in this way they form the Cosmic Path along which the yogi ascends, absorbing as he does so, the lower elements into the higher, thus strengthening and extending his unifying awareness (anusamdhdnd) of the configuration of the Path. Thus, moving from the gross elements constituting the outer physical body, to pure sensations (tanmdtra), then to the senses and mind back to their primordial source, the yogi rises from the **embodied subjectivity** of the waking state to the Fourth State (turiya) of contemplation where he is one with the pervasive intent which initiates the creative vision of consciousness. Abhinava writes: Once [the yogi] has known [this] Path in its completeness, he must then dissolve it into the deities who sustain it and these successively into the body, breath, mind [and emptiness] as before, and all these into his own consciousness. Once this is full and an object of constant worship, it destroys, like the fire at the end of time, the ocean of transmigration.Thus, the second method Ksemaraja teaches to dissolve away the diversity of sensory, mental and physical energies into the unity of consciousness is a meditation on the Fire of Consciousness (dahacintd) which the yogi visualises as burning away all division. At the Divine level (sambhavopaya) the yogi witnesses the sudden and violent withdrawal of all objectivity into the pure ego (aharri), like the pouring of fuel into a raging fire. He does not need to visualise this process but merely attend to it with a passive, receptive attitude. At the Individual level the yogi must exert his imagination to induce this process and so rise to the Divine level through the Empowered. The Vijnanabhairava teaches: Visualise the fortress [of your body] burning with the Fire of Time (kalagni) risen from the Abode of Time; then at the end peace manifests. The Fire of Time {kalagni) resides underneath the hell worlds at the bottom of the Cosmic Egg (Brahman^a). It issues from Ananta — a form of Siva who presides over the lower regions. He floats on a boat in the causal waters supporting the Egg, his mind all the while fixed on Bhairava. The flames of the Fire of Time rise up to the hell-worlds heating them intensely and radiate its energy throughout the universe. At the end of each period of creation the flames rise higher and destroy the old cosmic order to make room for a new one. At the microcosmic level the yogi reproduces this process by mentally placing the letters of the alphabet, in the prescribed order, on the limbs of his body starting from the left toe to the top of the head. As his attention progresses upwards, he visualises the Fire of Time moving with it in such a way that his bodily consciousness, together with the universe of differentiated perceptions, is gradually burnt away leaving in its place the white ashes of the undivided light of consciousness. Ksemaraja considers this meditation (dhyana) to be a limb of a programme of yogic practice at the Individual level 291 of which the remaining limbs are

as follows: Posture (A sana). The yogi fixes his attention on the centre between the inhaled and exhaled breath, absorbing in this way the flux of his awareness into the unfolding power of knowledge which rises initially as the upward flowing breath (udanaprana) in the Central Channel (su§umna) between the other two breaths. The Prank aspect of this flow disappears as it moves upward and the yogi experiences the spontaneous rise of the omniscience of consciousness within himself. The mind reverts back to its original, pervasive conscious nature and understands the infinite fact of Siva's omnipresence. This is the firm seat (asana) upon which the yogi sits to practice. Regulation of the Breath (Pranayama) .To regulate the movement of the breath, the yogi must first cleanse the right and left channels of the ascending and descending breath by blocking the left nostril while exhaling and the right while inhaling a few times. This ensures that the movement of the breath is firm and evenly distributed. Next, without attempting to control it in any way, he attends to the flow of his breathing. As the mind becomes steadier and in closer harmony with the rhythm of its movement, the duration of each inhalation and exhalation gradually alters until they become equal. At this stage they unite and merge in the upward flowing current of vitality in the Central Channel (Susumna). This is when true Pranayama begins. The yogi's mind pure and tranquil, he returns, as it were, to a prenatal state and the external breathing cycle is internalized, so that it no longer moves through the lungs but passes directly to the universal source of vitality. The yogi, now at the Empowered level of practice, experiences this movement as travelling from the Heart centre upwards to a point distant twelve fingers above the head where it merges in the void of consciousness. Free of its outer gross form, the breath moves freely through the Central Channel and soon transcends even this subtle movement to become one with the supreme vibration of consciousness. In this way, the yogi's breathing becomes one with the spontaneous rise and fall of energy from the bosom of the absolute. Abhinava quotes the Tantra of the Line of Heroes (Viravalitantra) as saying: When, by constantly merging the mind in Siva, Who is the pure conscious nature, the Sun and Moon [of the two breaths] have dissolved away and the Sun of Life, which is one's own consciousness, has reached the twelve-finger space, this is termed liberation. Breath control [at this stage] serves no useful purpose. Breath control which merely inflicts pain on the body is not to be practised. He who knows this secret is both themselves liberated and liberates others. **Focusing of Attention (Dhara nA):** Attention is fixed on the psychic centres in the body corresponding to the five gross elements In this way the vital breath is successively directed to these centres from the Heart of consciousness to refresh and stimulate their activity. First it moves to the Earth centre in the throat which regulates the firmness of the bones and flesh of the body; then to the Water centre in the glottis responsible for the balance of the bodily fluids. After this it travels to the navel which is the Fire centre dealing with digestion and anabolisms and catabolesis in general. It then moves to the Wind centre in the toe of the left foot which governs the movement of gases to and from the cells via the circulatory system. When the yogi has thus achieved control over these forces, the breath rises from the Heart to the top of the head and he becomes master of the Ether element and so attains every yogic power. **Meditation (Dhyana)** The highest form of meditation stills the flux of the qualities (guna) and induces the mind into a state of contemplative absorption. The object of this meditation is the supreme and pervasive divinity of the pure subject whose true nature is known to none but himself alone (svasatpvedya). The yogi attains him by merging into the constant flow of awareness that streams into the Light which illumines his own nature. Contemplation (Samadhi) .The yogi rises to the level of contemplation when the awareness he has of himself and the things around him become one and he realises his own identity with Siva, **(or Rama the omniscient: cosmic general ledger)** the sole reality. The aim of this Yoga in all its phases is to achieve the Fourth State of consciousness (turiya) beyond the three states of waking, dreaming and deep sleep and to then ultimately reach the liberated state Beyond the Fourth (turiyatita). These five states correspond to: (a) Siva's activity (vy apara), that is, His power of action; (b) Siva's Lordship (adhipatya), which is His power of knowledge; (c) the absence of these two, which corresponds to Siva's power of will; (d) His exertion (prerakatva), which contains all the cycles of creation and destruction and, (e) the rest Siva enjoys in His own nature, which is His power of consciousness. The first three states, when divorced from the last two, belong to the sphere of transmigratory existence. The Fourth and Beyond the Fourth on the other hand are higher, supramundane (alaukika) states of consciousness in which the yogi enjoys bliss and repose (visranti) in his own nature by penetrating (samavesa) into the universal consciousness of the Self, through which he ultimately

becomes liberated (jivanmukta). Beyond the Fourth is the state of awareness ParamaSiva Himself enjoys when duality has entirely disappeared and everything is realised to be one with consciousness. The Fourth is the state of awareness of the yogi who, catching hold of the pure subjectivity (upalabdhrta) flowing through the lower three states, is still actively eliminating his sense of duality. While the former is the supreme subject as T consciousness (aham), the latter is the pure awareness (prama) or I-nesses (ahanta) of the subject which encompasses the lower states, giving them life and uniting them together As such, the Fourth State is the reflective awareness of one's own nature shining in all three states at one with them. The fact that we recall that we slept well is proof that this state of consciousness persists even in deep sleep. Indeed, if the flow of Turiya could somehow be brought to a halt, all the other states of consciousness would come to an end in the absence of the pure subjectivity which makes them, and their contents, manifest. The states of waking, dreaming and deep sleep correspond to the form of awareness consciousness assumes when it predominantly manifest as the object, means of knowledge and individual subject, respectively. Turiya is the pure awareness (prama) that both transcends them and merges them all into itself. 300 As such, it appears as the triad of deed, means and agent in the pure act (vy apara) of consciousness unsullied by any outer reality. Abhinava explains: {Turiya) transcends the three aspects of *form\ 'sight' and T consisting as it does of the pure act of 'seeing'; therefore any means [by which this state could be realised] has merely a [provisional] instrumental value. It is, in other words, pure subjectivity of the nature of absolute freedom, independent of all external means. This is the state of consciousness called Turiya, luminous with its own light. Turiya is thus not just a psychological state but the supreme creative power (para sakti) of consciousness, the Goddess (samviddevi) who generates and withdraws the entire universe of subject, object and means of knowledge. In the Heart of Recognition Ksemaraja explains: Whenever the extroverted [conscious] nature rests within itself, external objectivity is withdrawn [and consciousness] is established in the inner abode of peace which threads through the flux of awareness in every [externally] emanated [state]. Thus Turiya, the Goddess of Consciousness, is the union of creation, persistence and destruction. **(Read as cosmic consciousness; Kshemaraja does not distinguish like we have done, individual, collective and cosmic and of three states)** She emanates every individual [cycle] of creation and withdraws it. Eternally full [of all things] and [yet] void [of diversity] She is both and yet neither, shining radiantly as non-successive [consciousness] alone. The yogi is fully absorbed in this state of consciousness and takes possession of its power when he is able to rise from contemplation (samadhi) carrying with him the abiding awareness of Turiya throughout his waking, dreaming and deep sleep. When he achieves this constantly, he continues to experience these states individually, but they no longer obscure the insight (pratibha) he has acquired because he realises that they are all aspects of the bliss of Turiya. Thus, while the common man calls this state the 'Fourth' (turiya) because he cannot experience it directly and knows only that it is beyond the other three, the yogi calls it 'Beyond Form' (rupatita) because it transcends the detachment of the state of deep sleep which, devoid of objective content, is the naked form' of the individual subject tending towards the fullness of consciousness. Those who are on the path of knowledge (jnaniri) call it the 'Whole* (pracaya) because, in this state, they see the entire universe gathered together in one place. 'Supra-mental Awareness' (manonmana) is the name given to the experience of Turiya in the waking state. The yogi in this state moves and lives in the world of waking experience free of all disturbing thoughts while abiding in the transcendental silence beyond the activities of the mind. 'Infinite' is the name of the experience of Turiya while dreaming because, free of the limitations imposed upon the body by time and space, the yogi enjoys the unlimited expanse of the Self. When Turiya is experienced in deep sleep, the yogi's state is called 'All things' (sarvdrtha) because in it he discovers his freedom from limitations in this, the most contracted state of human consciousness. The yogi who manages to maintain Turiya- consciousness comes to experience the three states of waking, dreaming and deep sleep as the constant flow of the bliss of consciousness in which all traces of the relative distinction between these states and their contents is eradicated. Following the stream of Turiya to its highest level (para katfha), he reaches the state Beyond the Fourth (turiydtita), which is the universal consciousness (caitanya) of the Self. Here the yogi comes to rest within his own nature. Plunged in the vast, waveless ocean of the consciousness and bliss (ciddnanda) of the state Beyond the Fourth, the yogi becomes Siva, the Free One (svacchanda), and thus wanders freely, practising the Yoga of Freedom. K§emaraja equates the Fourth

State with the pure (suddha), innate (sahaja) knowledge that one's own conscious nature is all things. It is the Supra- mental State (unmana) in which Siva's pervasive presence is experienced once the Yoga practised at the Individual level attains fruition at the Empowered. What the yogi must do, once consciousness is elevated to grasp the Fourth State, is make it constant. He must forcefully lay hold of it within himself and not release his grip until it becomes permanent. Then he travels 'Beyond the Fourth' to enlighten- ment. Before this ultimate attainment the yogi inevitably falls. The forces operating within consciousness that limit and obscure it throw him down whenever they possibly can. The only way the yogi can defend himself against them is to maintain a constant attentive awareness of the Fourth State. He falls when he is distracted but when he attends carefully to his pure conscious nature, he realises that every aspect of his state of being, including the forces that lead him astray, are one with the pulsing flux of his own consciousness and so cannot affect him. These powers, which are the energies of Matrka we have already discussed, are not the only obstacles the yogi must overcome. He must, for example, also resist the temptation to rest content with the miraculous yogic powers (siddhi) he acquires in the course of his spiritual develop- ment. Again to do this he must practice Yoga. Similarly, in order to pervade the Fourth State gradually through the other states in the manner proper to practice at the Individual level, the method is the same. He must practise the higher yoga of the Tantras which, turning his mind inwards and freeing it from discursive representations, allows him to penetrate into the Supreme Principle. Once the yogi has attained this contemplative state, his main problem is to make it permanent. In the introvered state the gross external movement of the breath is suspended and with it the activity of the intellect, mind, individualised consciousness, powers of the senses and the ego. 313 When the yogi rises out of this state, he is liable to fall again into the lower order of creation generated by Maya if he does not maintain his awareness of the higher reality he has experienced and allows his awakened, illumined insight to be obscured by the dream-like vision of thought-constructs. 314 Naturally, the yogi must rise out of the introverted condition of suspension. It is inherent in the very nature of reality that it should move out of itself. 315 Pure, universal consciousness initially transforms itself into the vital breath 316 charged with the impression iyasana) of the power of awareness attained through introversion. By attending to the pulse (spanda) of the breath as it moves out of the absolute, the yogi can develop an intuitive sense of the inherent unity of all he will perceive in the mental and physical spheres created by the outpouring of consciousness. In this way he realises that his own nature is everywhere present in all he perceives and that all things thus reside within him. Blessed with this insight his consciousness remains free and unlimited even at the individual level where the breath, mind, senses and body are active. If the yogi fails to do this, he finds himself once again beset by the strictures of his embodied existence and must, as before, try to pervade all his other states of consciousness with the aesthetic delight (rasa) and wonder (camatkara) of the Fourth State he experienced in contemplation. Again this means that he must strengthen his pure, empowered awareness that his universal nature manifests as all things. In this way he discovers Siva's presence in every sphere of individualised consciousness ranging from the breath to outer objectivity. The yogi's mind then becomes tranquil and undistracted because wherever it may wander, the yogi perceives only Siva, his authentic nature. Consciousness is thus freed of all external referents and the yogi's subjectivity is purified of all identification with the body or anything else that belongs to the objective sphere. The yogi then becomes detached from the opposites of pleasure and pain and is transcendentally free (kevalin). The yogi is again, however, liable to fall if he allows himself to get entangled in the play of opposites. This fall is more serious than the others because, although he is caught by the confining restrictions of individual- ised consciousness as before, he is now also affected by karma. Fleeing from pain in the pursuit of pleasure he is bound to act (karma) to minimise one and maximise the (a **maximaization problem is mentioned elsewhere about the svabhava of human beings to enhance pleasure reduce deprivation)** other and so is thrown down to the lowest level of embodied subjectivity (sakala). In order to regain his lost state, he must ascend gradually, by Siva's grace, from one order of subjectivity to the next and so free himself progressively of the limitations of the lower levels to gain the greater freedom and expansion of the higher. As he progresses, the objective sphere also evolves from the grossest perceptions of physical objects outside the lowest order to subjectivity, through to the subtler inner, mental perceptions to finally reach the order of subjectivity that contains objectivity within itself and is free to externalise it at will. The degree to which this process develops

depends, as before, on the yogi's awareness of the Fourth State. In consonance with the general principle that the remedy should suit the defect, the yogi is instructed to seek this higher state of consciousness in the wonder (camatkara) or delight (ananda) he feels in moments of intense physical pleasure. At first he experiences this subtle consciousness for an instant in the subjective sphere. If he manages to catch hold of it, it becomes more intense as the cognitive and objective spheres are also gradually pervaded and vitalised by it. Occasions for this practice are, for example, the sense of satisfaction one feels after a good meal or the aesthetic delight one experiences when listening to good music or the pleasure of sexual union with the Tantric consort or even solitary sexual excitation. In these moments of delight the yogi can penetrate momentarily into his own authentic Siva-nature (sambhavavesa) through the empowered contact (saktasparsa) m he makes with it in the freedom of the pure subjectivity of the Fourth State. If the yogi develops his awareness of this higher level of consciousness and maintains it, he eventually experiences it constantly. Clearly, what prevents the yogi from attending to his state of consciousness rather than the circumstances which induce it is the craving for pleasure (abhila§a) born of ignorance — the source of every impurity which clouds consciousness. Craving directs the yogi's attention towards outer, worldly things and so he is caught in the **net of thought- constructs**. To free himself of his worldly desires and reverse this binding extroversion, the yogi must eradicate its cause. To be freed of all the ups and downs of the path and no longer be tormented by the possibility of a fall, the yogi must see reality perfectly and completely. This insight is itself liberation and the moment it dawns the yogi is instantly freed. This sudden realisation is the goal of Tantric Yoga. Accordingly the Tantra declares: "He, who perceives reality directly, even for the brief moment it takes to blink, is liberated that very instant and never reborn again." Although the yogi's body and mind continue to function as before, they are like mere outer coverings which contain, but do not obscure, the mighty, universal consciousness which operates through them. The yogi's body is the universe, the senses the energies that vitalise it, his mind Mantra, the rhythm of his breath the pulse of time and his inner nature pure, dynamic consciousness. Raised above all practice, and hence all possibility of falling to lower levels, the yogi realises that he has always been free and that his journey through the dark land of Maya was nothing but a dream, a construct of his own imagination. **(Models are given in one of the papers of the series due to spatial constraints) Relationship betwixt string theory (note the spade remarks above) and holographic principle has been enucleated and expatiated in literature**. Some exemplar work is sententiously mentioned in the following:The ratio of shear viscosity to volume density of entropy can be used to characterize how close a given fluid is to being perfect. Using string theory methods, we show that this ratio is equal to a universal value of $\hbar/4\pi$kB for a large class of strongly interacting quantum field theories whose dual description involves black holes in anti–de Sitter space. **P K. Kovtun, D. T. Son, and A. O. Starinets** provide evidence that this value may serve as a lower bound for a wide class of systems, thus suggesting that black hole horizons are dual to the most ideal fluid **Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics Phys. Rev. Lett. 94, 111601 – Published 22 March 2005 P K. Kovtun, D. T. Son, and A. O. Starinets** According to't Hooft the combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored on a two-dimensional projection much like a holographic image. The two-dimensional description only requires one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena. After outlining't Hooft's proposal we give a preliminary informal description of how it may be implemented. One finds a basic requirement that particles must grow in size as their momenta are increased far above the Planck scale. The consequences for high-energy particle collisions are described. The phenomenon of particle growth with momentum was previously discussed in the context of string theory and was related to information spreading near black hole horizons. The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the rate of spreading is found to saturate the bound from causality. Perception is not reality. It needs some hidden variables to posit the reality. (see earlier models).We Make Sense Of Objects, Events Or State Of Affairs When We Pin Them Down And Describe Them, State Propositions About Them. We Can Then Develop These Propositions Into Logical Forms To Draw Conclusions, Make Predictions Or Whatever. The Trouble Is, Neither Object As We See Them, Nor Propositions As We Commonly Use Them, Are At All Simple. Objects As We Perceive Them, For Example,

Clearly Have All Sorts Of Characteristics As Well That We Don't Perceive. They Have A History, And A Potential For The Future, For Example. The Worst Problem Is That Objects As We See Them Are Actually Mixtures Of Things, The Affects Of Complex Causes: They Are 'Overdetermined', To Use The Phrase Associated With Both Freud And Marx. Deleuze Actually Draws On Resources In Classical Philosophy To Argue This, Especially The Work Of The Stoics, Who Saw Objects And Events As A 'Mixture Of Bodies', This Mixture Emanating From Somewhere In The Depths, Below Conscious Sense Making. Incidentally, This Reliance On The Stoics Also Makes Deleuze A Bit Stoical, In The Sense That He Thinks That These Mixed Bodies Are Part Of Some Unified Whole (He Was Going To Call This Being), And The Role Of Destiny Is Associated With The Whole. It Follows That Ethically And Conceptually, All Human Beings Can Do This Discover The Ways In Which Being Works. There's No Point Complaining About It Either. This Is the Basis of Badiou's Critique Of Deleuze. It Is Also The Basis Of Deleuze's Anti-Humanism: We Don't Understand Reality By Trying To Develop A Conscious Synthesis, Tracking Analogies Based On Some Notion Of A Shared Essence, Or Seeing God As An Ultimate Synthesiser For That Matter. Events And States Of Affairs Are Also More Complex Than They Look, And Deleuze Illustrates This Discussion With The Extraordinary Notion Of 'Compossibility' In Leibniz. Basically, The Idea Is That Singularities Can Produce Events In All Sorts Of Divergent Ways, All Of Them Equally Possible, And This Potential Is Shrunk If We Just Study The Actual Course Of Events. Objects Are More Complex Than They Look, With All Sorts Of Bits Of Their Being Unavailable To Us, But Language Is Equally Complex. Here, Deleuze Tries To Demonstrate This By Looking At People Who Use All Kinds Of Linguistic Anomalies Such As Paradoxes, Or, In The Case Of Lewis Carroll Especially, Portmanteau Words Or Esoteric Words. Although These Constructions Offend Logic, And Are Strictly Speaking Nonsense, They Still Make Sense. This Also Points To Some Powers Of Language That Are Not Immediately Available To Inspection. More Conventionally, We Know That The Use Of Particular Words Can Have Different Functions In Any Language System—They Can Denote, They Can Manifest Inner Thoughts, And They Can Signify (In The French Sense, That Is Indicate The Existence Of A Structured Language System Which Enables Us To Communicate With Each Other. Deleuze Proposes To Modify Classical Saussurian Semiotics By Adding A Moveable Element, An 'Empty Space' Or Floating Signifier'). Again Wordplay Can Deliberately Confuse These Different Functions, As When I Use The Term 'It' To Denote A Specific Object, Only To Refer To A Whole Process As 'It', To Express Myself By Saying Something About 'It' And So On (Actual Easy Examples Are Thin On The Ground In Deleuze!). Then There Is One Of Those (Many) Philosophical Diversions Into Various Types Of Explanation For The Extra Elements That Are Not Available To Actual Objects Or Speech Acts. Platonists Thought The Extra Bits Referred To Some Universal Form Or Idea, Other Philosophers Like Husserl Or Kant Suggest There Was Some Transcendental Realm Beyond The Immediate. Deleuze Offers Objections To Both For Those Approaches, And Suggests That The Real Is Actually Divided Into Virtual And Actual Levels (Delanda's Commentary Is Invaluable Here). The Objects And Words Of Our Immediate Perception Are Actualised Are Condensed Out From Much More Complex Objects And Events At The Virtual Level. In This Book, About The Only Candidate For These Complex Objects Is The Singularity, Which Actualizes Itself In Various Partial Ways. Deleuze Goes On To Develop This Idea By Saying That Singularities Themselves, And The Events And Objects They Actualize, Are Produced In A Random Or Arbitrary Fashion, The Chance Is At The Heart Of Actualisation. We Learn That The Virtual Also Has Its Own Sense Of Time, Which Is Not The Usual One Which Is Highly Limited By The Insistence On The Present Tense In Human Operations. It Is A Very Abstract Discussion, Which Finally Comes To An Example I Could Understand—The Emergence Of Language In Freud. As Infants Develop, Their Biological Urges Produce Certain Infantile Concepts Like Part Objects. These Are Classically Mixed, Deriving From Contradictory Drives In The Depths Of The Unconscious. The Sounds The Infant Makes Our Initially Just Bodily Emanations, But They Gradually Come To Take On The Form Of Linguistic Units, Like Phonemes. There Are Elements Of Adult Language Available Too, Of Course, But These Are Initially Unintelligible. Then A Process Described As The Phantasm Manages To Combine Various Infantile Concepts Together Into A'disjunctive Unity', A Mixture Of Heterogeneous Elements Which Deleuze Thinks Characterises Most Objects. A Kind Of Primitive Narrative Develops In The Phantasm, Partly Driven By Biological Drives, And

Partly Driven By Emerging Linguistic Competencies (Such As The Oedipal Scene). The Key To This Is The Freudian Notion Of The Phallus, Which Both Refers To Biological Organs Like Penises, And To Linguistic And Cultural Functions To Do With Authority And Value. The Phallus Is The Ideal Ambiguous Object (Or Empty Object As Deleuze Insists On Calling It), Able To Zigzag Between Bodies And Language, And Thus Make Divergent Series 'Resonate'. The Phantasm Therefore Develops An Energy Of Its Own, Which Permits A Relative Disengagement From Sexual Energy (Sublimation) And, In The Final Stages, The Proper Development Of Language In The Form Of Symbolisation. At Last We Have Language And Events Brought Together On The Surface, Which Is Of The Level Of Consciousness. If I Have Read The Appendices Correctly, This Phantasmic Form Is Then Generalised To Include All The Operations Of Making Sense As Describes Right At The Beginning. I Have Lots Of Reservations About This Whole Schema, In Fact, Ranging From The General Structure Of The Argument, Where Philosophical Issues Are Introduced As If They Were Necessary, Whereas Their Role Is Been Predetermined All Along (For Example, Deleuze Has Always Admired Stoics, And Here They Are As Offering The Best Account), Right Down To The Curious Insistence On The Phantasm As The Adult Form Of Making Sense—In My View, Dreams Are A Lot More Like A General Model Of Thinking, And They Include The More Sophisticated Linguistic Operations Of Metonymy (Condensation), And Metaphor (Displacement). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. Finally **Leonard Susskind** considers string theory as a possible realization of't Hooft's idea. The light front lattice string model of Klebanov and Susskind is reviewed and its similarities with the holographic theory are demonstrated. The agreement between the two requires unproven but plausible assumptions about the nonperturbative behavior of string theory. Very similar ideas to those in this paper have long been held by Charles Thorn. © 1995 American Institute of Physics **The world as a hologram Leonard Susskind J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249 Ofer Aharonya, , Steven S. Gubserb, , Juan Maldacenab, c, , Hirosi Oogurid, e, , Yaron Ozf,** review the holographic correspondence between field theories and string/M theory, focusing on the relation between compactifications of string/M theory on Anti-de Sitter spaces and conformal field theories. They review the background for this correspondence and discuss its motivations and the evidence for its correctness. They describe the main results that have been derived from the correspondence in the regime that the field theory is approximated by classical or semiclassical gravity. We focus on the case of the supersymmetric gauge theory in four dimensions, but discuss also field theories in other dimensions, conformal and non-conformal, with or without supersymmetry, and in particular the relation to QCD. We also discuss some implications for black hole physics. **Physics Reports Volume 323, Issues 3–4, January 2000, Pages 183–386 Large N field theories, string theory and gravity Ofer Aharonya, , Steven S. Gubserb, , Juan Maldacenab, c, , Hirosi Oogurid, e, , Yaron Ozf, , doi:10.1016/S0370-1573(99)00083-6 T. Banks, W. Fischler, S. H. Shenker, and L. Susskind** suggest and motivate a precise equivalence between uncompactified 11-dimensional M theory and the $N=\infty$ limit of the supersymmetric matrix quantum mechanics describing D0 branes. The evidence for the conjecture consists of several correspondences between the two theories. As a consequence of supersymmetry the simple matrix model is rich enough to describe the properties of the entire Fock space of massless well separated particles of the supergravity theory. In one particular kinematic situation the leading large distance interaction of these particles is exactly described by supergravity. The model appears to be a nonperturbative realization of the holographic principle. The membrane states required by M theory are contained as excitations of the matrix model. The membrane world volume is a noncommutative geometry embedded in a noncommutative spacetime. DOI: http://dx.doi.org/10.1103/PhysRevD.55.5112 **M theory as a matrix model: A conjecture Phys. Rev. D 55, 5112 – Published 15 April 1997 T. Banks, W. Fischler, S. H. Shenker, and L. Susskind** These TASI lectures review the Holographic principle. The first lecture describes the puzzle of black hole information loss that led to the idea of Black Hole Complementarity and subsequently to the Holographic Principle itself. The second lecture discusses the

holographic entropy bound in general space-times. The final two lectures are devoted to the ADS/CFT duality as a special case of the principle. The presentation is self contained and emphasizes the physical principles. Very little technical knowledge of string theory or supergravity is assumed. Report number: SU-ITP 99-14, KUL-TF-2000/03 Cite as: arXiv: hep-th/0002044 **TASI lectures on the Holographic Principle Daniela Bigatti, Leonard Susskind** The notion of a space-time uncertainty principle in string theory is clarified and further developed. The motivation and the derivation of the principle are first reviewed in a reasonably self-contained way. It is then shown that the nonperturbative (Borel summed) high-energy and high-momentum transfer behavior of string scattering is consistent with the space-time uncertainty principle. It is also shown that, as a consequence of this principle, string theories in 10 dimensions generically exhibit a characteristic length scale which is equal to the well-known 11 dimensional Planck length gs1/3 ℓs of M-theory as the scale at which stringy effects take over the effects of classical supergravity, even without involving D-branes directly. The implications of the space-time uncertainty relation in connection with D-branes and black holes are discussed and reinterpreted. Finally, we present a novel interpretation of the Schild-gauge action for strings from the viewpoint of noncommutative geometry. This conforms to the space-time uncertainty relation by manifestly exhibiting a noncommutativity of quantized string coordinates between, dominantly, space and time. We also discuss the consistency of the space-time uncertainty relation with S and T dualities. Copyright (c) 2000 Progress of Theoretical Physics **String Theory and the Space-Time Uncertainty Principle Tamiaki Yoneya Oxford Journals Science & Mathematics Progress of Theoretical Physics Volume 103, Issue 6Pp. 1081-112** In this Letter a recently proposed gravity dual of noncommutative Yang-Mills theory is derived from the relations between closed string moduli and open string moduli recently suggested by Seiberg and Witten. The only new input one needs is a simple form of the running string tension as a function of energy. This derivation provides convincing evidence that string theory integrates with the holographical principle and demonstrates a direct link between noncommutative Yang-Mills theory and holography.DOI: http://dx.doi.org/10.1103/PhysRevLett.84.2084 **Holography and Noncommutative Yang-Mills Theory Phys. Rev. Lett. 84, 2084 – Published 6 March 2000 Miao Li and Yong-Shi Wu Petr Hořava** suggests that M theory could be nonperturbatively equivalent to a local quantum field theory. More precisely, we present a "renormalizable" gauge theory in eleven dimensions, and show that it exhibits various properties expected of quantum M theory, most notably the holographic principle of't Hooft and Susskind. The theory also satisfies Mach's principle: A macroscopically large space-time (and the inertia of low-energy excitations) is generated by a large number of "partons" in the microscopic theory. **Petr Hořava** argues that at low energies in large eleven dimensions, the theory should be effectively described by eleven-dimensional supergravity. This effective description breaks down at much lower energies than naively expected, precisely when the system saturates the Bekenstein bound on energy density. He shows that the number of partons scales like the area of the surface surrounding the system, and discuss how this holographic reduction of degrees of freedom affects the cosmological constant problem. proposal is put forth for the holographic field theory as a candidate for a covariant, nonperturbative formulation of quantum M theory. DOI: http://dx.doi.org/10.1103/PhysRevD.59.046004 **M theory as a holographic field theory Phys. Rev. D 59, 046004 – Published 26 January 1999 Petr Hořava** The recent proposal on the correspondence between the super-Yang–Mills theory and string theory in the Penrose limit of the AdS5×S5 geometry involves a few puzzles from the viewpoint of holographic principle, especially in connection with the interpretation of times. To resolve these puzzles, we propose to interpret the PP-wave strings on the basis of tunneling null geodesics connecting boundaries of the AdS geometry. Our approach predicts a direct and systematic identification of the S-matrix of Euclidean string theory in the bulk with the short-distance structure of correlation functions of super-Yang–Mills theory on the AdS boundary, as an extension of the ordinary relation in supergravity–CFT correspondence. Holography requires an infinite number of contact terms for interaction vertices of string field theory and constrains their forms in a way consistent with supersymmetry. Copyright © 2003 Elsevier B.V. All rights reserved. **Nuclear Physics B Volume 665, 18 August 2003, Pages 94–128 Holographic reformulation of string theory on AdS5×S5 background in the PP-wave limit Suguru Dobashi, Hidehiko Shimada, Tamiaki Yoneya doi:10.1016/S0550-3213(03)00460-7** The strongest adversary in quantum information science is

decoherence, which arises owing to the coupling of a system with its environment1. The induced dissipation tends to destroy and wash out the interesting quantum effects that give rise to the power of quantum computation2, cryptography2 and simulation3. Whereas such a statement is true for many forms of dissipation, **Frank Verstraete1, Michael M. Wolf2 & J. Ignacio Cirac3** show here that dissipation can also have exactly the opposite effect: it can be a fully fledged resource for universal quantum computation without any coherent dynamics needed to complement it. The coupling to the environment drives the system to a steady state where the outcome of the computation is encoded. In a similar vein, they show that dissipation can be used to engineer a large variety of strongly correlated states in steady state, including all stabilizer codes, matrix product states4, and their generalization to higher dimensions5. **Letter abstract Nature Physics 5, 633 - 636 (2009) Published online: 20 July 2009 | doi: 10.1038/nphys1342 Quantum computation and quantum-state engineering driven by dissipation Frank Verstraete1, Michael M. Wolf2 & J. Ignacio Cirac3** Nonlocal twist operators are introduced for the O (n) and Q-state Potts models in two dimensions which count the numbers of self-avoiding loops (respectively, percolation clusters) surrounding a given point. Their scaling dimensions are computed exactly. This yields many results: for example, the number of percolation clusters which must be crossed to connect a given point to an infinitely distant boundary. Its mean behaves as $(1/33\sqrt{\pi})\,|\ln(p_c-p)|$ as $p\to p_c-$. As an application **John Cardy** computes the exact value $3\sqrt{/2}$ for the conductivity at the spin Hall transition, as well as the shape dependence of the mean conductance in an arbitrary simply connected geometry with two extended edge contacts. DOI: http://dx.doi.org/10.1103/PhysRevLett.84.3507 **Linking Numbers for Self-Avoiding Loops and Percolation: Application to the Spin Quantum Hall Transition Phys. Rev. Lett 84, 3507 – Published 17 April 2000 John Cardy Ferenc Iglói, Loïc Turban, Dragi Karevski, and Ferenc Szalma** consider the Ising model and the directed walk on two-dimensional layered lattices and show that the two problems are inherently related: zero-field thermodynamical properties of the Ising model are contained in the spectrum of the transfer matrix of the directed walk. The critical properties of the two models are connected to the scaling behavior of the eigenvalue spectrum of the transfer matrix which is studied exactly through renormalization for different self-similar distributions of the couplings. The models show very rich bulk and surface critical behaviors with nonuniversal critical exponents, coupling-dependent anisotropic scaling, first-order surface transition, and stretched exponential critical correlations. It is shown that all the nonuniversal critical exponents obtained for the aperiodic Ising models satisfy scaling relations and can be expressed as functions of varying surface magnetic exponents. DOI: http://dx.doi.org/10.1103/PhysRevB.56.11031 **exact renormalization-group study of aperiodic Ising quantum chains and directed walks Phys. Rev. B 56, 11031 – Published 1 November 1997 Ferenc Iglói, Loïc Turban, Dragi Karevski, and Ferenc Szalma** Using exact expressions for the persistence probability and for the leading eigenvalue of the Fokker-Planck operator of a random walk in a random environment, we establish a fundamental relation between the statistical properties of anomalous diffusion and the critical and off-critical behavior of random quantum spin chains. Many exact results are obtained from this correspondence, including the space and time correlations of surviving random walks and the distribution of the gaps of the corresponding Fokker-Planck operator. In turn **Ferenc Iglói and Heiko Rieger** derives analytically the dynamical exponent of the random transverse-field Ising spin chain in the Griffiths-McCoy region. DOI: http://dx.doi.org/10.1103/PhysRevE.58.4238 **Anomalous diffusion in disordered media and random quantum spin chains Phys. Rev. E 58, 4238 – Published 1 October 1998 Ferenc Iglói and Heiko Rieger** To gain deeper insight into the dynamics of complex quantum systems we need a quantum leap in computer simulations. We cannot translate quantum behaviour arising from superposition states or entanglement efficiently into the classical language of conventional computers. The solution to this problem, proposed in 1982 (ref. 1), is simulating the quantum behaviour of interest in a different quantum system where the interactions can be controlled and the outcome detected sufficiently well. **A. Friedenauer1, H. Schmitz1, J. T. Glueckert, D. Porras & T. Schaetz** study the building blocks for simulating quantum spin Hamiltonians with trapped ions2. They experimentally simulate the adiabatic evolution of the smallest non-trivial spin system from paramagnetic into ferromagnetic order with a quantum magnetization for two spins of 98%. We prove that the transition is not driven by thermal fluctuations but is of quantum-mechanical origin

(analogous to quantum fluctuations in quantum phase transitions3). We observe a final superposition state of the two degenerate spin configurations for the ferromagnetic order (|↑↑right fence+|↓↓right fence), corresponding to deterministic entanglement achieved with 88% fidelity. This method should allow for scaling to a higher number of coupled spins2, enabling implementation of simulations that are intractable on conventional computers. **Letter Nature Physics 4, 757 - 761 (2008) Published online: 27 July 2008 | doi:10.1038/nphys1032 Subject Categories: Quantum physics | Atomic and molecular physics Simulating a quantum magnet with trapped ions A. Friedenauer1, H. Schmitz1, J. T. Glueckert, D. Porras & T. Schaetz** This article gives a comprehensive description of the fractal geometry of conformally-invariant (CI) scaling curves, in the plane or half-plane. It focuses on deriving critical exponents associated with interacting random paths, by exploiting an underlying quantum gravity (QG) structure, which uses KPZ maps relating exponents in the plane to those on a random lattice, i.e., in a fluctuating metric. This is applied to critical models, like O(N) and Potts models, and to the Stochastic L\"owner Evolution (SLE). The multifractal (MF) function f(alpha, c) of the harmonic measure near any CI fractal boundary, is given as a function of the central charge c of the associated CFT. The Hausdorff dimensions D_{H} of a non-simple scaling curve or cluster hull, and D_{EP} of its external perimeter or frontier, are shown to obey the duality equation (D_{H}-1)(D_{EP}-1)=1/4, valid for any c. The universal mixed MF spectrum f(alpha,lambda;c) describing the local spiralling rate lambda and singularity exponent alpha of the potential near any CI scaling curve is given. The duality between simple and non-simple random paths is established via symmetry of the KPZ quantum gravity map. An extended dual KPZ relation is introduced for the SLE_{kappa}, which commutes with the kappa to kappa'=16/kappa duality. This gives the SLE exponents from simple QG rules, established from the general structure of correlation functions of arbitrary interacting random sets on a random lattice. Journal reference: {\it Fractal Geometry and Applications: A Jubilee of Beno\^{i}t Mandelbrot} (M. L. Lapidus and M. van Frankenhuysen, eds.), Proc. Symposia Pure Math. vol. 72, Part 2, 365-482 (AMS, Providence, R.I., 2004) arXiv: math-ph/0303034(or arXiv: math-ph/0303034v2 for this version) **Conformal Fractal Geometry and Boundary Quantum Gravity Bertrand Duplantier** Hume, For Deleuze, Considers The Mind To Be A System Of Associations Alone, A Network Of Tendencies (ES 25): "We Are Habits, Nothing But Habits – The Habit Of Saying 'I'. Perhaps There Is No More Striking Answer To The Problem Of The Self." (ES X.) The Mind, Affected By The Natural Principle Of Association, Becomes Human Nature, From The Ground Up: Empirical Subjectivity Is Constituted In The Mind Under The Influence Of The Principles Affecting It; The Mind Therefore Does Not Have The Characteristics Of A Preexisting Subject. (ES 29) These Associations Account Not Only for Experience in the Basic Sense, But up To the Highest Level of Social and Cultural Life: This Is The Basis For Hume's Rejection (E) Of A Social Contract Model Of Society (Such As Hobbes'), In Favour Of Convention Alone. Morals, Feelings, Bodily Comportment, All Of These Elements Of Subjectivity Are Explained, Not By Transcendental Structures, Such As Kant Will Propose, But By The Immanent Activity Of Association. Once This Habitual Structure Of The Self Is In Place, Deleuze Suggests, The Humean Concept Of Belief Comes Into Play , Which Is Resolutely A Central Part Of Human Nature. It Describes The Particularly Human Way Of Going Beyond The Given. When We Expect The Sun To Come Up Tomorrow, We Do Not Do So Because We Know That It Will, But Because Of A Belief Based On A Habit. This In Turn Reverses The Hierarchy Of Knowledge And Belief, And Results, For Deleuze, In A, "Great Conversion Of Theory To Practice." (PI 36) Every Act Of Belief Is A Practical Application Of Habit, Without Any Reference To An A Priori Ability To Judge. Not Only Is The Human Being Thus Habitual, On Deleuze's Reading, But Also Creative Even In The Most Mundane Moments Of Life. Finally, Deleuze Insists That One Of Hume's Greatest Contributions To Modern Philosophy Is His Insistence That All Relations Are External To Their Terms: This Is The Essence Of Hume's Anti-Transcendental Stance. Human Nature Cannot Unite Itself, There Is No 'I' Which Stands Before Experience, But Only Moments Of Experience Themselves, Unattached And Meaningless Without Any Necessary Relation To Each Other. A Flash Of Red, A Movement, A Gust Of Wind, Dashing Of A Woman Against You, Obscene Gestures, Spitting At You, Talking Extempore About You (My Addition) And These Elements Must Be Externally Related To Each Other To Create The Sensation Of A Tree In Autumn. In The Social World, This Externality Attests To The Always-Already Interested Nature Of Life: No

Relation Is Necessary, Or Governed By Neutral Laws, So Every Relation Has A Localised And Passional Motive. The Ways In Which Habits Are Formed Attests To The Desires At The Heart Of Our Social Milieu. Subjectivity, As Deleuze Describes It Through His Reading Of Hume, Is A Practical, Passional, And Empiricist Concept, Immediately Located At The Heart Of The Conventional, Which Is To Say The Social. (Stanford Encyclopedia of Philosophy). The Buddha Always Used The Terms Void, No Rising And Falling, Calmness And Extinction To Explain The Profound Meaning Of Sunyata And Cessation. For Example, Sunyata And The State Of Nirvana Where There Is No Rising Nor Falling, Are Interpreted By Most People As A State Of Non-Existence And Gloom. They Fail To Realise That Quite The Opposite, Sunyata Is Of Substantial And Positive Significance. The Sutras Often Use The Word "Great Void" To Explain The Significance Of Sunyata. In General, We Understand The "Great Void" As Something That Contains Absolutely Nothing. However, From A Buddhist Perspective, The Nature Of The "Great Void" Implies Something Which Does Not Obstruct Other Things, In Which All Matters Perform Their Own Functions. Materials Are Form, Which By Their Nature, Imply Obstruction. The Special Characteristic Of The "Great Void" Is Non-Obstruction. The "Great Void" Therefore, Does Not Serve As An Obstacle To Them. Since The "Great Void" Exhibits No Obstructive Tendencies, It Serves As The Foundation For Matter To Function. In Other Words, If There Was Neither “Great Void” Nor Characteristic Of Non-Obstruction, It Would Be Impossible For The Material World To Exist And Function. The "Great Void" Is Not Separated From The Material World. The Latter Depends On The Former. We Can State That the Profound Significance of Sunyata and the Nature of Sunyata in Buddhism Highlight The "Great Void’s" Non-Obstructive Nature. Sunyata Does Not Imply The "Great Void". Instead, It Is The Foundation Of All Phenomena (Form And Mind). It Is The True Nature Of All Phenomena, And It Is The Basic Principle Of All Existence. In Other Words, If The Universe’s Existence Was Not Empty Nor Impermanent, Then All Resulting Phenomena Could Not Have Arisen Due To The Co-Existence Of Various Causes And There Would Be No Rising Nor Falling. The Nature Of Sunyata Is Of Positive Significance. Calmness And Extinction Are The Opposite Of Rising And Falling. They Are Another Way To Express That There Is No Rising And Falling. Rising And Falling Are The Common Characteristics Of Worldly Existence. All Phenomena Are Always In The Cycle Of Rising And Falling. However, Most People Concentrate On Living (Rising). They Think That The Universe And Life Are The Reality Of A Continuous Existence. Buddhism On The Other Hand, Promotes The Value Of A Continuous Cessation (Falling). This Cessation Does Not Imply That It Ceases To Exist Altogether. Instead, It Is Just A State In The Continuous Process Of Phenomena. In This Material World, Or What We May Call This "State Of Existence", Everything Eventually Ceases To Exist. Cessation Is Definitely The Home Of All Existences. Since Cessation Is The Calm State Of Existence And The Eventual Refuge Of All Phenomena, It Is Also The Foundation For All Activities And Functions. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) Studies by Eric V. Linder about geometric dark energy bring out the quintessential prerequisites about the extreme physics ad dark matter**.** The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function w(z) and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as “geometric dark energy” from the Ricci scalar. We investigate these interrelationships, including for the case of super acceleration or phantom energy where the fate of the universe may be gentler than the Big Rip. DOI: http://dx.doi.org/10.1103/PhysRevD.70.023511 **Probing gravitation, dark energy, and acceleration Phys. Rev. D 70, 023511 – Published 28 July 2004 Eric V. Linder. In the same vein he continues to question the number of parameters that exists in dark energy** .How many dark energy parameters? For exploring the physics behind the accelerating Universe a crucial question is how much we can learn about the dynamics through next generation cosmological experiments. For example, in defining the dark energy behavior through an effective equation of state, how many parameters can we realistically expect to tightly constrain? Through both general and specific examples (including new parametrizations and principal component analysis) we argue that the answer is two. Cosmological parameter analyses involving a measure of the

equation of state value at some epoch (e.g. w0) and a measure of the change in equation of state (e.g. w′) are therefore realistic in projecting dark energy parameter constraints. More elaborate parametrizations could have some uses (e.g. testing for bias or comparison with model features), but do not lead to accurately measured dark energy parameters. DOI: http://dx.doi.org/10.1103/PhysRevD.72.043509 **Phys. Rev. D 72, 043509 – Published 11 August 2005 Eric V. Linder and Dragan Huterer D.F. Mota, C. van de Bruck Oxford Univ** study the spherical collapse model in dark energy cosmologies, in which dark energy is modelled as a minimally coupled scalar field. We first follow the standard assumption that dark energy does not cluster on the scales of interest. Investigating four different popular potentials in detail, we show that the predictions of the spherical collapse model depend on the potential used. We also investigate the dependence on the initial conditions. Secondly, they investigate in how far perturbations in the quintessence field affect the predictions of the spherical collapse model. In doing so, we assume that the field collapses along with the dark matter. Although the field is still subdominant at the time of virialisation, the predictions are different from the case of a homogeneous dark energy component. This will in particular be true if the field is non--minimally coupled. **D.F. Mota, C. van de Bruck** conclude that a better understanding of the evolution of dark energy in the highly non--linear regime is needed in order to make predictions using the spherical collapse model in models with dark energy. Astrophys 421 (2004) 71-81DOI:10.1051/0004-6361:20041090arXiv: astro-ph/0401504 **on the spherical collapse model in dark energy cosmologies D.F. Mota, C. van de Bruck (Oxford Univ)** Studies by Eric V. Linder about geometric dark energy bring out the quintessential prerequisites about the extreme physics ad dark matter**.** The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function w(z) and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as “geometric dark energy” from the Ricci scalar. We investigate these interrelationships, including for the case of super acceleration or phantom energy where the fate of the universe may be gentler than the Big Rip. DOI: http://dx.doi.org/10.1103/PhysRevD.70.023511 **Probing gravitation, dark energy, and acceleration Phys. Rev. D 70, 023511 – Published 28 July 2004 Eric V. Linder. In the same vein he continues to question the number of parameters that exists in dark energy** .How many dark energy parameters? For exploring the physics behind the accelerating Universe a crucial question is how much we can learn about the dynamics through next generation cosmological experiments. For example, in defining the dark energy behavior through an effective equation of state, how many parameters can we realistically expect to tightly constrain? Through both general and specific examples (including new parametrizations and principal component analysis) we argue that the answer is two. All Existences Exhibit Void-Nature And Nirvana-Nature. These Natures Are The Reality Of All Existence. To Realise The Truth, We Have To Contemplate And Observe Our Worldly Existence. We Cannot Realise The Former Without Observing The Latter. Consider This Heart Sutra Extract, "Only When Avalokiteshvara Bodhisattva Practised The Deep Course Of Wisdom Of Prajna Paramita Did He Come To Realise That The Five Skandhas (Aggregates, And Material And Mental Objects) Were Void." Profound Wisdom Leads Us To The Realisation That All Existences Are Of Void-Nature. The Sutras Demonstrate That The Profound Principle Can Be Understood By Contemplating And Observing The Five Skandhas. We Cannot Realise The Truth By Seeking Something Beyond The Material And Mental World. The Buddha, Using His Perfect Wisdom, Observed Worldly Existence From Various Implications And Aspects, And Came To Understand All Existences. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) .But From The Profound Contemplation And Wisdom Of The Buddha And Mahabodhisattvas, We Know There Is No Such Reality. Instead, Egolessness (Non-Self) Is The Only Path To Understand The Reality Of The Deluded Life. All Existences Are Subject To The Law Of Causes And Conditions. These Include The Smallest Particles, The Relationship Between The Particles, The Planets, And The Relationship Between Them, Up To And Including The Whole Universe! From The Smallest Particles To The Biggest Matter, There Exists No Absolute Independent Identity. Egolessness (Non-Self) Implies The Void Characteristics Of All Existence. Egolessness (Non-Self)

Signifies The Non-Existence Of Permanent Identity For Self And Existence (Dharma). Sunyata Stresses The Voidness Characteristic Of Self And Existence (Dharma). Sunyata And Egolessness Possess Similar Attributes. As We Have Discussed Before, We Can Observe The Profound Significance Of Sunyata From The Perspective Of Inter-Dependent Relationships. Considering Dharma-Nature And The Condition Of Nirvana, All Existences Are Immaterial And Of A Void-Nature. Then We See Each Existence As Independent Of Each Other. But Then We Cannot Find Any Material That Does Exist Independent Of Everything Else. So Egolessness Also Implies Void-Nature! From The Observation Of All Existences, We Can Infer The Theory Of Nirvana And The Complete Cessation Of All Phenomena. From The Viewpoint Of Phenomena, All Existences Are So Different From Each Other, That They May Contradict Each Other. They Are So Chaotic. In Reality, Their Existence Is Illusionary And Arises From Conditional Causation. They Seem To Exist On One Hand, And Yet Do Not Exist On The Other. They Seem To Be United, But Yet They Are So Different To One Another. They Seem To Exist And Yet They Do Cease! Ultimately Everything Will Return To Harmony And Complete Calmness. This Is The Nature Of All Existence. It Is The Final Resting Place For All. If We Can Understand This Reality And Remove Our Illusions, We Can Find This State Of Harmony And Complete Calmness. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) All Our Contradictions, Impediments And Confusion Will Be Converted To Equanimity. Free From Illusion, Complete Calmness Will Be The Result Of Attaining Nirvana. The Buddha Emphasised The Significance Of This Attainment And Encouraged The Direct And Profound Contemplation On Void-Nature. He Said, "Since There Is No Absolute Self-Nature Thus Every Existence Exhibits Void-Nature. Because It Is Void, There Is No Rising Nor Falling. Since There Is No Rising Nor Falling, Thus Everything Was Originally In Complete Calmness. Its Self-Nature Is Nirvana." Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia).The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars Of Xian Shou Say, "It Is Arising From Primal Nature", And The Scholars Of Chan (Zen) Say, "It Is Nature That Causes The Rising Of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia). In Other Words, All Dharma Arises From Causes And Conditions, And All Dharma Is Empty In Nature. The Law of Dependent Origination (Existence) And The Nature Of Emptiness Is Neither The Same Nor Different. They Exist Mutually. The Truth Of "Sunyata" And "Existence", And "Nature" And "Phenomena" Are Not In Conflict With Each Other. Unlike The Scholars Of The Dharmalaksana Sect Who Explain The Dharma Only From The Aspect Of Dependent Origination, Or The Scholars Of Dharma-Nature That Explain The Existence Of Dharma Only From The Aspect Of Dharma-Nature, The Scholars Of Madhyamika Explain The Truth Of The Dharma From Both Aspects. Hence This Is Called The Middle Path Which Does Not Incline To Either Side. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) **C S Lent, P D Tougaw, W Porod and G H Bernstein** formulate a new paradigm for computing with cellular automata (CAS) composed of arrays of quantum devices-quantum cellular automata. Computing in such a paradigm is edge driven. Input, output, and power are delivered at the edge of the CA array only; no direct flow of information or energy to internal cells is required. Computing in this paradigm is also computing with the ground state. The architecture is so designed that the ground-state configuration of the array, subject to boundary conditions determined by the input, yields the computational result. proposal is put forth for a specific realization of these ideas using two-electron cells composed of quantum dots. The charge density in the cell is very highly polarized (aligned) along one of the two cell axes, suggestive of a two-state CA. The polarization of one cell induces a polarization in a neighboring cell through the Coulomb interaction in a very non-linear fashion. Quantum cellular automata can perform useful computing. The authors show that AND gates, OR gates and inverters can be constructed and interconnected. **C S Lent et al 1993 Nanotechnology 4 49 doi:10.1088/0957-4484/4/1/004 Quantum cellular automata C S Lent, P D Tougaw, W Porod and G H Bernstein** The basic features of the dynamics of open quantum systems, such as the dissipation of energy, the decay of coherences, the

relaxation to equilibrium or non-equilibrium stationary state, and the transport of excitations in complex structures are of central importance in many applications of quantum mechanics. The theoretical description, analysis and control of non-Markovian quantum processes play an important role in this context. While in a Markovian process an open system irretrievably loses information to its surroundings, non-Markovian processes feature a flow of information from the environment back to the open system, which implies the presence of memory effects and represents the key property of non-Markovian quantum behaviour. **Heinz-Peter Breuer** review recent ideas developing a general mathematical definition for non-Markovianity in the quantum regime and a measure for the degree of memory effects in the dynamics of open systems, which are based on the exchange of information between system and environment. **Heinz-Peter Breuer** further study the dynamical effects induced by the presence of system–environment correlations in the total initial state and design suitable methods to detect such correlations through local measurements on the open system. **Heinz-Peter Breuer 2012 J. Phys. B: At. Mol. Opt. Phys. 45 154001 doi:10.1088/0953-4075/45/15/154001 Foundations and measures of quantum non-Markovianity** A fundamental step towards atomic- or molecular-scale spintronic devices has recently been made by demonstrating that the spin of an individual atom deposited on a surface1, or of a small paramagnetic molecule embedded in a nanojunction2, can be externally controlled. An appealing next step is the extension of such a capability to the field of information storage, by taking advantage of the magnetic bistability and rich quantum behaviour of single-molecule magnets3, 4, 5, 6 (SMMs). Recently, a proof of concept that the magnetic memory effect is retained when SMMs are chemically anchored to a metallic surface7 was provided. However, control of the nanoscale organization of these complex systems is required for SMMs to be integrated into molecular spintronic devices8, 9. Here we show that a preferential orientation of Fe4 complexes on a gold surface can be achieved by chemical tailoring. As a result, the most striking quantum feature of SMMs—their stepped hysteresis loop, which results from resonant quantum tunnelling of the magnetization5, 6—can be clearly detected using synchrotron-based spectroscopic techniques. With the aid of multiple theoretical approaches, **M. Mannini et al** relate the angular dependence of the quantum tunnelling resonances to the adsorption geometry, and demonstrate that molecules predominantly lie with their easy axes close to the surface normal. Findings of **M. Mannini et al prove** that the quantum spin dynamics can be observed in SMMs chemically grafted to surfaces, and offer a tool to reveal the organization of matter at the nanoscale. **Quantum tunnelling of the magnetization in a monolayer of oriented single-molecule magnets M. Mannini et al** Nature 468, 417–421 (18 November 2010) doi: 10.1038/nature09478 Supposing That There Were Indeed An "Energy Of Contraction" Constant In All Centers Of Force Of The Universe, It Remains To Be Explained Where Any Difference Would Ever Originate. It Would Be Necessary For The Whole To Dissolve Into An Infinite Number Of Perfectly Identical Existential Rings And Spheres, And We Would Therefore Behold Innumerable And Perfectly Identical Worlds COEXISTING [Nietzsche Underlines This Word Twice] Alongside Each Other. Is It Necessary For Me To Admit This? Is It Necessary To Posit An Eternal Coexistence On Top Of The Eternal Succession Of Identical Worlds? The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO"The Cyclical Hypothesis, So Heavily Criticized By Nietzsche (VP II 325 And 334), Arises In This Way." In Fact, Nietzsche Was Not Criticizing The Cyclical Hypothesis But Only The Particular Form Of That Hypothesis Presented In Vogt's Work. All Of Nietzsche's Texts Without Exception Speak Of The Eternal Return As The Repetition Of The Same Events Within A Cycle Which Repeats Itself Eternally. If Deleuze's Interpretation Holds That The Eternal Return Is Not A Circle, Then What Is It? A Wheel Moving Centrifugally, Operating A "Creative Selection," "Nietzsche's Secret Is That The Eternal Return Is Selective" Says Deleuze: The Eternal Return Produces Becoming-Active. It Is Sufficient To Relate The Will To Nothingness To The Eternal Return In Order To Realize That Reactive Forces Do Not Return. However Far They Go, However Deep The Becoming-Reactive Of Forces, Reactive Forces Will Not Return. The Small, Petty, Reactive Man Will Not Return. Affirmation Alone Returns, This That Can Be Affirmed Alone Returns, Joy Alone Returns. Everything That Can Be Denied, Everything That Is Negation, Is Expelled Due To The Very Movement Of The Eternal Return. We Were Entitled To Dread That The Combinations Of Nihilism And Reactivity Would Eternally Return Too. The Eternal Return Must Be Compared To A Wheel; Yet, The Movement Of The Wheel Is Endowed With Centrifugal Powers That Drive Away The Entire

Negative. Because Being Imposes Itself On Becoming, It Expels From Itself Everything That Contradicts Affirmation, All Forms Of Nihilism And Reactivity: Bad Conscience, Ressentiment..., We Shall Witness Them Only Once. [...] The Eternal Return Is The Repetition, But The Repetition That Selects, The Repetition That Saves. Here Is The Marvelous Secret Of A Selective And Liberating Repetition. There Is No Need To Remind The Reader That Neither The Image Of A Centrifugal Movement Nor The Concept Of A Negativity-Rejecting Repetition Appears Anywhere In Nietzsche's Writings, And Indeed Deleuze Does Not Refer To Any Text In Support Of This Interpretation. Further, One Could Highlight That Nietzsche Never Formulates The Opposition Between Active And Reactive Forces, Which Constitutes The Broader Framework Of Deleuze's Interpretation. For Some Years, Marco Brusotti Has Called Attention To The Fact That Deleuze Introduced A Dualism That Does Not Exist In Nietzsche's Writings. To Be Sure, The German Philosopher Describes A Certain Number Of "Reactive" Phenomena (For Example, In The Second Essay Of The Genealogy Of Morality, § 11, He Talks About "Reactive Affects" [Reaktive Affekte], "Reactive Feelings" [Reaktive Gefühlen], Reactive Men [Reaktive Menschen]); But These Are Nonetheless The Result Of Complex Ensembles Of Configurations Of Centers Of Forces That Remain In Themselves Active. Neither The Word Nor The Concept Of "Reactive Forces" Ever Appears In Nietzsche's Philosophy. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO Some Parts Are Deleted Due To Spatial Constraints. Please Pardon Me For The Deletion. Deleuze Opposes The Historical Course Of The Hegelian Notion That Confronts Struggles And Finally Dialectizes The Negative And Results In A Consoling Teleology Leading To The Triumph Of The Idea Or The Liberation Of The Masses With The Centrifugal Movement Of The Wheel, Which Simply Ejects The Negative. It Is Still A Case Of A Consoling And Optimistic Teleology, Which, Instead Of Confronting The Weight Of History, The Grief And The Negative, Makes It Disappear In One Centrifugal Stroke Of A Magic Wand. There Is Reason To Worry That This Be A Case Of Repression, Which, Unable To Dialectize Or Accept The Negative, Simply Seeks To Exorcise It In One Gesture Of "Creative Selection." But Exorcism Is A Feat Of Magic And Not Of Philosophy: It Is Unfortunately Not Enough To Make The Negative Disappear. In All Probability, The Negative Will Come Back With A Vengeance. In Contrast To Deleuze's "Affirmation Of Affirmation", Which Affirms Only Affirmation, Nietzsche Conceives Of The Eternal Return From A Rigorously Non-Teleological Perspective As The Accomplishment Of A Philosophy Strong Enough To Accept Existence In All Its Aspects, Even The Most Negative, Without Any Need To Dialecticize Them, Without Any Need To Exclude Them By Way Of Some Centrifugal Movement Of Repression. It Denies Nothing And Incarnates Itself In A Figure Similar To The One Nietzsche, In Twilight Of The Idols, Draws Of Goethe: Such A Spirit, Who Has Become Free Stands In The Middle Of The World With A Cheerful And Trusting Fatalism In The Belief That Only The Individual Is Reprehensible, That Everything Is Redeemed And Affirmed In The Whole-He Does Not Negate Anymore. Such A Faith However, Is The Highest Of All Possible Faiths: I Have Baptized It With The Name Of Dionysus. The Eternal Return: Genesis and Interpretation BY PAUL D'IORIO The Most Complete Theory Of Mind That Comports With The Holographic Principle Of Mind, As Developed Here, Is The Microgenetic Theory Of Neurologist Jason Brown. Brown (2005) Presents An Evolutionary Theory Of Values, Morals, And Ethics, Building On The Microgenetic Theory And Process Theory Of His Earlier Works (1988, 1991, 1996, 1998, And 2000). Brown Is Work Has Been A Progression That Began With The Study Of Brain Processes In Neurological Subjects With Brain Injuries Or Lesions. In His Earlier Work (Brown, 1988; 1991) He Described The Process Of Microgenesis As A Process Of Elaboration Of Mental Contents In An Evolutionary And Developmental Hierarchy Of Brain Structures Within The Process Of The Genesis Of The Mental State, And Related The Hierarchy Of The Genesis Of The Mental State To Disturbances Of Language Comprehension Or Expression (Aphasia), Of Knowledge (Agnosia), And Or Purposeful Movement (Apraxia). The Fundamental Neurology Of These Disturbances Had Been Described In Brownís Previous Work (1972, 1988), And In The Seminal Work Of The Russian Neurologist And Neuropsychologist, Alexander Luria (1966). Brown (1988, 1991, 1996, 1998) Created A Process Theory Of Mind On The Basis Of His Neurological Observations, And Went On From His Neurological Work To Incorporate Process Theory In Philosophy, As Developed Principally By Henri Bergson (1911/1998). And Alfred North Whitehead (1925/1953, 1929/1978). Bergson Had Pioneered The Concept Of An Irreducible Duration

Of Experience. This Concept Was Elaborated By Whitehead, In Keeping With Discontinuities Of Particles, As "Actual Entities" Of A Rudimentary Sort, As They Make "Quantum Leaps" Along Their Trajectories, Halting Or Persisting At A Given Location For A Short Period Of Time Or Duration Before Leaping To The Next. Whitehead Employed These "Quantum Leaps" To The Process By Which Experience Arises In "Actual Entities" At The Most Fundamental Level Of Actuality, Through The Epochal Or Halting Duration Of A Discontinuous Process (Whitehead, 1925/1953, 1929/1978). This Halting Allows Actual Entities To Participate In A Process Of Internal Relations, Relations On The "Inside" Of Things. According The Holographic Principle, This "Inside" Is In The Holographic Surface Of The Relevant Region Of Spacetime Of The Actual Entity, And Within Other Surfaces That Are Non-Locally Connected With That Surface. The Holographic Surface Is Thus The Locus Of Non-Local Information And Experience, In Our Interpretations Of Whitehead Is Metaphysics. Internal Relations Occur Through Feeling, Or "Prehension," Which Is Nothing More Than The Subjective Quality Of Non-Local Experience. The Internal Relations Of The Becoming Entity Occur In A Seriality Of Prior Becomings Of The Actual Entity, With The Entity Inheriting All Of Its Causal Past As Well As Its Relevant Feelings Or Prehensions From Prior "Occasions" Or Quanta Of Experience. This Process Is, In The Holographic Model, The Process By Which Former Surfaces, Now-Pasts, Are Elaborated On To The Now-Present. Prehensions Also Occur With Other Actual Entities Throughout The Universe, With Which They Are Non-Locally Connected, And With Neo-Platonic "Eternal Objects," Existing In An Eternal Heaven. The "Ingression" Of Eternal Objects Is Fundamental To Each Duration Of Becoming Of The Entity, And, On This Basis, Whitehead Formulated The Notion Of An Eternal Heaven In Constant Intercourse All Of Its Creation. There Are Clear Parallels Between Whitehead's Cosmology And The Cosmology Introduced Here According To The Holographic Principle, In That The "Heaven" May Be Regarded As The Holographic Boundary, In Constant Intercourse With The Relevant Volume Of Spacetime. This Interconnection Of Feelings Or Prehensions, According To Their Relevance, Produces Intensities And Contrasts Which Give Rise To The Creative Advance, Which, For Whitehead (1929/1978) Is The Process Which Impels The Movement From The Physical To The Mental Pole Of Process. With Each Halting, Internally-Timeless Duration There Is A Concrescence Or Integration Of These Feelings, Which, When Complete, Constitutes A New Actual Entity Or Actual Occasion (They Are The Same Thing) At A New Locus In Spacetime. This Movement To A New Locus, Or Transition, Is, Fundamentally, The Quantum Leap. MICROGENESIS AND THE HOLOGRAPHIC PRINCIPLE The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science If the Holographic Principle is true, than it must be the fundamental principle of mind. The brain has no way around the Holographic Principle. For the reductionist, the Holographic Principle is the ultimate reduction. It applies to the most minuscule level of what we can observe, and beyond. For the Universalist, the Holographic Principle gives us the ultimate universal. It extends to the limit of our Universe, the Universal holographic boundary, and beyond. For the Phenomenalist, the Holographic Principle gives us the ultimate ground of our phenomenal perception, our grounding in the Universal "now," in the now-present of the Universal holographic boundary, moving outward from now-pasts to now-futures. Wheeler (1988) has said that all of reality is information, and that other physical quantities are "mere incidentals." Information monism is gaining popularity in physics. By breaking the dichotomy of between information and experience, we find a deep connection between Wheeler is monism and the experiential monism of Whitehead, sometimes called panexperientialism, which relates to the later theological concept of his student, Charles Hartshorne, panentheism, God inside of everything (Hartshorne, 1964). There thus seems to be a convergence of the concepts of information, experience, and spirituality. There is not one universe, but many parallel universes, or, more accurately, a vast superposition of universes called the multiverse (Penrose, 2004). However, as explained in our treatment of the double-slit experiment, in the observation of events on the quantum level, the individual observer sees only one of these vast superpositions, not a summation of a vast number of potentials. This has been explained in terms of the many worlds or many minds theories, in which there are multiple copies of the same observing individual, but this particular issue remains unresolved, leaving physics ungrounded in reality (Penrose, 2004). The concept of One Mind is not only more parsimonious than that of many minds, but also lets us out of the bizarre and counterintuitive idea that there are multiple copies of our own selves, which exist as

mere potential, and are thus not actual. There are many possible universes, but there is only One Mind, which determines events on the quantum level, and thus creates our Universe. As we had discussed previously, the quantum level provides the essential ground of the Holographic Principle, such that quantum-level holographic surfaces are elaborated at higher levels, manifesting higher orders of information from the quantum "world." In this process, there is a reduction of the wave function or potentialities of quantum fields. At the level of consciousness, this entails freedom to choose which observations we make (Stapp, 1997), which gives us the capacity to think and to make decisions. These capacities are the basis of individuality, self-determination, judgment, and values. The progressive evolution of the manifestation of Mind through higher orders of experience, leading to higher orders of consciousness, is entailed by the Holographic Principle. Again, we are dealing here with levels of description, with the multiverse of all potentials fundamentally supporting the single Universe we collectively observe. The multiverse is the wave function of the Universe (Penrose, 2004). The recursive integration of nested hierarchies of holographic surfaces brings out a single actuality in consciousness from a wave function that is unconscious. Consciousness thus gives us information at a level of experience that is causal. In this sense, we partake in the creation of the Universe. We participate in creation, and this participation, when fully realized, leads us to higher levels of consciousness and of realization. As a system, the biosphere that we live in has a holographic surface, creating a deep sense of ecology as we collectively move toward a planetary consciousness. It is only when the Universal Holographic Boundary reaches the information storage and processing capacity needed for the requisite biological and biochemical complexity that consciousness evolves in living things. This evolution is natural and spontaneous, since consciousness gives rise to what is actual, as opposed to what is merely potential. In this sense, consciousness is still in the process of creating our Universe, and levels of higher consciousness will continue to evolve. We are all in the same "now," and that now is defined by the present Universal Boundary. This assures us that our experience is Universal, and does not pass with time. This is the fundamental basis of memory and of cognition. The identity of mind and brain is a myth. We have a continual, internal or non-local relation with the Universe, as it has with us. Once this mystery is resolved, the myth is no longer needed, and there is a confluence of science and spirituality.Experience is primary, information is secondary. We can only gather information from experience, whether it is in the laboratory or in life. We cannot measure the information on the surfaces of systems. The physicists that have formulated the Holographic Principle for all systems are quite aware of this, or else the principle would have been established or discredited. We cannot measure what we experience. It is intangible, yet it is all we know to be actual. Everything else is inferred. Because it cannot be measured, it has been fundamentally disregarded by mainstream science. Materialism is considered scientific, while idealism is considered unscientific. But arenít ideas, fundamentally, made of information? Brain science has mistaken the representation of information for information itself, and has tied those representations to matter and energy. Consciousness, the highest order of information, has generally been regarded as superfluous, something that needs to explained away, or altogether ignored. Yet it is the only "thing" that reaches our awareness. Consciousness comes at a price. For everything that becomes conscious, there must be something that becomes conscious. Consciousness is certainty, and its complement, unconsciousness uncertainty. Consciousness is the particle nature of experience. It has definiteness about it. The unconscious is the wave nature of experience, it is like the metaphysical cloud of unknowing. If we are the most conscious of animals, then we must also be the most unconscious. Perhaps this is the predicament of humankind. The Holographic Principle of Mind leads us naturally from the most fundamental experiences, existing as quantum potentials from the conformations of proteins down through the fields of electrons, through their manifestation upward through a recursion or successive applications of the same holographic process, through higher levels of experience, to the emergence of consciousness as higher and higher orders of experience. The quantum Holographic Principle Mind does not require anything more quantum than is obviously present at the microscopic and submicroscopic levels, as it represents successive orders of manifestations from these levels. Recursion also applies here in the sense that it is used in computer science, in which the function of the part depends on the function of the whole. A program, as a part, cannot work without a functioning whole, the operating system. Recursive wholes which are, for us, supra-conscious, are on the group, species, planetary, and universal levels. As

individuals, our consciousness cannot function without recursion to the Universal Consciousness, even though we may be unaware that such Universal Consciousness exists. Reaching upward to these supra-conscious levels is a spiritual process, making transcendence the ultimate solution to the unconscious human predicament. What was once supra-conscious becomes unconscious through a process of conditioning? We are born as creatures of the Earth and of the Universe, as evidenced by the beliefs and practices of "primitive peoples." There is evidence from cave paintings that our hominid ancestors experienced a kind of holographic perception (Combs, 1996), which could constitute our early connection with the holographic subtext of reality, and which we might then propose existed in our animal ancestors, and in extant animal species. If this is the case, than microgenesis would entail the recapitulation of this holographic experience as it progresses through our ancestral past. The supra-conscious mind seems to envelope perinatal experiences, and Stanislav Grof (1994) has developed techniques to gain access to these experiences, as well as to the earlier experiences of our human and animal ancestral lineage, and of our universal history. Grof (1994) concludes: "Our consciousness seems to have the amazing capacity to directly access the earliest history of the universe ñ witnessing dramatic sequences of the Big Bang, the formation of the galaxies, the birth of the solar system, and the early geophysical processes on this planet billions of years ago." The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science. There Are Similarities Between The Patterns Of Holography And Of Psychological Transference, Where Holography Is The Process Of Recording And Reconstructing Holograms. Employing A Theoretical Perspective Using A Hermeneutic Method, This Dissertation Parallels Holography With Transference, Offering Another Way To Encounter Transference By Showing Similarities Between The Processes Of Each And The Results Of Each. Though Complex, Infinitely Varying, And Unique, Their Patterns Are Clearly Identifiable. Thus They Are A Metaphorical Fit To The Concept Of Strange Attractors In Physics And A More Literal Fit To The Concept Of Archetypes In Depth Psychology Or Dynamic Psychology, Psychology Which Attends To The Living, Autonomous Unconscious. This Study Explains How Holography Models Transference, What A Hologram Is And How It Works, And How Depth Psychology Understands Of The Interaction Between Consciousness And The Unconscious Is Related To The Hologram. It Describes Transference And Related Psychological Processes As Understood In Six Different Schools Of Depth Psychological Thought. It Shows That The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology, Processes Such As Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. By Identifying The Similar Patterns Of These Processes, This Study Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole: The Visual Image Of The Overall Hologram Is Present In Each Component Part Of The Hologram, The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. By Noting Differences As Well As Similarities In These Processes, It Suggests An Inventory Of The Qualities Of The Holographic Archetype. This Study Furthers Understanding Of The Pervasiveness, Force, And Autonomy Of The Unconscious Acting Through Transference And Projection By Identifying A Group Projection Of Domestic Violence Lying At The Core Of The Christian Myth. This Study Also Furthers Understanding Of The Concept Of Transference By Providing A Reflection Hologram Of The Human Psyche As An Artistic Work And As A Visual Metaphor Of Transference. Strange Attractors: Transference, Holography, And An Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003). Onticology, Like All Variations Of Object-Oriented Ontology, Is Realist In Its Orientation. In Defending A Realist Ontology Onticology Holds That The Vast Majorities Of Objects, Actants, Beings, Or Entities Are Independent Of Humans And Are What They Are Regardless Of Whether Any Humans Regard Them Or Register Them. In Short, Onticology Rejects Any Anthropomorphic, Idealist, Or Anti-Realist Thesis To The Effect That To Be Is To Be The Correlate Of Mind, Spirit, The Body, The Human, And Language Or Otherwise. While It Is Certainly The Case That Knowledge Is Necessarily Dependent On The Object To Which It Relates, The Converse Does Not Hold True. Objects Are Not Dependent On Being Known, Regarded, Perceived, Or Spoken About. As Such, And To Put It In Aristotelean

Terms, Knowledge Is An Accident Of Objects, Not Objects An Accident Of Knowledge. As Althusser So Nicely Puts It, "[N]O Doubt There Is A Relation Between Thought-About-The-Real And This Real, But It Is A Relation Of Knowledge, A Relation Of Adequacy Or Inadequacy Of Knowledge, Not A Real Relation, Meaning By This A Relation Inscribed In That Real Of Which The Thought Is The (Adequate Or Inadequate) Knowledge" (Reading Capital, 96). Althusser Goes On To Remark That "[T] He Distinction Between A Relation Of Knowledge And A Relation Of The Real Is A Fundamental One: If We Did Not Respect It We Should Fall Irreversibly Into Either Speculative Or Empiricist Idealism" (Ibid.). Onticology Categorically Endorses Althusser's Verdict It Is A Fundamental Necessity To Distinguish Between Those Relations That Belong To The Object And Those That Belong To Knowledge. Contemporary Philosophy Continuously Confuses These Two Very Different Sorts Of Relations. Naturally The Question Arises Of How It Is Possible To Surmount Our Relation To The Object So As To Determine Whether Objects Themselves Possess The Properties We Encounter In Relating To Objects. In Other Words, Given That We Can Only Ever Relate To The Object In Relating To The Object How Is It Possible To Surmount This Relation To Get At The Being Of The Object Itself? Much More Will Have To Be Said About This Later– And The Answers Will Be Surprising With Respect To Standard Prejudices About Realism –However, For The Moment It Can Be Said That Onticology Takes Its Epistemological Inspiration From The Transcendental Realism Of Roy Bhaskar. Among Other Things, Bhaskar Sought To Provide A Transcendental Grounding For The Sciences. Insofar As Onticology Defends The Thesis That The Field Of Being Is Much Faster Than The Field Of Objects Investigated By The Natural Sciences, It Parts Way With The Thesis That The Domain Of Being Is Exhausted By The Domain Of Natural Objects. However, The General Form of Bhaskar's Argument Holds for Our Realist Purposes. A Transcendental Argument Seeks To Elucidate The Conditions Under Which Certain Acknowledged Practices And Forms Of Cognition Are Possible. Kant, For Example, Asked What Must Be The Case For Mathematical Judgments To Be Possible. How Is It Both That We Are Able To Extend Our Knowledge, As If By Magic, Through Mathematical Judgments And, More Significantly, That These Judgments Are Able To Provide Genuine Knowledge Of The World Despite The Fact That These Forms Of Reasoning Are Not Based On Experience? Part of Kant's Argument Consisted in Claiming That Mind Imposes the Forms of Space and Time on the Data of Experience. In Other Words, Space And Time Are Not Attributes Of Being Itself But Rather Of The Mind That Regards Being. Insofar, Kant Argues, As Mathematics Is Ultimately A Rumination On The Nature Of Space And Time Taken In Their Most Abstract Form And Insofar As The Mind Imposes Space And Time On The Manifold Of Sensation, It Thus Follows That A Priori Judgments About The Nature Of Spatio-Temporal Relations Are Possible That Anticipate The Structure Of Actual-Space Times Without Directly Experiencing These Space-Times. Why? Because Any Manifold Of Sensation Must Necessarily Be Structured By These Forms Imposed By Intuition Onticology– A Manifesto For Object-Oriented Ontology Part I Posted By Larval Subjects Under Object-Oriented Philosophy. ……. it is necessary to distinguish the being of objects from the manifestation of objects. While objects are acts, these acts are not identical with their performance in either nature (events where no humans are about to perceive them) or with their performance for humans. Rather, the proper being of the object is not its performance or manifestation, but the generative mechanism that serves as the condition under which these performances or manifestations are possible. As Graham Harman will argue– though in a very different theoretical constellation –the being of objects is essentially withdrawn or hidden. No one has ever perceived a single object, but we do perceive all sorts of effects of objects. Traditional epistemology has confused these effects with the objects themselves. Fortunately we do occasionally manage to cognize objects through a sort of detective work that infers these generative mechanisms from their effects; without, for all this, ever exhausting the infinity of a single object. At any rate, if objects were not withdrawn in this way, the practice of experiment would be unintelligible. This leads to Bhaskar's answer to the first question: What must the world be like for science to be possible? Note, this is not a question about mind or culture, but about the world itself, regardless of whether or not humans exist. Once again, knowledge is an accident of objects, not objects an accident of consciousness or cognition. If science is to be possible– and I would argue, if any human practice is to be possible –then the world must be structured and differentiated. The world must have joints or, as Harman puts it, the world must be composed of

"chunks". Why is this case? Let us return to the question of experimentation and the conditions under which experiment is possible. We will adopt two possible hypotheses pertaining to the ontological nature of the world independent of humans: 1) As certain mystics and contemporary crypto-mystics would have it, the world is an undifferentiated One-All that is only subsequently segmented or partitioned into discrete beings by some form of human agency whether this be through cognition or language (in the case of language we might think of Saussure's and Hjelmslev's undifferentiated "sonorous matter"). 2) Entities are the sum totality of their relations to all other entities in the universe. The first hypothesis is easily dispatched on two grounds: First, this hypothesis fails to register the contradiction in its own utterance. At the level of explicit content, it claims that the world is an undifferentiated One-All that is only subsequently segmented into discrete beings, yet what it misses at the unconscious level of its own utterance is that it registers at least one structured differentiation that is not undifferentiated within this One-All: Namely, the agency through which the One-All subsequently comes to be differentiated. Certain anti-realist, transcendental philosophers will, in a gesture that is all too cute, claim that the agency by which the world is segmented cannot properly be said to exist, thus attempting to resolve this contradiction through a sleight of hand. However, as Meillassoux has shown in After Finitude, all attempts on the part of transcendental anti-realist philosophies to treat the transcendental subject as a non-existing or non-objectile agency that does not itself exist end up, all too clearly, attaching that conception of finitude and the segmentary work with which it is charged to the body (a differentiated being or generative mechanism). Second, suppose the anti-realist transcendental philosophers were to convince us through his appeals to the non-existence of the transcendental, would we still encounter problems? Like Atlas, transcendental subjectivity is charged with the monumental task of segmenting the formless Apeiron of the One-All from out of primal chaos into a supremely segmented world. But this world appears to us to be too slippery for even a titan like Atlas to grasp. Were the world, prior to and independent of humans genuinely a formless Apeiron it would contain no differences providing hand-holds for Atlas to grasp in his segmenting activity. Consequently, no segments could ever come into being. Yet everywhere we encounter segments, so the world must not be a formless One-All, but must rather be structured and differentiated even if structure and differentiation are transformed in their encounter with the human. Onticology– A Manifesto For Object-Oriented Ontology Part I Posted By Larval Subjects Under Object-Oriented Philosophy. The neural networks of the human brain act as very efficient parallel processing computers co-ordinating memory related responses to a multitude of input signals from sensory organs. Information storage, update and appropriate retrieval are controlled at the molecular level by the neuronal cytoskeleton which serves as the internal communication network within neurons. Information flow in the highly ordered parallel networks of the filamentous protein polymers which make up the cytoskeleton may be compared to atmospheric flows which exhibit long-range spatiotemporal correlations, i.e. long-term memory. Such long-range spatiotemporal correlations are ubiquitous to real world dynamical systems and are recently identified as signature of self-organized criticality or chaos. The signatures of self-organized criticality i.e. long-range temporal correlations have recently been identified in the electrical activity of the brain. A recently developed non-deterministic cell dynamical system model for atmospheric flows predicts the observed long-range spatiotemporal correlations as intrinsic to quantum-like mechanics governing flow dynamics. The model visualizes large scale circulations to form as the result of spatial integration of enclosed small scale perturbations with intrinsic two-way ordered energy flow between the scales. Such a concept maybe applied for the collection and integration of a multitude of signals at the cytoskeletal level and manifested in activation of neurons in the macroscale. The cytoskeleton networks inside neurons may be the elementary units of a unified dynamic memory circulation network with intrinsic global response to local stimuli. Cite as: arXiv: chao-dyn/9809003.The neural network of the human brain responds as a unified whole memory bank to a multitude of input signals from the environment and functions with a high degree of robustness and stability. The three aspects of neural networks memory bank are, storage, real-time update and retrieval. The memory is believed to be embedded in the strength of the numerous connections or synapses in the network. Sensory inputs (electrical) produce particular patterns of activity in groups of neurons which then trigger optimal response to the input signal. The cooperative response of millions of neurons to a multitude of input signals has been compared to a very efficient

parallel processing computer with neurons and their synaptic connections as fundamental units of information processing, like switches within computers. However, recent studies by Hameroff et al [1,2] and Rasmussen et al [3] show that neurons and synapses are extremely complex and resemble entire computers, rather than switches. The interiors of neurons (and other eucaryotic cells) are now known to contain highly ordered parallel networks of filamentous protein polymers collectively termed the cytoskeleton. Information storage, update and appropriate retrieval are controlled at the molecular level by the neuronal cytoskeleton which serves as the internal communication network within neuron. Organization of information at the molecular level in the cytoskeletal network contributes to the overall response of each neuron and the collective activity pattern of neurons then governs the response to the environmental stimuli. The general awareness or consciousness of the individual to environment may also be governed by the overall background activity pattern of the neurons and their cytoskeletal networks. Coherent signal flow patterns in neural networks may form the basis for general consciousness and response to stimuli (external or internal). Inputs signals trigger spontaneous appropriate coherent pattern formation in the activity of the neurons with implicit spatial correlations in the activity pattern. The time variation of electrical activity of the brain as recorded by the Electro Encephalogram (EEG) exhibits fluctuations on all scales of time, i.e. a broadband spectrum of periodicities (frequencies) contribute to the observed fluctuations [4]. Power spectral analysis which is used to resolve the component frequencies (f) and their intensities shows that the intensity (power) of the component frequencies follows the inverse power law form 1/fB where B is the exponent. Inverse power law form for power spectra of temporal fluctuations imply long-range temporal correlations, i.e. long - term memory of short - term fluctuations. The signatures of short - term fluctuations are carried as internal structures of long - term fluctuations. Time variation of spatial activity pattern in neural networks therefore has inbuilt long - term memory. Neural network activity patterns therefore exhibit long - range spatial and temporal correlations. Such non-local connections in space and time are ubiquitous to time evolution of spatially extended dynamical system in nature and are recently identified as signature of self-organized criticality [5]. Examples of dynamical systems, i.e. systems which change with time include atmospheric flows, electrical activity of the brain, heart rhythms, stock market price fluctuations, etc. Extended dynamical systems in nature have self similar fractal geometry. Self similarity implies that submits of a system resemble the whole in shape. The world fractal coined by Mandelbrot [6] means fractional or broken Euclidean geometry appropriate for description of non-Euclidean structure generic to natural phenomena. The fractal dimension D is given by dlnM/dlnR where M is the mass contained within a distance R from a point within the extended object. A constant value for D implies uniform stretching on logarithmic scale for length scale range R. Objects in nature exhibit multifractal structure, i.e. the fractal dimension D varies with length scale R. Fractal architectures generic to nature support functions which exhibit fluctuations on all time scale, i.e. the fluctuations are irregular (nonlinear) and apparently chaotic. The association of fractal structures with chaotic dynamics has been identified in all dynamical systems in nature. Fractals, chaos and nonlinear dynamics or Chaos Science is now an area of intensive research in all branches of science [7]. Incidentally, Chaos Science began in 1963 with identification of sensitive dependence on initial conditions resulting in chaotic solution for computer realizations of deterministic nonlinear mathematical model of atmospheric flows and named appropriately deterministic chaos .The computed trajectory of time evolution exhibits fractal geometry. The discipline of nonlinear dynamics and chaos began with investigation of universal characteristics of deterministic chaos in nonlinear mathematical models of dynamical systems in all branches of science. In mathematics, the Cantorian fractal space-time is now associated with reference to quantum mechanical objects [8,9,10]. Further, El Naschie has shown that fractal structures (space-time) incorporate the golden mean equal to (1+Ö 5)/2 =1.618 in their architecture signifying ordered signal /information flow in the fractal network. The golden mean is incorporated in the fractal architecture of the cycloskeleton network [11] which plays a very important role in sub-consciousness to consciousness process integration [12,13]. Surprisingly similar chaotic behavior in space and time was found to be exhibited by all real world dynamical systems. Fractal structure to the spatial pattern concomitant with chaotic (irregular) dynamics has now been identified to be intrinsic to physiological and biological systems [14,15]. The branching interconnecting networks of neurons and intra-neuronal cytoskeleton networks are fractal structures which generate electrical signal

pattern with self-similar fluctuations on all scales of time characterised by 1/fB power law behavior for the power spectrum. Such inverse power law form for spectra of temporal fluctuations implies long-range temporal correlations, i. e., long term memory of short term fluctuations or events. Fractal architecture of neural networks supports and coordinates information (fluctuations) flow on all time and space time scales in a state of dynamic equilibrium, now identified as self-organized criticality, is ubiquitous to natural phenomena (living and non-living) and is independent of the exact details of the dynamical processes governing the space-time evolution. The physics of self-organized criticality or deterministic chaos is not yet identified. The physical mechanism governing self-organized criticality should be universally applicable to diverse biological, physical, chemical and other dynamical systems. In this paper a universal cell dynamical system model for self-organized criticality applicable to neural networks of the brain is summarised [16, 17, and 18]. This model was originally developed to explain the observed self-organized criticality in atmospheric flows [19, 20, and 21]. Therefore a brief description of the model with respect to atmospheric flows in first described followed by application to neural networks. Atmospheric flows exhibit self-organized criticality or long-range spatiotemporal correlations manifested in the self similar fractal geometry to the global cloud cover pattern concomitant with inverse power law form 1/fB for power spectrum of temporal fluctuations in meteorological variables such as temperature, pressure, etc. documented by Tessier et al [22]. The co-operative existence of fluctuations ranging in size (duration) from the turbulence scale of millimetres (seconds) to the planetary scale of thousands of kilometres (years) contribute to coherent weather pattern in atmospheric flows. Townsend [23] postulated that large eddies (waves) form in atmospheric flows as a chance configuration (envelope) of enclosed turbulent (small scale) eddies. A hierarchical continuum of eddies is therefore generated with larger eddies enclosing smaller eddies. Since large eddy is but the integrated mean of enclosed turbulent eddies, atmospheric eddy energy (kinetic) distribution follows normal distribution characteristics according to the Central Limit Theorem in Statistics. The eddy kinetic energy represented by square of eddy amplitude then represents the probability density. Such a result that the additive amplitudes of eddies, when squared, represent probability densities is observed in the subatomic dynamics of quantum system such as the electron or photon. Atmospheric eddy energy spectrum therefore follows quantum-like mechanical laws [19,20,21]. Condensation of water vapour in updraft regions of large eddies give rise to cloud formation while adjacent downdraft regions are associated with evaporation and cloud dissipation, thereby accounting for the discrete cellular structure to cloud geometry.\ Under the head of applications authors' describe the following: Frohlich [24] had described analogous self-organization of vibrational modes of all frequencies triggering coherent activity in biological functions. Insinna [25] has summarized Froehlich's coherent excitation concept as follows. More than 20 years ago Frohlich [24,26-28] introduced the concept of cooperative vibrational modes between proteins. Coherent oscillations in the range of 1010- 1012 Hz involving cell membranes, DNA and cellular proteins could be generated by interaction between vibrating electric dipoles contained in the proteins as a result of nonlinear properties of the system. Through long-range effects proper to Froehlich's nonlinear electrodynamics a temporospatial link, is, in fact, established between all molecules constituting the system. Single molecules may thus act in a synchronized fashion and can no longer be considered individual. New unexpected features arise from such a dynamic system, reacting as a unified whole entity [25]. Coherent Frohlich oscillations may be associated with the dynamical pattern formation of intraneuronal cytoskeletal architecture which coordinates and integrates information flow into the neuron and generates output signal. Hameroff and colleagues [1,29-31] have simulated such interaction in their cellular automata model. Conclusion they draw in is herein below: Fractal architecture to information flow path results in spatiotemporal integration of signals so that the fractal system responds as a unified whole to a multitude of input signals. Two disparate examples for such self-organized information flow networks are atmospheric flows and the neural networks of the human brain. **Cantorian Fractal Spacetime and Quantum-like Chaos in Neural Networks of the Human Brain A. M. Selvam (for reference s please see the original article. Models are given elsewhere in the papers series .** If The Holographic Principle Is True, Than It Must Be The Fundamental Principle Of Mind. The Brain Has No Way Around The Holographic Principle. For The Reductionist, The Holographic

Principle Is The Ultimate Reduction. It Applies To The Most Minuscule Level Of What We Can Observe, And Beyond. For The Universalist, The Holographic Principle Gives Us The Ultimate Universal. It Extends To The Limit Of Our Universe, The Universal Holographic Boundary, And Beyond. For The Phenomenalist, The Holographic Principle Gives Us The Ultimate Ground Of Our Phenomenal Perception, Our Grounding In The Universal "Now," In The Now-Present Of The Universal Holographic Boundary, Moving Outward From Now-Pasts To Now-Futures. Wheeler (1988) Has Said That All Of Reality Is Information, And That Other Physical Quantities Are "Mere Incidentals." Information Monism Is Gaining Popularity In Physics. By Breaking The Dichotomy Of Between Information And Experience, We Find A Deep Connection Between Wheeler Is Monism And The Experiential Monism Of Whitehead, Sometimes Called Panexperientialism, Which Relates To The Later Theological Concept Of His Student, Charles Hartshorne, Panentheism, God Inside Of Everything (Hartshorne, 1964). There Thus Seems To Be A Convergence Of The Concepts Of Information, Experience, And Spirituality. There Is Not One Universe, But Many Parallel Universes, Or, More Accurately, A Vast Superposition Of Universes Called The Multiverse (Penrose, 2004). However, As Explained In Our Treatment Of The Double-Slit Experiment, In The Observation Of Events On The Quantum Level, The Individual Observer Sees Only One Of These Vast Superpositions, Not A Summation Of A Vast Number Of Potentials. This Has Been Explained In Terms Of The Many Worlds Or Many Minds Theories, In Which There Are Multiple Copies Of The Same Observing Individual, But This Particular Issue Remains Unresolved, Leaving Physics Ungrounded In Reality (Penrose, 2004). The Concept Of One Mind Is Not Only More Parsimonious Than That Of Many Minds, But Also Lets Us Out Of The Bizarre And Counterintuitive Idea That There Are Multiple Copies Of Our Own Selves, Which Exist As Mere Potential, And Are Thus Not Actual. There Are Many Possible Universes, But There Is Only One Mind, Which Determines Events On The Quantum Level, And Thus Creates Our Universe. As We Had Discussed Previously, The Quantum Level Provides The Essential Ground Of The Holographic Principle, Such That Quantum-Level Holographic Surfaces Are Elaborated At Higher Levels, Manifesting Higher Orders Of Information From The Quantum "World." In This Process, There Is A Reduction Of The Wave Function Or Potentialities Of Quantum Fields. At The Level Of Consciousness, This Entails Freedom To Choose Which Observations We Make (Stapp, 1997), Which Gives Us The Capacity To Think And To Make Decisions. These Capacities Are The Basis Of Individuality, Self-Determination, Judgment, And Values. The Progressive Evolution Of The Manifestation Of Mind Through Higher Orders Of Experience, Leading To Higher Orders Of Consciousness, Is Entailed By The Holographic Principle. Again, We Are Dealing Here With Levels Of Description, With The Multiverse Of All Potentials Fundamentally Supporting The Single Universe We Collectively Observe. The Multiverse Is The Wave Function Of The Universe (Penrose, 2004). The Recursive Integration Of Nested Hierarchies Of Holographic Surfaces Brings Out A Single Actuality In Consciousness From A Wave Function That Is Unconscious. Consciousness Thus Gives Us Information At A Level Of Experience That Is Causal. In This Sense, We Partake In The Creation Of The Universe. We Participate In Creation, And This Participation, When Fully Realized, Leads Us To Higher Levels Of Consciousness And Of Realization. As A System, The Biosphere That We Live In Has A Holographic Surface, Creating A Deep Sense Of Ecology As We Collectively Move Toward A Planetary Consciousness. It Is Only When The Universal Holographic Boundary Reaches The Information Storage And Processing Capacity Needed For The Requisite Biological And Biochemical Complexity That Consciousness Evolves In Living Things. This Evolution Is Natural And Spontaneous, Since Consciousness Gives Rise To What Is Actual, As Opposed To What Is Merely Potential. In This Sense, Consciousness Is Still In The Process Of Creating Our Universe, And Levels Of Higher Consciousness Will Continue To Evolve. We Are All In The Same "Now," And That Now Is Defined By The Present Universal Boundary. This Assures Us That Our Experience Is Universal, And Does Not Pass With Time. This Is The Fundamental Basis Of Memory And Of Cognition. The Identity Of Mind And Brain Is A Myth. We Have A Continual, Internal Or Non-Local Relation With The Universe, As It Has With Us. Once This Mystery Is Resolved, The Myth Is No Longer Needed, And There Is A Confluence Of Science And Spirituality.Experience Is Primary, Information Is Secondary. We Can Only Gather Information From Experience, Whether It Is In The Laboratory Or In Life. We Cannot Measure The Information On The Surfaces Of

Systems. The Physicists That Have Formulated The Holographic Principle For All Systems Are Quite Aware Of This, Or Else The Principle Would Have Been Established Or Discredited. We Cannot Measure What We Experience. It Is Intangible, Yet It Is All We Know To Be Actual. Everything Else Is Inferred. Because It Cannot Be Measured, It Has Been Fundamentally Disregarded By Mainstream Science. Materialism Is Considered Scientific, While Idealism Is Considered Unscientific. But Aren't Ideas, Fundamentally, Made Of Information? Brain Science Has Mistaken The Representation Of Information For Information Itself, And Has Tied Those Representations To Matter And Energy. Consciousness, The Highest Order Of Information, Has Generally Been Regarded As Superfluous, Something That Needs To Explained Away, Or Altogether Ignored. Yet It Is The Only "Thing" That Reaches Our Awareness. Consciousness Comes At A Price. For Everything That Becomes Conscious, There Must Be Something That Becomes Conscious. Consciousness Is Certainty, And Its Complement, Unconsciousness Uncertainty. Consciousness Is The Particle Nature Of Experience. It Has Definiteness About It. The Unconscious Is The Wave Nature Of Experience, It Is Like The Metaphysical Cloud Of Unknowing. If We Are The Most Conscious Of Animals, Then We Must Also Be The Most Unconscious. Perhaps This Is The Predicament Of Humankind. The Holographic Principle Of Mind Leads Us Naturally From The Most Fundamental Experiences, Existing As Quantum Potentials From The Conformations Of Proteins Down Through The Fields Of Electrons, Through Their Manifestation Upward Through A Recursion Or Successive Applications Of The Same Holographic Process, Through Higher Levels Of Experience, To The Emergence Of Consciousness As Higher And Higher Orders Of Experience. The Quantum Holographic Principle Mind Does Not Require Anything More Quantum Than Is Obviously Present At The Microscopic And Submicroscopic Levels, As It Represents Successive Orders Of Manifestations From These Levels. Recursion Also Applies Here In The Sense That It Is Used In Computer Science, In Which The Function Of The Part Depends On The Function Of The Whole. A Program, As A Part, Cannot Work Without A Functioning Whole, The Operating System Recursive Wholes Which Are, For Us, Supra-Conscious, Are On The Group, Species, Planetary, And Universal Levels. As Individuals, Our Consciousness Cannot Function Without Recursion To The Universal Consciousness, Even Though We May Be Unaware That Such Universal Consciousness Exists. Reaching Upward To These Supra-Conscious Levels Is A Spiritual Process, Making Transcendence The Ultimate Solution To The Unconscious Human Predicament. What Was Once Supra-Conscious Becomes Unconscious Through A Process Of Conditioning? We Are Born As Creatures Of The Earth And Of The Universe, As Evidenced By The Beliefs And Practices Of "Primitive Peoples." There Is Evidence From Cave Paintings That Our Hominid Ancestors Experienced A Kind Of Holographic Perception (Combs, 1996), Which Could Constitute Our Early Connection With The Holographic Subtext Of Reality, And Which We Might Then Propose Existed In Our Animal Ancestors, And In Extant Animal Species. If This Is The Case, Than Microgenesis Would Entail The Recapitulation Of This Holographic Experience As It Progresses Through Our Ancestral Past. The Supra-Conscious Mind Seems To Envelope Perinatal Experiences, And Stanislav Grof (1994) Has Developed Techniques To Gain Access To These Experiences, As Well As To The Earlier Experiences Of Our Human And Animal Ancestral Lineage, And Of Our Universal History. Grof (1994) Concludes: "Our Consciousness Seems To Have The Amazing Capacity To Directly Access The Earliest History Of The Universe Ñ Witnessing Dramatic Sequences Of The Big Bang, The Formation Of The Galaxies, The Birth Of The Solar System, And The Early Geophysical Processes On This Planet Billions Of Years Ago." The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science There Are Similarities Between The Patterns Of Holography And Of Psychological Transference, Where Holography Is The Process Of Recording And Reconstructing Holograms Employing A Theoretical Perspective Using A Hermeneutic Method, This Dissertation Parallels Holography With Transference, Offering Another Way To Encounter Transference By Showing Similarities Between The Processes Of Each And The Results Of Each. Though Complex, Infinitely Varying, And Unique, Their Patterns Are Clearly Identifiable. Thus They Are A Metaphorical Fit To The Concept Of Strange Attractors In Physics And A More Literal Fit To The Concept Of Archetypes In Depth Psychology Or Dynamic Psychology, Psychology Which Attends To The Living, Autonomous Unconscious. This Study Explains How Holography Models Transference, What A Hologram Is And How It Works, And How Depth Psychology

Understands Of The Interaction Between Consciousness And The Unconscious Is Related To The Hologram. It Describes Transference And Related Psychological Processes As Understood In Six Different Schools Of Depth Psychological Thought. It Shows That The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology, Processes Such As Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. By Identifying The Similar Patterns Of These Processes, This Study Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole: The Visual Image Of The Overall Hologram Is Present In Each Component Part Of The Hologram, The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. By Noting Differences As Well As Similarities In These Processes, It Suggests An Inventory Of The Qualities Of The Holographic Archetype. This Study Furthers Understanding Of The Pervasiveness, Force, And Autonomy Of The Unconscious Acting Through Transference And Projection By Identifying A Group Projection Of Domestic Violence Lying At The Core Of The Christian Myth. This Study Also Furthers Understanding Of The Concept Of Transference By Providing A Reflection Hologram Of The Human Psyche As An Artistic Work And As A Visual Metaphor Of Transference. **Strange Attractors: Transference, Holography, And An Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003).** A host of observed, but very basic human phenomena, including consciousness itself,have eluded rigorous scientific description by all disciplines of science. This is true, not because of insufficient evidence for a particular phenomenon's existence, but rather forlack of a theoretical construct, which could fit within the prevailing paradigms of science. For the past century eminent men and women of science have accumulated thousands of pages of data on mind/mind and mind/matter interactions. Many of the most telling experiments have been criticized, perfected and repeated numerous times during the past five decades, using increasingly sophisticated technologies. Meta analysis of these experiments produce accumulated probabilities against chance occurrences exceeding trillions to one (Radin, 1997). It has required, however, that quantum science mature for seventy-five years and during that period, test, validate and synthesize a number of seemingly outrageous physical concepts arising from quantum theory, before testable theories could arise which offer hope that anomalous mind and consciousness data can be explained (Mitchell, 1996).The missing concepts that prevented the earliest investigators of consciousness from succeeding in their quest were 1) a generalized theory of information, and 2) quantum science itself, with the associated phenomena of non-locality, the zero point energy field and the quantum hologram. These associated phenomena are still not well understood butare sufficiently validated today by both theory and experiment to provide a basis for postulating a necessary condition for the existence of consciousness phenomena, as experienced in the observable four dimensional space/time universes. A third concept, chaos theory, is also necessary to understand the nonlinear evolutionary processes that caused consciousness to evolve toward the anthropic consciousness experienced by humans. In particular, chaos theory maps far from equilibrium systems and demonstrates the irreversibility of nonlinear processes and thus the irreversibility of time in the macro-scale universe. Another class of phenomena, including normal sensory perception and evolution, to cite but two, have explanatory theories in classical science, but which in view of current developments in late quantum physics and in chaos theory may be incomplete approximations to the correct theory. Information concepts have been examined by Weiner, von Neuman and Shannon in well-known seminal works and by Frieden more recently (1998) to produce theories useful to physics, to computation and to communications technologies. These theories, although accurate and mathematically useful in their domains, fall short of being sufficiently encompassing when considering the problem of consciousness, its evolution and its associated phenomena. Even relatively simple perceptual organisms utilize patterns of energy, that is, information, not completely described by existing mathematical theories. Theory and experimental evidence for the zero point energy field has been published by many authors, but I shall cite Haisch, Rueda and Puthoff, (1997, 1998), as the most contemporary and relevant work for this paper. Theory and experimental evidence concerning the quantum hologram has been developed by Schempp (1992, 1993) andMarcer (1996, 1997, 1998), separately and jointly,

based upon a new understanding ofquantum mechanics. (See previous work by Cramer [1986], Berry [1988], Anandan [1992] and Resta [1997]).Non-locality, although predicted by the earliest work in quantum theory and decisively demonstrated by Aspect in 1982, has been thought to be a curious property of particle physics but of little relevance to macro-scale reality until discovery of the quantum hologram. Further, it is widely believed that non-local quantum information represented by entanglement of particles could not be recovered locally as useable information (Eberhard's theorem). However recent work both in theory and experiment (e.g. see Nature, 1997, 11th December, vol. 390, Sudbury T pp551-552, and Bouwmeester D. et alp 575-579) is in line with the work by Berry, Resta, Schempp and Marcer and makes it clear that this is not the general case for quantum information processing andcommunication.It has been widely accepted in science, until recently, particularly in the field of artificialintelligence, that the brain was likely a complex classical computer, incapable of supporting quantum processes. The work by Hammeroff (1994) and Penrose in isolating and describing microtubules in brain tissue have caused a re-examination of this dogma, and renewed interest in uncovering the quantum processes involved.Based upon this earlier work I postulate and examine the evidence in this paper for thefollowing theories: 1. The basis of subjective experience is rooted in the quantum attribute of naturecalled non-locality. I will use the word "perception" in its most generic sense to denote a basic subjective experience at all levels of complex matter. Thus the non-local quantum correlation between entangled quantum particles is considered the root cause of the phenomenon experienced as perception in more complex matter, but the non-local quantum hologram is the non-local carrier of information for molecular and larger scale matter. Thus, perception is not an object but rather the label for a nonlinear process involving an object, a percipient and information.2. The experience of humans is that they sometimes, perhaps often, perceive information from or about physical objects that is not available through normal, local, sensory mechanisms, nor classical space/time information. Objective testing data in overwhelming abundance provides evidence that this is true, though an explanatory mechanism has until contemporary times remained elusive. I shall call this intuitive information or intuitive perception. I postulate that a quantum hologram is the source of this intuitive perception and that the percipient is at that time in phase-conjugate-adaptive-resonance (pcar) with the entity or object associated with the quantum hologram.3. The phenomenon of "learning" in humans is a subjective process that involves perception, memory, intentionality, and evaluation of outcome and behavior change. This may be viewed as a classical nonlinear feedback loop. Although we cannot know precisely the subjective experience of another entity, presumably in the successful training of animals, an analogous subjective process is in effect. Sheldrake (1981) has published a successful theory of morphic resonance related to animal learning based upon non-local information. Marcer has published papers (1996, 1997) theorizing a mechanism by which the quantum hologram causes learning to take place in both DNA molecules and prokaryote cells as an adaptation process of environmental resonance, rather than mutation and adaptation solely by random processes. I postulate that Marcer's concept can be generalized to nature alarge and that the quantum hologram is the information structure suitable to explain Sheldrake's morphic resonance. The non-local quantum correlations observed in particles, and the non-local quantum hologram associated with molecular and larger scale objects, serve the purpose of providing information at all scale sizes to guide evolutionary processes. That is to say, that quantum non-locality is the basis of perception, and thus fundamental and necessary to the complex organizations of matter and information in the universe. Further, since learning is an observed property of complex systems such as animals and, via the quantum hologram, is theorized to be a property of simple cells and molecules, one can also postulate the generalization that nature evolves through a learning process rather than because of random mutations.4. Marcer (1997) has proposed that the condition of phase-conjugate-adaptive-resonance (pcar) is a necessary condition for an object in three-dimensional reality to be perceived as it really is. That is, resonance requires a virtual path mathematically equal but opposite to the incoming sensory information about the object. Further, that it is the incoming electromagnetic (space/time) information (visual, acoustic, etc), which decodes the information of the quantum hologram and establishes the condition of pcar so that accurate three-dimensional perception is possible. That is to say, both quantum information and space/time information aroused in the act of perception by organisms having sensory preceptors. I propose that the two equal but opposite paths required by the pcar condition are the mathematical

equivalent of perception and attention (or intention). (I shall distinguish between attention and intention in following pages.) Discussion The anecdotal evidence for humans perceiving non-local information dates to prehistory. The data were sufficiently robust that both experiences and philosophers, from Plato and Aristotle forward, accepted that both physical and non-physical realms of reality must exist. Non-physical was thought to explain the subtle, ephemeral and mystical subjective experiences ubiquitously reported in human culture. After Descartes and Newton, however, classical western science rapidly discarded the non-physical hypothesis and systematically began to ignore all evidence for perception of non-local information Field theories and point particles were created to preserve the concept of physical contact between particles and to explain obvious examples of "spooky action at a distance" such as gravitation and electromagnetic interactions. Information, broadly defined as patterns of energy, reemerges however, in non-local form in the mysterious quantum spin correlations of double slit experiments, although it has been widely believed that such non-local information could not be recovered and utilized by sensory systems. With validation of theory and experiments concerned with the non-local quantum hologram, information, including non-local information, suddenly acquires a more important status in physical theory, a status as important as energy itself. This is true because information is the basis of the cognition and knowing by which creatures perceive reality, and non-local information can now be seen as a ubiquitous and useful property of the cosmos, rather than a unique attribute of particles (and human animals). It is likely that most, if not all, subtle, ephemeral and unexplained phenomena associated with subjective experience are connected, directly or indirectly, with the phenomenon of non-locality. The brain is clearly a quantum computer (Schempp & Marcer, 1996) which utilizes both quantum and space/time information. This discovery alone almost certainly sets a necessary, but not sufficient condition, for intelligent life to have arisen in the cosmos, wherever environmental conditions permit. Many volumes have been written in this century by scientists experimenting with remote viewing, ESP, telepathy, clairvoyance, precognition, etc. Police agencies routinely use “psychics" to assist in criminal cases often with success. Intelligence agencies of governments have clandestinely utilized the findings to successfully gain information about an adversary. Many reports of these activities have been recently declassified and printed in open professional journals, even though no explanatory physical mechanism has yet been reported which is acceptable to mainstream science. The most succinct modern summary of this activity and analysis of results have been published by Radin (1997). Quantum Holography Non-locality and the non-local quantum hologram provide the only testable mechanism discovered to date which offer a possible solution to the host of enigmatic observations and data associated with consciousness and such consciousness phenomena. Schempp (1992) has successfully validated the concept of recovery and utilization of non-local quantum information in the case of functional Magnetic Resonance Imaging (fMRI) using quantum holography. Marcer (1995) has made compelling arguments that a number of other chemical and electromagnetic processes in common use have a deeper quantum explanation that is not revealed by the classical interpretation of these processes. Hammeroff (1994) and Penrose have presented experimental data on microtubules in the brain supporting quantum processes. The absorption/re-emission phenomena associated with all matter is well recognized. That such re-emissions are sufficiently coherent to be considered a source of information about the object is due to the theoretical and experimental work of Schempp and Marcer, based upon the transactional interpretation of quantum mechanics of Cramer (1986), the Berry geometric phase analysis of information (Berry, 1988; Anandan, 1992) and the ability of quantum phase information to be recovered and utilized (Resta, 1997). The mathematical formalism appropriate to these analyses is consistent with standard quantum mechanical formalism and is defined by means of the harmonic analysis on the Heisenberg nilpotent Lie group G, algebra g and nilmanifold (see Schempp (1986) for a full mathematical treatment). The information carried by a quantum hologram encodes the complete event history of the object with respect to its three dimensional environment. It evolves overtime to provide an encoded non-local record of the "experience" of the object in the four-dimensional space/time of the object as to its journey in space/time and the quantum states visited. The question of the brain’s ability, as a massively parallel quantum processor, to decode this information is addressed by Marcer and Schempp in "Model of the Neuron Working by Quantum Holography" (1997) and "The Brain as a Conscious System" (1998). They argue that an organism’s ability to perceive objects as they are and where they actually are in three-

dimensional reality requires the phase conjugate relationship provided by quantum holography. It is not sufficient for the incoming electromagnetic illumination (or acoustic signal) carrying object information to present tothe brain a wave front in the manner presented to a flat photographic plate. Rather, a virtual signal as mapped by the phase conjugation of quantum holographic formalism is required to decode the information in order for perception and cognition to exist as we experience it in three dimensional realities. The percipient and the source of information are in a resonant relationship for the information to be accurately perceived. Many investigators have proposed a holography mechanism as a basis for brain functioning, beginning with Pribram, and indeed, others have proposed holography as a construct forth universe itself, but discovery of the non-local quantum hologram created by the absorption/remission phenomenon and characteristic of all physical objects provides the first quantum physical mechanism compatible with macro-scale three dimensional worlds we experience it. The existence of a quantum hologram associated with each physical object provides each physical object with the non-local waveform predicted by quantum theory's wave/particle duality and extends quantum theory to all physical matter. It allows, for the first time, possible approach for understanding the mysterious world of consciousness. Postulating that this is globally true, we inhabit a quantum world where non-local effects should be expected at all levels of functioning, not just as a curious artifact of the subatomic level of reality. Existence of the non-local quantum hologram suggests that nature has utilized non-local information from the big bang forward, throughout its evolutionary history; and long before planetary environments self organized to permit living matter and complex space/time sensory systems to evolve. The papers of Marcer and Schempp on learning inherent in DNA and prokaryote cells using quantum holography, when generalized, helps explain the ubiquitous appearance in nature across distances, scale sizes and species, of similar processes, organs and sensory systems. This certainly conforms to the fractal geometry of chaos theory. Certainly the similarities, of DNA, cell structure, organs and brains across species are easier to reconcile with a non-local learning process than with a theory of localized random mutation and natural adaptation. It is important to observe that in standard particle physics experiments the object is to discover the quantum characteristics of the individual types of particles, and the conditions under which they split and recombine. In quantum holography the object is to treat the entire group of re-emitted quanta as a whole, and as in laser holography, to examine the information carried in the interference pattern and phase relationships. These represent two quite different levels of approach to quantum information. In particle experiments, it is considered that the eigenvalues of the applicable matrix represent measurable values; and that information is lost during measurement due to decoherence of the particles and energy exchange. But in the quantum holographic formalism, the information is carried in the phase relationships, which are represented by off-diagonal terms in the matrix, and the information is recoverable under the proper conditions as Berry and Resta have predicted, and as Schempp has demonstrated with fMRI. The quantum mathematics is consistent with standard quantum theory in both cases. In decoding the quantum holographic information, however, the energy exchange is insignificant. The similarity of the mathematical treatment in these various experiments is important tothe thesis of this paper. In examining the quantum non-locality of particles it is spin numbers and/or polarization that are the parameters of interest. A standard technique of analysis is to use the Fourier transform to map the state of the particles into the frequency domain. In the formalism of the quantum hologram, mapping into the frequency domain is also fundamental; however, the requirement for pcar assures that the phase relationships are matched so that the percipient (sensory system) is able to decode the information carried in the phase relationships. It is precisely the pcar requirement that permits the encoded holographic information to be decoded by the percipient. Mathematically, decoding is simply reversing the rotation of the phase vector in phase space. Physically, it is matching the frequencies and phase of the information such that resonance results. Frequency, phase matching and resonance are an operational characteristic of every type receiver technology. Pribram's earliest proposal that the brain stored information encoded as in a hologram and mapped by the Fourier transform is in complete agreement with the evidence presented by quantum holographic mathematical formalism. It is the spin and polarization attributes of particles (both are mapped by wave mathematics) that represent the puzzling non-local property of subatomic matter. It is the phase relationships that carry the information in holography (again mapped by waveform mathematics). And it appears that the brain stores and manages

information not as a classical digital machine, but rather as an analog device using non-local properties of thequantum hologram, which can be analyzed by wave form mathematics (harmonic analysis on the Heisenberg Lie group). In the cosmological evolutionary scheme of things this similarity of appropriate mapping techniques is too bizarre a coincidence to be ignored Asa cosmic accident. Thus there is ample evidence that the **non-local attribute of** nature is much more than just a curious artifact of subatomic particle interactions, but rather is a more fundamental phenomenon that appears at all scale sizes and is, in particular, associated with the utilization of information in nature, and associated with the fact that information has a causal effect independent of distance. It is precisely information, however, that is the basis of the phenomena of perception, cognition, memory, learning etc, that is to say, consciousness and the subjective experience. Though the evidence is quite ample to postulate that non-locality is the unique, universal basis for perception and the subjective experience, the evidence though compelling is not sufficient to be conclusive that such is indeed the case. The next steps are to validate more completely with experimental evidence that non-locality plays a major role at all scale sizes and that all physical objects are quantum objects and thus interconnected by information in this strange way. **Non-Locality in Nature**: There is experimental evidence to strongly suggest that simple organisms perceive and respond to information non-locally as well. Cleve Backster was perhaps the earliest to experiment with plants and simple life forms in electromagnetic isolation in late 1960'sand early 1970's. His work was not confirmed through replication by others at that time. Other investigators have had mixed results replicating non-local information perception by simple organisms and living tissues. In the area of human experimentation, results likewise have been mixed and controversial for three-quarters of a century. However Meta analysis by Radin (1997) and independently by Utts (1991) across a large and appropriate spectrum of experiments demonstrates compelling statistics that the **perception of non-local information exists and is real.** Perhaps was there a larger body of experimental evidence for simple life forms, similar Meta patterns would emerge. Failure to replicate results in well constructed experiments does not, in the case of subtle consciousness phenomena, prove that the phenomenon is missing but rather that a **hidden mechanism** below the threshold of classical measurement is operating. For example, the most telling experimental evidence to explain the sometimes inconsistent results relates to direct non-local observer and/or experimenter effects. Gertrude Schmiedler isolated the **"sheep/goat "**effect in human experimentation decades ago (1972). Experimenters and/or participants in a human telepathy (or similar non-local) experiments exhibited results statistically above or below chance results depending upon their subjective bias toward the experiment. (In other words, 100% wrong answers would be as statistically significant as 100% correct answers in such tests, and in addition betrays the mind set or intention of the subject; whereas only chance results would be inconclusive.) More recently, a series of experiments by Marilyn Schlitz (1997) investigating "intentionality" clearly demonstrated that experimenter bias (intentionality) affected the outcome even of double blind experiments. Thus, in the subtle realms of mind and consciousness studies, bias, belief and intention clearly have an effect. The lack of an existing theoretical structure in classical science to support any type perception of non-local information, much less to support bias, belief or intention as having a non-local effect, when in fact it does have a non-local effect, is quite sufficient to account for anomalous results in many scientific experiments. Further validation and acceptance of the non-local thesis will have strong positive repercussions for the prevailing scientific paradigm and particularly the theory of measurement. **Attention and Intention:** A powerful and telling series of experiments conducted by Dean Radin (1997) at University of Nevada at Las Vegas following a decade long set of equally significant experiments by Brenda Dunne and Robert Jahn at Princeton University (1988) provide insight as to the subtleties involved in this level of mind/brain functioning. Jahn and Dunne provided overwhelming evidence that subjects could intentionally produce statistically skewed results in mechanical processes normally thought to be driven by random processes. Radin went further; he discovered that audiences watching a stage performance would skew the output of nearby random number generators during periods of high emotional content in the stage performance. Further, in a wide-ranging audience participation experiment, he recorded the output of computer random number generators during the television broadcasts of the O.J. Simpson murder trial. Most television media reported this event for weeks on end and tens of millions of humans were watching the results. Again, the results of the random number generators were skewed corresponding to emotional

peaks during the trial drama and corresponding to the number of people watching television. The thesis in the Princeton experiments was that participant intentionality created a non-random effect to bias the skewed distribution. In the Radin experiments the results were not intentional, as the participants were unaware of the experiment, but the hypothesis was that attention (in particular, rapt attention) drove the system away from chaos (randomness) and toward greater order (reduced entropy). These results suggest that attention and intention provide closely correlated outcomes, further, that randomness may not be a general property of nature, but that what is perceived as random noise in a system may be information (a pattern of energy) that is not in resonance at that moment with the particular perceptual system. William Tiller, emeritus professor at Stanford also has performed experiments (1997) that are consistent with these results, though his interpretation of the operating mechanism is somewhat different. These different types of mind/mind, mind/matter experiments have been rigorously and routinely conducted for decades with statistically compelling results but just as routinely dismissed or ignored by main stream science because the implications of non-local action are so foreign to the classical paradigm. However, if we consider that the condition of phase-conjugate-adaptive-resonance is necessary to completely specify the act of perception as described in the mathematical formalism of the non-local quantum hologram by Marcer, then we may also consider the perceived object and the percipient's perceptual system as locked in a resonant feedback loop. The incoming wave front carrying information may be labeled as "perception" from the point of view of the percipient, and the return path required by the resonant relationship may be labeled "attention" (or for subsequent discussion, "intention"). It is a well established principle in the meditative practices of esoteric disciplines that prolonged focused attention on a object of meditation causes the percipient and the object to appear to merge so that a deeper level of information about the object is obtained; information such as history or internal functioning, that would not be available through classical space/time information. The concept of the quantum hologram adequately and completely describes how this phenomenon might take place. Further, it is accepted that the mind/brain is a massively parallel processor, capable of performing many tasks simultaneously and subconsciously (in the right, intuitive part of the brain). Attention (meaning conscious, focused attention) is a unique and singular task that must take place sequentially, mostly in the left cognitive part of the brain. The condition of attention deficit disorder (ADD) is precisely the problem of a percipient being unable to maintain a singular focus for sufficient time to complete a desired task or observation. Thus, the action of focusing attention by apercipient may be construed as a necessary condition for pcar to be established with the perceived object. Non-Locality, Near and Far Marcer has presented the case for the pcar requirement in normal sensory perception (visual and acoustic). A frequent modality used by psychic sensitive individuals to gain information is to physically touch an object. Touching an object satisfies the pcar requirement and presumably allows the percipient access to information about the object not available from space/time information. Police agencies frequently use this modality with psychic sensitives to gain information about a crime scene, much as they utilize bloodhound to track the scent of an individual, often with considerable success. If, as in the theory of the quantum hologram, the object has been in the presence of the individual about whom information is desired, the event history of the object and that of the individual intersect. The Berry phase information of the object contains its journey in three dimensional space and time, as well as the quantum states through which it has passed on this journey. The sensitive individual, with a honed talent, seems often able to decode useful Berry phase information from the object about the individual sought. It may also be the case with the blood hound, which additional non-local information has been gained about the subject, even though the classical explanation is that the animal is operating only with heightened olfactory sensing. Although perception in the three dimensional world requires and utilizes pcar, most humans, however, do not bring to conscious awareness non-local information when weare routinely operating in three-dimensional reality. We perceive objects as presented by space/time information, that is, shape, color, function (tree, chair, table, etc) but are not usually aware of the additional non-local information. It takes training as provided by many of the esoteric traditions and/or certain naturally sensitive individuals to routinely perceive the non-local holographic information associated with a particular object. There is massive evidence to suggest, however, that the brain has these latter capabilities at birth. Suppression by cultural conditioning in childhood and subsequent lack of practice cause the natural ability for

conscious, intuitive perceptions to atrophy. Particularly in western tradition, educational interest has been on the left brain, rational functions rather than right brain, intuitive functions. However, mystic adepts and natural psychics' routinely demonstrate that non-local information is perceptible from physical objects by focusing attention, quieting the left brain and allowing intuitive perceptions to appear. It is the left brain cognitive ability in humans that provides canonical labeling of the intuitive and artistic processes taking place in the right brain. The fact that with training and practice, individuals can recover, deepen and label their individual cognitive access to intuitive, on-local information demonstrates that learning is taking place within the whole brain itself and involves enhanced coherence and coordination between the hemispheres. This process is different and distinct from the left brain function of extending and extrapolating factual data and logical deduction to leap to an "intuitive" conclusion, while omitting the intermediate steps leading to that conclusion. Experimental protocols for remote viewing normally provide clues to the location of the object such as adescription, a picture, or location by latitude and longitude, that is to say, an icon representing the object. These clues seem to be sufficient for the percipient to establish a resonance with the object. Normal space/time information (visual, acoustic, tactile) about the object is not being directly perceived by the percipient, nor does the object usually appear at its physical location in space/time like a photograph or map in the mind. Rather, the information is perceived and presented as internal information and the percipient must associate the perceptions with his/her internal data base of experience in order to cognize and to describe the object's perceived attributes. In the case of complex objects being remotely viewed, the perceived information is seldom so unambiguous as to be instantly recognizable as correct. Sketches, metaphors and analogies are usually employed to cognize and communicate the non-local information. A considerable amount of training, teamwork and experience are necessary to reliably and correctly extract complex non-local information from a distant location. The information appears to the percipient as sketchy, often dream-like, and wispy, subtle impressions of the remote reality. Very skilled individuals may report the internal information as frequently vivid, clear and unambiguous. The remote viewing information, being strictly non-local, and in this hypothesis, the information perceived by quantumholography, is missing the normal space/time components of information necessary to completely specify the object. It has been demonstrated that this intuitive mode of perception can be trained in most individuals. Perhaps additional training and greater acceptance of this capability will allow percipients to develop greater detail, accuracy and reliability in their skill. In principle, training will not only increase the skill and accuracy but should cause the appropriate neural circuitry to become more robust as well. In the absence of space/time (electromagnetic) signals to establish the pcar condition and to provide a basis for decoding the quantum hologram, an icon representing an object seems to be sufficient to allow the brain to focus on the object and to establish the pcar condition. However, a reference signal is also required to provide decoding of the encoded holographic phase dependent information. Marcer (1998) has established, using Huygen's principle of waves and secondary sources, which any waves reverberating through theuniverse remain coherent with the waves at the source, and are thus sufficient to serve as the reference to decode the holographic information of any quantum hologram emanating from remote locations. The Zero Point Field: The results of the Michelson/Morley experiment banished the concept of aether from early twentieth century physics. However, it left a void as the nature of interstellar space and nothing for propagating waves to wave in. Quantum physics reincarnated the aether as the zero point fields, a seething cauldron of quantum potential and unmanifest energy where particles and antiparticles spontaneously arise and then disappear. The very fabricant structure of space/time itself is again in question; its structure and its metric under intense investigation with far more questions than answers having emerged to date. For the purposes of this paper the relevant issues are two: 1) the emission/absorption phenomenon, and 2) the structure and mechanics of non-locality. Zero point (zero degrees Kelvin) emission and absorption of quanta from all physical objects is a well established phenomenon. It is our view that the zero point fields are the plenum (or cauldron) which supports this absorption and re-emission, and makes the phenomenon of the quantum hologram possible at all temperatures. Although particle experiments are carried out under rigid conditions of temperature and pressure, Schempps experimental work with the fMRI requires no such constraints. There are deep and difficult questions yet to be answered about how the information of thequantum hologram maintains its integrity and is

propagated, about how resonance takes place at extremely large distances. There is considerable evidence that intuitively perceived information is truly non-local. It does not obey the inverse square law for space/time energy propagation; it is time independent and cannot be shielded by electromagnetic shielding. Such characteristics are the mark of non-locality. But understanding the mechanics of non-locality (or a visual picture) is missing from standard models. Some physicists turn to superluminal speed of propagation, others to the zero point as a zero dimension, which is resonant with all parts of the universe simultaneously. The issue of instantaneous communication (or at least superluminal communication) of non-local effects on a cosmic scale remains a problem, even though the phenomenon itself is well validated. Perhaps it is a problem of topology. What shape can the universe have such that one point can be in simultaneous contact with all other points? In this regard it is clear that certain problems between quantum mechanics, special and general relativity remain inexistence. Haisch, Puthoff and Rueda continue to investigate the **metrics of the zero point fields** with regard to better defining the unanswered questions about, mass, gravitation and inertia. Perhaps these investigations will also bring answers for how phase related information is propagated non-locally, likely within the zero point fields, and thereby unveil the mechanics of the resonance phenomenon. Further, new investigations reported by Van Flandern (1998) on measurements from orbiting Global Positioning System (GPS)clocks indicate the predictions from Lorentzian relativity to be approximately four times more accurate (0.7% opposed to 3%) than predictions of special relativity. If these measurements are further validated, it implies that Lorentzian relativity with a Hubble absolute rest reference frame, an aether (zero point field), and instantaneous propagation of non-local effects may be the preferred one. If this is the case, then many questions about non-locality would be resolved. Intentionality I have argued that by establishing pcar between a percipient and an object, the phase conjugate (equal but opposite) paths connecting the two can be labeled "perception" and "attention". In the case where the object is a simple physical object (rock, flower, etc.), our interest is on the non-local information perceived by the percipient about the object. However, from the point of view of the object, information about the percipient is also available to the object. The pcar condition is a reciprocal relationship, mathematically. Quantum holographic formalism predicts that the history of events of quantum objects is carried in the quantum hologram, thus we must conclude that the **"attention" focused upon the object causes that event to be recorded in that object's quantum hologram.** Although we cannot query the object about its experience perhaps an experiment such as one utilizing the Aharonov-Bohm effect would detect a phase shift in the object's holographic field. (In this discussion I use anthropic labeling as we are discussing human perception. The phenomena however, are rooted in natural (and primitive) non-local physical processes, which are fundamental. The evolved complexities of perception, cognition, etc, associated with a brain obviously, as yet, have no analogous label to describe the experience of simple objects.)Once the pcar condition is established, the percipient can change its mind state with regard to the object. The perceived information can be operated upon by the brain's function so that cognition occurs with respect to the perceived information and meaning assigned. Cognition and meaning require finding a relationship between the perceived informationand the information residing in the percipient's memory. The percipient can then form intent with respect to the object. In such case the path I have labeled "attention" could Figure 1 In self aware animals (those with a brain) cognition, meaning and intent with respect to an object can often be described in simple terms, for example: enemy, fight or flight; food, eat; friend, greet, etc. The non-local component of information, although present and creating effect, is operating below the level of conscious perception in humans and results in "instinctual" subconscious behaviors in all animals. The brain as a massive parallel computer is simultaneously performing numerous tasks to accomplish the desired intention. Classical modelings of this autonomous activity describe it only in terms of classical information and energy flow in the central nervous system and the brain. However, if non-locality is operating at all levels of activity, as this theory suggests, certainly there are resonances involving non-local information operating throughout the body of an organism in parallel with classical space/time functions. Subsequent experimental work will surely uncover these quantum processes where non-local resonance is involved in the functioning of an animal's internal processes. In the case of non-local effects at a distance, outside the body, simple spin correlations of entangled particles is the most basic. The spin coherence is reciprocal. Action on one particle creates an effect on other entangled particles. The non-local information is

causal of affects at large distances. It is no less important for macro-scale objects. Sheldrake (1995) proposed and others conducted experiments with dogs whereby the animals correctly anticipated their owners' departure from work to return home. He proposed other successful experiments where rats learning a new maze benefited non-locally from the experience of others that had previously learned the maze, in the total absence of classical space/time information. It is not surprising then, that human's exhibit an even wider range of reactions to non-local information. The evidence suggests that humans can perceive, cognize, and give meaning to non-local information across a range of complexity, from inanimate objects, simple organisms, animals and other humans. The existence of quantum holography provides an adequate informational structure to permit a theory for the observed results. The case is a classic case in phenomenology, where results are repeatedly observed over time that fall outside the prevailing paradigm, and must await new developments in science before an explanation is forthcoming. The results for intentional effects of non-locality should be no more difficult to accept than the results for perception. The pcar relationship implies symmetry, that is, information flows in both directions between object and percipient such that each is object and each is percipient. Only the complexity of the more ordered perceiving system suggests a non-symmetrical relationship. We humans have great difficulty in accepting that thoughts, specifically intentionality, can cause action at a distance. Yet, it has been observed for centuries and in recent decades subjected to scientific scrutiny. Were not prayer to have produced some positive results, religion would have been abandoned centuries ago. That cause was ascribed to supernatural agency rather than non-locality is simply, again, phenomenology needing to wait while science caught up. Modern studies by Dossey (1993), Byrd (1988) plus many others have attempted to document the efficacy of prayer, particularly healing prayer. The results in most cases are very suggestive of non-local effects, and some claim they establish the case for healing prayer. However, the difficulties of controlling all variables in such clinical studies leave many avenues for valid criticism. The fact that Radin's several studies (1997) demonstrated that attention alone produced non-local results in machines, i.e., reduced randomness (increases order) does confirm that information has non-local effect and may be correctly formulated asnegentropy. These results apply directly to healing prayer as well.The case for pcar conditions to create remote effects by transfer of non-local information between equally complex percipients, humans for example, is not difficult to understand. Indeed, hundreds of successful experiments establish the case. In these cases no energy transfer is required, only non-local information, as each percipient/object has access to its own energy source. The case for intentionally creating remote physical effects in inanimate objects is more puzzling. Even though teleportation of quantum states has been successfully accomplished for particles, and numerous studies (Radin, 1997; Dunne and Dunne, 1988) show that macro-scale objects can also be changed or moved, the energy transfer mechanism by which the classical states of a remote object are affected remains elusive. Conclusions The case for mind/mind and mind/matter interactions is impressively well documented over many decades as studies in phenomenology, with staggering probabilities against chance having produced the results. The discovery of the non-local quantum hologram, which is theoretically sound and experimentally validated in at least one application, thé fMRI, is sufficient to postulate that the quantum hologram is a solution to the foregoing enigma. Further, recognition that the quantum hologram is a macro-scale, non-local, information structure described by the standard formalism of quantum mechanics extends quantum mechanics to all physical objects including DNA molecules, organic cells, organs, brains and bodies. The discovery of a solution which seems to resolve so many phenomena, and also that points to the fact that in many instances classical theory is incomplete without including the subtle non-local components involved, suggests a major paradigm change must be forthcoming. The papers already published by Marcer and Schempp proposing a learning model both for DNA and prokaryote cells, which uses quantum holography, suggests that evolution in general is driven by a learning feedback loop with the environment, rather than by random mutations. This solution to biological evolution was proposed by Lamarck in 1809 but discarded for the mechanistic solution of random mutations by the colleagues of Darwin. The fact that non-local correlations and non-local quantum information can now be seen as ubiquitous in nature leads to the conclusions that the **quantum hologram can properly be labeled as "nature's mind"** and that the intuitive function we label in humans as the "sixth sense" should properly be called the "first sense". The perception of non-local information certainly preceded

and helped to shape, through learning feedback, the sensory systems that evolved in planetary environments, and which we currently label as the five normal senses. We must conclude that evolved, complex organisms, which can form intent, can produce and often **do produce non-local causal effects associated with that intent.** Further, that attention alone produces coherence in nature that in some measure reduces randomness. Finally, I conclude that the cited experiments and current understanding of non-locality in nature is sufficient to postulate that **non-locality is the antecedent attribute of energy and matter which permits perception and is the root of the consciousness which manifests in the evolved organisms existing in three dimensional realities**. ReferencesAnandan, J. "The Geometric Phase"; Nature, 360,26, 307-313 (1992)Berry, M. V. "The Geometric Phase" Scientific American, December, 26–32 (1988)Byrd, R. C. 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National Institute for Discovery Science: Nature's Mind: the Quantum Hologram by Edgar Mitchell, Ph.D. National Institute for Discovery Science Home > Resources > Consciousness Studies Nature's Mind: the Quantum Hologram Edgar Mitchell, Sc.D. Institute of Noetic Sciences, Sausalito, Calif. Fax: 561-641-5242, edgarmitchell@msn.comIntroduction** My Answer To Quora Question "If Consciousness Has No Evolutionary

Advantage, Doesn't That Imply That It Is An Emergent Property? " While Human Consciousness Has Certainly Been Shaped By Evolution, Which Does Not Mean That Consciousness Itself Could Have Evolved From Non-Consciousness. Whether We Are Talking About Other Species Of Animals Or Cells Or Organic Molecules, The Same Issues Which We Run Into In Explaining Human Consciousness Are Still Present At Any Scale. The Issues Of The Hard Problem Of Consciousness, Explanatory Gap, Binding Problem, And Symbol Grounding Problem Make The Mind-Body Split Just As Relevant Wither The 'Body' Is A Brain, Neuron, Or Subatomic Particle. No Matter What, You Have To Explain How An 'Interior World' Can 'Exist' In A Physical Structure Whose Behavior Is Causally Closed. Whatever Way You Slice It, If We Accept That T-Cells Can Be Effective In Detecting And Neutralizing Threats On A Cellular Level Without Having Consciousness, Or That DNA Can Create Cellular Machines Which Build A Brain Without Consciousness, And Then We Are Admitting That Consciousness Doesn't Make Sense As A Functional Adaptation. The Rest Of The Universe Already Works Too Well Without It. There Is Nothing Especially Interesting about A Hominid's Need for Food and Shelter Which Would Demand Rich Awareness to Develop out Of Blind Reflex. Single Celled Organisms Chase Food, Avoid Danger, Etc Also. We Are Then Left With Considering That Either Consciousness Could Somehow Be An Accident Of Evolution, Or That Consciousness May Be Intrinsic To All Physical Phenomena In Some Sense (Panpsychism, Panexperientialism) Or Even That Consciousness Is The Universal Substrate Upon Which All Phenomena Depends (Idealism, Idealist Monism). If Consciousness Is A Mutation That Has No Functional Role (A Spandrel), We Have To Ask Why It Would Even Be A Possibility. Remember That If Consciousness Is A Mutation, We Are Assuming That There Is A Whole Universe Already In Place Which Is Overflowing With Processes, Biological And Otherwise, Which Are Perfectly Capable Of Directing Themselves Effectively While Being Unconscious. It's Actually A Radically Anthropocentric Cosmology Since We Are Privileging Our Tiny Piece Of History In The Universe As The Only Piece Which Is Not Devoid Of Experience. We Are Saying That Everything That Existed Before Humans Was Unconscious, Therefore An Invisible, Intangible, Silent Void With No Memory Etc. If We Are Not Intending That, And Prefer To Think That The Universe Looked, Sounded, Felt, And Tasted Just Like It Does For Homo Sapiens Since The Dawn Of Time, Then We Would Have To Ask Exactly What We Think Consciousness Is Adding To That Kind Of Eternal-Universal 'Unconsciousness'. If Consciousness Is Intrinsic to Physical Phenomena (As in Penrose-Hameroff's Microtubule-Based Quantum Consciousness) or Is Intrinsic to Information Integration (As in Tononi-Koch's IIT), We Still Have the Same Kind Of Mind-Body Problem. A 'Body' Which Is A Statistical Function Rather Than A Literal Form In Space Is Still Falls Short Of Explaining Why And How There Is Any Such Thing As Consciousness. In My View, Only The Idealist Monist View Or What I Call Pansensitivity Makes Sense Ultimately As The Parent Of Both Physics And Information. Just As We Learn To Count On Our Fingers, All Forms Of Information Are Representations Of Experiences Which Have An Aesthetic Foundation – A Seeing, Feeling, Touching, Thinking, Etc. Without That Sensory-Motive Context From The Start, There Would Nothing To Evolve; Only Abstractions In The Dark (Or Not Even Dark). Once We Can Get Over Ourselves As A Species And Recognize That Consciousness Doesn't Begin And End With Us, I Think That Awareness Will Be Seen As The Container Of Relativity Itself, With Quantum Mechanics And Evolutionary Biology As A Consequence Of Deeper Stories Rather Than Their Originator. Multisense Realism A Cosmology Of Sense, Essence & Existence For Models See One Of The Papers In The Series Procrastinated Due To Constraints On Space. Events Occur In Time, But There Is No Time Except That Which Has Been Produced By The Big Bang. It Makes No Difference Whether The Big Bang Is The Only Beginning Of The Only Universe Or Just The Beginning Of One Of Many Universes, Either Way It Doesn't Explain The Beginning Of Existence. Some Say That Everything Following The Big Bang Is Random, Not Designed, But Why Is 'Designed' Even An Option? If That Word Means Anything, And Anything Has Ever Been Designed By Anything, Then What Difference Does It Make Whether That Capacity Of Intention Came Early On In The Universe Or More Recently? How Do We Know The Difference Between What We Design And What Is Random? How Do We Know That Randomness Even Exists? We Can't Generate True Randomness Computationally, We Can Only Grab Onto Some Pattern That Seems Nearly Random By Some Arbitrary Duration Of Measurement And Say That It Is Good Enough. Random Is A Concept, and the Difference between Random and

Intentional May Ultimately A Matter of Perspective. Some Interesting Possibilities Arise When We Consider The Observation That The More Something Seems Intentional, The More It Is 'Like Me' And The More Unlike Me Something Appears, The More It Seems Mechanistic. In A Way That Is Similar To How Any Object Appears As A Dot Or Smudge If It Is Too Small Or Distant For Us To See, The Notion That Perceptual Relativity Dictates The Quality Of Intention And Unintention Is A Provocative Hypothesis Of MSR. Random Events Cannot Follow A Series Nor Have Results. That Doesn't Mean That There Has To Be An Entity Making Everything Happen, But It Suggests That The Universe Is A Phenomenon Of Appearance, And Part Of Appearance Is Oscillation Between Intentional And Unintentional Attributes. The Universe Speaks In Both Entropy And Significance. Randomness Doesn't Do Anything, Can't Be Anywhere Or Feel Anything. Randomness Is An Abstraction Derived From An Expectation Of The Contrary. By Random, All We Mean Is That It Lacks Pattern And Intent – Both Of Which Must Be Implicitly Present Before They Can Be Hypothetically Absent. There Is Sense And There Is Non-Sense, Order And Dis-Order, It Is Not Non-Randomness And Dis-Chaos. Many People Believe That Physical Law Excludes The Possibility Of Free Will Because Of Strong Causal Closure. By This, What Is Meant Is That There Is No Room In Physics For Any Force That Causes A Physical Effect That Has Not Been Accounted For, Whether Or Not This Is Actually True Or Not Is Questionable In The First Place. The Sudden Addition Of Dark Matter And Dark Energy In The 1990s, Which Together Must Account For 95.1% Of Mass-Energy Of The Universe, Doesn't Seem To Have Encountered Nearly As Much Resistance From The Scientific Community Than Conscious Intention Has. Even If It Were The Case, However, And The Typing And Reading Of These Words Will Be Someday Explained By Purely Neurochemical Mechanisms, The Question Of Why The Feeling Of Intention Exists, And How It Can Be Produced Will Remain Unanswered. My Free Will Demands That Your Free Will Admit That There Is No Free Will. In All Of The Contemporary Debates On Free Will There Seems To Be A Blind Spot, In Which Free Will Does Not Exist, Except Where It Concerns The Application Of The Results Of Scientific Experiments. There Is Always A Call For The Educated And Enlightened To Voluntarily Change Their Own Minds, To Be Persuaded By The Argument Of Their Own Free Will. The Hypocrisy Is Hard To Overlook Once You See It. It Seems That The Anti-Free Will Assertion Makes An Exception For People Who Are 'Right'. Being Right Seems To Give Us A Right To Have Opinions Which Others Cannot, Due To Their Enslavement To Physical Causes. An Issue Which Often Comes Up In Free Will Debates Is How Our Stance On Free Will Impacts Criminal Prosecution. Again, The Blind Spot Of The Anti-Free Will Philosopher Projects That While Criminals May Not Be Held Liable For Their Actions Because Their Neurology Is To Blame, Then We Can't Allow Society To Take Credit Or Blame For Its System Of Justice Either. If Guilt Or Innocence Is Irrational, Then Believing That We Can Modify Our Own Attitudes Toward Guilt And Innocence Must Also Be Irrational. What Seems More Irrational Is To Codify Law Into A Mechanistic Formula Which Diminishes Our Capacity For Thought And Feeling In Consideration Of The Fate Of Others (Or Others Considering Our Fate).Zero-Tolerance Determinism: Another Pillar Of The Anti-Free Will Position Is That It Must Be An All-Or-Nothing Phenomenon. This Seems To Be More Of A Hasty Generalization Than An Honest Look At The Phenomena. In A More Tolerant Analysis, It Should Be Clear That Free Will Doesn't Have To Be Absolutely Free. The Fact That Our Conscious Mind Thinks That It Was Presented With Certain Options Still Requires That We Choose Freely From Among Those Options. It's True That We Are Always, In a Sense, Only Able to Choose What Our Mind Thinks Is the 'Best' Option (Even If That Choice Is Based on Bad Advice from Our Stomach or Emotions), But Still There Is an Intentional Participation Which Cannot Be Explained by Statistical Functions. No Matter How Constrained We Are By The Rules Of The Road, Our Car, The Limits Of Our Driving Skill, Etc, There Is Still A Difference Between Driving And Being Asleep At The Wheel, Or Between Driving And Being Forced To Drive Somewhere At Gunpoint. We Should Not Look Only At How Our Freedom Seems To Vanish Upon Thorough Inspection; We Must Also Look At How It Appears In The First Place – Unbidden And Self-Evident. Look At The Universal Appeal Of Freedom And The Universal Disparagement Of That Which Is Unfeeling, Robotic, And Mechanistic. One Thing That Is Seldom Mentioned In These Endless Debates Is Creativity. Were The Egyptians Destined To Build Pyramids And Not Megalithic Circles? Was It Inevitable That The Star Spangled Banner Was Written In Reference To The United States? Did Deterministic Processes Have No Choice But To

Create The Illusion Of Free Will? Why? The Work of Benjamin Libet Is Frequently (Compulsively?) Cited As Well, Work Which Even Libet He Later Made Clear Did Not Show That Free Will Didn't Exist. The Fact That A Neurological Signal Can Be Detected Before The Various Parts Of Us (The Self Who Makes The Decision, The Self Who Knows They Make The Decision, The Self Who Reports That They Make The Decision) Can Arrive At A Consensus Does Not Mean That The Initial Impulse Doesn't Correspond To Conscious Experience. There Is Also The Matter Of Focusing On Repetitive, Reflex Actions Which Minimize Free Will And Maximize Predictive Expectation. These Kinds Of Experiments Are Like Proving That Chefs Lack Imagination By Studying The Fry Cooks At Mcdonalds. Since Physics And Neuroscience Has No Theory At All As To The Origin Or Utility Of Consciousness, We Cannot Give Inanimate Instruments The Benefit Of The Doubt When It Comes To Our Subjectivity. Because We Are A Single Zygote Which Has Reproduced Itself, In Some Sense, Every Cell In Our Body Is 'Us' Just As Much As Any Organ Or Process In Our Body. We Are Complex, But Not In The Way That A Machine Is Complex. We Are Not Assembled From Specialized Parts; We Are A Single Whole, Divided Into Relative Specialization. Just Because Every Part Of Us Doesn't Know What Every Other Part Of Us Is Doing At All Times, Doesn't Mean That It Isn't All 'Us' Doing It Pansensitive Relativity: If The Perceptual Relativity Hypothesis Is Correct, And Pansensitivity Is The Engine Of Both Physics And Subjectivity, Then Cause Itself Is Relativistic. Intention Is Real And Primitively Creative In The First Person, And Unreal/Recombinant-Derivative In The Third Person…But Third Person Is Only Real Because There Is A First Person Participant Present. **Free Will, Determinism, And the Big Bang: Multi Realism Blog: Wikipedia http://multisenserealism.com/2012/08/27/deleuzes-the-logic-of-sense-part-i/.** From A Whiteheadian Perspective, However, There Are At Least Two Key Sources Of Misunderstanding Latent In Some Of These "Returns" To "The Body", "The Object" And "Materiality". There Is A Rather Too Hasty Dismissal Of The Concept Of Subjectivity As Such And There Is A Related Tendency To "Flatten Out" Any Would-Be Distinctions Between Human And Non-Human Entities. Such Positions Thus Risk A Return To A Bleak Anti-Subjectivism That Mocks Those Who Might Cling To The Idea That "Humans Are Very Different From Knives Or Paper" (Harman, 2002, Cited In Thrift, 2008). With Respect To The Latter, For Example, Nigel Thrift Invokes The ANT Principle Of The Democracy Of Things And States That In His Theory "Things [And Human Beings] Are Given Equal Weight, And I Do Mean Equal" (Thrift, 2008, P. 9). Much Of This Is Done In The Name Of A Whiteheadian Inspiration Since The Book Begins With Strong References To Whitehead, Including Key Concepts Such As The "Actual Occasion". The Point Concerning The Dismissal Of Subjectivity Can Be Brought Out Most Starkly By Contrasting One Of Thrift's Statements About His Non-Representational Theory With A Statement From Whitehead Himself. "Thus Things", Writes Thrift, "Are Not Just Bound By Their Brute Efficacy To The Visible Termini Of Humans In Some Form Of Latent Subjectivism Such As 'Concern' Or 'Care'". Now, It Is Certainly True That For Whitehead A "Thing" Is, By Definition, "Describable Without Reference To Its Entertainment" In An Occasion Of Experience (Whitehead, 1933/1935, P. 226). In This Sense, Things Are Not "Bound By Their Brute Efficacy To The Visible Termini Of Humans". But It Is Certainly Problematic To Imply That His Work Is Based Upon A Move Away From The Subject–Object Relation As The Fundamental Structure Of Experience And Hence From Concepts Such As Concern And Care. On The Contrary, On This Matter Whitehead's Thinking Is Quite Comparable To That Of His Contemporary Martin Heidegger In Being Grounded In The Concept Of Concern. The Centrality Of This Concept Is Due To The Way In Which It Brings Together Subject And Object As Relative Terms In The Unity Of What He Calls An Actual Occasion Of Experience: Thus The Quaker Word "Concern", Divested Of Any Suggestion Of Knowledge, Is More Fitted To Suggest This Fundamental Structure. The Occasion As Subject Has a "Concern" For the Object. And The "Concern" At Once Places The Object As A Component In The Experience Of The Subject, With An Affective Tone Drawn From This Object And Directed Towards It. With This Interpretation, The Subject–Object Relation Is The Fundamental Structure Of Experience. (Whitehead, 1933/1935, P. 226) Granted, Thrift Does State That He Wishes To "Temper… The More Extreme Manifestations Of This Lineage, Which Can End Up By Positing A Continuity Of And To Experience About Which I Am Sceptical" (2008, P. 6) However, One Wonders What Is Left Of Whiteheadian Cosmology If This Radical Extension Of The Concept Of Experience And Hence Of

Empiricism Is Omitted. It Is A Little Like Being A Marxist Without The Dialectical Materialism. It Is Also Difficult Not To Be Sceptical About The Remaining Self-Consciously "Inhuman" Theoretical Framework "In Which Individuals Are Generally Understood As Effects Of The Events To Which Their Body Parts (Broadly Understood) Respond And In Which They Participate" (Thrift, 2008, P. 60). In Recent "Radical" Social Theory, It Seems The Baby Of Subjectivity Is At Risk Of Being Thrown Out With The Bathwater Of Representationalism, Leaving Only The Hollow Remainder Of A Reactive Ensemble Of "Body Parts". Some Clarity Is Therefore Needed If We Are Not Simply Going To Recruit Whitehead Into Concerns Alien To His Own. His Chief Problem Was Not the Notion of the Subject/Object Structure of Experience Itself, But It's Too Rapid Identification with the Difference between Knower and Known (Whitehead, 1933/1935, P. 225). It Is This Conflation Of The Deeper Subject/Object Relation With The More Superficial Distinction Between Knower And Known That Gives Rise To A "Representational" Style Of Thinking And Its Interminable Debates. I Will Argue That It Is Precisely By Way Of The Subject/Object Relation Of The Actual Occasion That We Are Best Positioned To Conceive Of "The Cumulation Of The Universe And Not A Stage-Play About It" (Whitehead, 1927–1928/1985, P. 237). **Palgrave Communications | Palgrave Macmillan Journals Journal Home > Archive > Original Articles > Full Text Original Article Subjectivity (2008) 22, 90–109 Doi:10.1057/Sub.2008.4 A.N. Whitehead And Subjectivity Paul Stenner1** 1University Of Brighton, Brighton, UK Correspondence: Paul Stenner, School Of Applied Social Science, University Of Brighton, Falmer Campus, Mayfield House, Brighton BN9 1PH, UK. E-Mail: P.Stenner@Brighton.Ac.Uk (For Models Please See One Of The Papers In The Series Attributed To Spatial Restraints It Is Accommodated Where It Could Be). In Sum, Whitehead Offers A Relational Process Ontology That Promises To Deepen The Constructivist Insights Associated With The Turn To Textuality, But Without Reducing The Universe To "Discourse" And "Materiality". In This Ontology, Things (Whether Occasions Or Assemblages) Are Definable As Their Relevance To Other Things And In Terms Of The Way Other Things Are Relevant To Them. Things Have Relational Essences. Likewise, Things Do Not Exist Independently Of Temporality But Are Constituted By The History Of Their Specific And Situated Encounters. Every Actual Thing Is Thus "Something By Reason Of Its Activity" (Whitehead, 1927/1985, P. 26). Importantly, This Talk Of "Things" Need Not Incline One Towards Denying The Relevance Of Subjectivity. I Have Thus Taken Issue With A Tendency Illustrated In The Work Of Thrift (2008), Who Appears To Define Cutting-Edge Social Theory As Concerned With "Flow" And "Play" Rather Than With Stability (Since "Non-Foundational Theory Takes The Leitmotif Of Movement", P. 5 And "Privileges Play", P. 7); As "A Means Of Going Beyond Constructivism" (P. 5); And As "Trading" In "Modes Of Perception Which Are Not Subject-Based" (P. 7). I Have Tried To Show That, In Fact, The Concept Of Process Is As Much About Stability As About Change. Stability Is To Be Thought Of As An Achievement Resulting From Particular Ways Of Actualizing Potential And Of Patterning Occasions Into Spatial And Temporal Co-Assemblies. Nevertheless, I Have Also Stressed That Becoming Is An Inherently Self-Creative Process, Albeit A Self-Creation Grounded In The Facticity Of A Concrete Inheritance. Whitehead's Category Of Subjective Unity Is Thus An Exemplary Statement Of Constructivism, Which States That: "Self-Realization Is The Ultimate Fact Of Facts. An Actuality Is Self-Realizing, And Whatever Is Self-Realizing Is An Actuality" (Whitehead, 1927–1928/1985, P. 222). The "Capture Of Intensity" And The "Clutch At Vivid Immediacy" Are Thus The Defining Characteristics Of Life (Whitehead, 1927–1928/1985, P. 105). Finally, I Have Suggested That It Is Only On The Basis Of A Deep Extension Of The Concept Of Experience Throughout Nature That Whitehead Is Able To Resoundingly Affirm His Reformed Notion Of The Subjectivist Principle: "That Apart From The Experiences Of Subjects There Is Nothing, Nothing, Nothing, Bare Nothingness" (1927–1928/1985, P. 167). "Scientific Reasoning Is Completely Dominated By the Presupposition That Mental Functionings Are Not Properly Part of Nature" (Whitehead, 1938/1966, P. 156). (Ibid). Lewis Carroll Is Wonderful In Exploring Some Of The Paradoxes Of Logic. Following Some Of These Paradoxes Will Lead To The Important Role Of 'Sense' In Understanding. [NB Weird Titles Are Deleuze's Own]. First Series Of Paradoxes Of Pure Becoming: We Need To Explore The Nature Of An Event. Events Assume Becoming, Since They Refer To States In The Past And The Future In A Way Which 'Eludes The Present'. This Is Paradoxical But Still Makes Sense. Plato Tried To Distinguish Between Limited Fixed Things And Pure Becoming, But The Two Cannot Be Separated.

Instead, A Dualism Is Hidden In Material Bodies. We Need To Introduce The Notion Of A Simulacrum Which Is Neither Copy Nor Model [Massumi Has A Useful Article On This—Roughly, The Issue Is That What We Take To Be Material Reality Is Actually A Simulacrum Of The Virtual, A Limited Condensation 'Beneath Things' (2)]. It Also Avoids The Problems Of Subsuming Reality Under The Idea. Sometimes, This Is Indicated By A Summer The Peculiarities Of Language, Which Can Seem To Flow Over Specific Referents. This Provides A Clue That There Is Some Dimension To Language Which Serves To Come To The Aid Of More Specific Attempts To Name And Describe. There Are Implications For Identity. Fully Grasping Becoming Means That Identities Are Infinite, Incorporating Future And Past, Active And Passive, And Even Cause And Effect. Language Attempts To Limit This Infinity, But Still Often Alludes To It [With Open-Ended Statements Or Generalisations]. As With The Alice Stories, This Also Disrupts The Conventional Notion Of The Personal Identity. Normally, This Is Maintained By Some Underlying Commonsense Or Knowledge, As When 'The Personal Self Requires God And The World In General' (3). Becoming Threatens This Stability With The Paradox Of Events, Which Can Penetrate Even Commonsense—It Is Not Just A Doubt About Reality, But A Clear Indication Of The 'Objective Structure Of The Event Itself, Insofar As It Moves In Two Directions At Once, And Insofar As It Fragments The Subject Following This Double Direction' (3). Second Series Of Paradoxes Of Surface Effects: Stoics Divided Things Into Bodies And States Of Affairs, 'Actions And Passions' (4). There Is Also Some Cosmic Unifying Quality, Always In The Present. Bodies Can Interact and Cause Effects in Each Other, But These Effects Are Incorporeal, 'Logical or Dialectical Attributes… Not Things or Facts but Events' (5). They Have The Kind Of Subsidiary Existence, Acting As Verbs, And They Are Infinitives—The Example Is A Cut Inflicted On The Body, Which Is Seen As An Incorporeal Surface Effect, Compared To The Actuality Of Bodies And Their Mixtures. This Argument Had Important Implications For Understanding The Causal Relation. Specific Bodily Causes Produce Other Bodies, Linked By Some Cosmic Unity Or Destiny. Similarly, Effects Can Be Seen As Having Bonds Between Them, But Effects Can Never Be Causes In Themselves. They Can Only Be '"Quasi-Causes' Following Laws Which Perhaps Express In Each Case The Relative Unity Or Mixture Of Bodies On Which They Depend For Their Real Causes' (6). These Combinations and Bonds to Provide For Some Emergent Qualities, Which Means That Destiny Can Be Avoided. An Alternative Is Offered By The Epicurean Classification Of Different Kinds Of Causes Which Are Relatively Independent [And So Can Interact], And This Is A Kantian Idea Too. There Is A Reference Back To The Capacities Of Language To Offer 'A Declension Of Causes... [And]... A Conjugation of Effects' (6). Stoic Philosophy Introduces The Notion Of A Something Behind Both Specific Material Beings And Incorporeal Events. The Idea Must Belong To 'This Impassive Extra-Being Which Is Sterile, Inefficacious, And On The Surface Of Things: The Ideational Or The Incorporeal Can No Longer Be Anything Other Than An "Effect"' (7). This In Turn Leads To A Change Of Metaphor From Surface/Depth To Just Surface, To A Series Of Effects Which Are Manifestations And Are Of Different Types. We Have A Notion of Possibilities, Of Ideality Itself, Rather Than the Platonic Idea, But With No 'Causal and Spiritual Efficacy' (7). The Simulacrum Now Appears On The Surface, Rather Than Being Hidden In The Depths. Events As Effects Combine Past And Present, Active And Passive, All Of Which Are Located Elsewhere As Causes. The Relation Between Events Can Only Be Quasi-Causes. Stoics Saw Dialectical Analysis Has Explorations Of These Combinations, Once They Had Been Expressed In Propositions—Dialectics As Conjugation. Language Also Enables Us To Go Beyond Events Into The Possible Or Becoming. The Relation Between Propositions And Specifics Is Itself Still Paradoxical—'Chryssipus Taught "If You Say Something It Passes Through Your Lips, So If You Say "Chariot", A Chariot Passes Through Your Lips' (8). It Is Deliberate Nonsense In The Anglo American Sense, Or Humourous Play On The Surface, As Opposed To An Ironic Exploration Of Depths And Heights. Lewis Carroll Did Something Similar In Alice. [A Commentary On Alice Ensues, Stressing The Surface Rather Than The Underground World, And Picking Up The Disdain Lewis Carroll Felt For Boys Who Did Not Like To Operate At The Surface. Left Handers And Stutterers Can Sometimes Remind Us Of The Paradoxes Of The Surface, However, Which Can Defeat Commonsense Understandings]. Third Series Of The Proposition: Describing Events As Propositions Raises The Question Of How Best To Analyse Surface Events. There Are Three Possibilities: Denotation [Roughly, A Direct Connection Between Words And

Images Which Represent States Of Affairs, As In Indexical Signs. Here, Propositions Are Either True Or False, And May Be True In All Cases]; Manifestation -- A Relation Between The Proposition And The Person Expressing It, Statements Of Desire And Belief. These Are Causal Relations: 'Desire Is The Internal Causality Of An Image With Respect To The Existence Of The Object Or The Corresponding State Of Affairs' (13). Belief Anticipates Production Of An Effect By A Cause. Manifestation Includes Denotation, Makes It Possible. '"I" Is the Basic Manifester' (13). Manifestation Is 'The Domain Of The Personal, Which Functions As The Principle Of All Possible Denotation' (13). The Issues Here Turn On Veracity Other Than Truth And Falsehood, The Avoidance Of Illusion. Signification, The Relation Of A Word To Universal Or General Concepts, And Connections To Implications Which Have To Follow The Rules Of Syntax. Again, Signifying Involves Conceptual Implications Referring To Other Propositions, As In Premises Or Conclusions. This Involves A Certain Indirect Process, Implication Or Assertion, Instead Of Truth Or Veracity, Which Remain As Possible In Certain Conditions. However, It Is Not Just Formal Logical Operations That Are Involved, But Notions Of Probability Or Even Moral Terms Such As Promise Or Commitment. Error Produces Not Falsehood But Absurdity [Looks Really Close To Habermas and the Three Validity Claims Here]. Signification May Not Be Primary, Since All Language Begins From The Standpoint Of The 'I', But There Is An Assumption That Propositions Must Be Understood By Others And Have A General Force. This Implies That Manifestation Has Primacy. But Signification Is Implied, And [In Social Relations] Would Be The Basis Of Manifestation. It Is The Difference Between Langue And Parole. Particular Utterances Only Make Sense Against The Background Of Constant Concepts. This Is Extended To Particular Desires And Beliefs, As Opposed To 'Simple Opinions' Which Would Not Signify (16). Actual Utterances Often Involve Truth Claims And General Signification [And Other Presuppositions, Which Are Technically Infinite]. This Is The Paradox Facing Pure Logic, Solved By A Form Of [Smuggling]: 'Implication Never Succeeds In Grounding Denotation Except By Giving Itself A Ready- Made Denotation, Once In The Premises And Again In The Conclusion' (16). So Actual Propositions Feature Circular Relations between Signification, Manifestation, and Denotation. There Might Even Be A Fourth Dimension—Sense, But Introducing This Will Depend On Making Relations Theoretically Consistent—It Is 'Not Simply A Question Of Fact' (17). To Approach The Issue, We Ask Whether Sense Might Be Located In One Of The Existing Three. Denotation Concerns Itself With Truth And Falsity, Which Is Too Narrow. The Mere Relation Between Words And Denoted Things Is To Paradoxical To Always Makes Sense, As In The Example Of Speaking The Term 'Chariot'. Instead, Denotation Presupposes Sense. Manifestation Does Involve Some Manifesting Subject Which Initiates, So May Be Sense Is Itself A Subjective Matter Of Beliefs And Desires Of Persons—But Subjects Only Possess This Ability To Speak Because Of A General System Of Signification In Language. It Looks Like Sense Must Be Identified With Signification—But Signification Is Linked In A Circular Relation With Denotation And Manifestation. Perhaps it's Necessary to Think Of Different Forms of Possibility of Propositions—Logical, Physical, Syntactic and So On. This Might Serve As Foundations For Sense, But This Would Be An External Foundation, Independent Of Speech [I Think The Problem Is The Connections Between Any Foundations And Actual Act Of Speech, Whether Anything Would Escape The Foundations]. The Concept Of Truth In Particular Implies Independence From Form. This Independence, Separate From Conceptual Possibilities In Signification, Is What Constitutes Sense. This Is The Fourth Dimension. It Is 'An Incorporeal, Complex, And Irreducible Entity, At The Surface Of Things' (19). There Is Philosophers Have Discovered And Rediscovered This Quality. It Is The Idea Of A Something Again, Beyond The Propositions And The Terms And The Objects Which Are Denoted, Beyond The Subjective I And Things Which Are Expressed. Sense Is Irreducible To Propositions, and It Is and Must Be '"Neutral," Altogether Indifferent To Particular and General, Singular and Universal, Personal and Impersonal' (19). There Is Been Little Agreement About This Possible Fourth Dimension, Whether It Exists Simply In The Form Of Some Enquiry. It Is Not Even Immediately Useful Because It Is Neutral. It Can Only Be Inferred Indirectly, By Questioning Characteristics Of Propositions As Above—This Is 'Inspired In Its Entirety By Empiricism… [Avoiding Notions of Essence or Idea, and Knowing]… Have To Track Down, Invoked And Perhaps Produce A Phantom At The Limit Of A Length Or Unfolded Experience' (20). It Might Be What Husserl Called 'Expression', Lingering In Terms

Such As The Noema, As Pure Appearance, Outside Denotation Or Manifestation, Linked In Complex Ways To Appearances. In The Same Way, Sense Does Not Exist Outside Propositions Exactly, But 'Inheres or Subsists' (21). It Is Not Just An Expression, But An Attribute, Not Just Of The Proposition, But 'Of The Thing Or State Of Affairs' (21), [The Potential, 'To Be Able To Be Green' Rather Than Just The Denotation 'Green' Is The Example Here]. It Is Said Of A Thing, So It Depends On Propositions Which Express It And Is Therefore Not Separate From The Proposition. It Is Something Else, Both the Expressible, And the State Of Affairs: 'It Turns One Side towards Things and One Side towards Propositions' (22). It Is What Joins Propositions And Things. [It Is A Becoming]. It Operates On The Surface, Rather As Mathematics Does, Or The Nonsense Of Carroll. It Is The Operation Of Sense That Produces [Meaningful] Paradox. Fourth Series Of Dualities: Important Dualities Exist Between Causes And Effects, And 'Corporeal Things And Incorporeal Events' (23). This Is Extended To A Duality Between Things And Propositions, Bodies And Language. This Is Expressed In Lewis Carroll As A Duality Between Eating And Speaking—The Former Is A Matter Of Bodies Actions And Passions, And The Latter Movements Of The Surface And 'Ideational Attributes Or Incorporeal Events' (23) [Lots Of Examples From Alice About Being Presented To Food And Having Food Presented To You]. The Normal Relationship Can Be Distorted By 'Verbal Hallucinations… Unrestricted Oral Behaviour… And Various Disorders of the Surface' As Bodily Matters Intrude—Stuttering, Left Handedness (24). Sense Is Always Expressed In Propositions, But It Lies In States of Affairs, It Happens To Things. In This Sense, Bodies And Language Are United In The Production Of Sense, Existing 'On The Two Sides Of The Frontier Represented By Sense', Which Constantly Articulates The Differences (24)—Things Include 'Ideational Logical Attributes Which Indicate Incorporeal Events', And Propositions Include Both Denotations And Expressions, Names And Adjectives, And Verbs [The Latter Indicating Becoming And Chains Of Events] (24) [Illustrated With Words By Humpty Dumpty]. This Duality In Propositions Represents Two Dimensions, The 'Denotation Of Things And The Expression Of Sense' (25) [Here, It Is Sense That Is Being Expressed Not Subjectivity, However]. This Means The Duality Is Inherent In Propositions As Well As Between Propositions And Things. [More From Lewis Carroll Page 26, Turning On The Universal Denotation Implied By The Term 'It', And Also The Ability Of The Term To Summarise The Sense Of An Earlier Proposition]. The Two Dimensions Can Emerge In an Esoteric Word, Or In A 'Non Identifiable Aliquid' (26). The Example Given Is The Word 'Snark' [Both A Limited Thing And A Representation Of Anything That Is Too Exotic To Exist?]. [The Section ends with an Extended Quotation from the Gardener's Song in Sylvie And Bruno—Pass] Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable to Constraints on Space Accomodation Model Is Provided in Some Paper in the series Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars of Xian Shou Say, "It Is Arising From Primal Nature", And the Scholars of Chan (Zen) Say, "It Is Nature That Causes the Rising of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). The reason for becoming, according to Whitehead, is always the actual entity itself. He called this the ontological principle. The entity "enjoys" its relations and is "satisfied" by its own becomings. Whiteheads theory of process and relations can be viewed as an experiential alternative view of quantum physics which incorporates a theory of mind. However, Whitehead's master work, Process and Reality (1929/1978), is nearly impossible to fully understand, has its own idiosyncratic vocabulary, and is not entirely self-consistent. This relative opacity of Whitehead's work has impeded the generalization of his quantum process theory, or metaphysics, to other areas of discourse. In his independent formulation of the microgenesis of the mental state, Jason Brown discovered a process theory of mind that had many similarities to Whiteheadian process. He took the concepts of duration and discontinuities of process and applied them to the duration of the mental state as it

recapitulates the contents of prior mental states. Through the introduction of novelty, and through the process of microgenesis during the progression of mental states, the Self gives rise to a percept of self, the ego, in the object world, at the termination of the microgenetic process. This termination is the conscious instant, or instantiation of the microgenetic process. It is all that we consciously see. We see the ego, not the Self, and thus mistake the ego for self, replacing self with selfhood. The process itself is obscured from our consciousness, but can be inferred from those neurological deficit syndromes, which expose lower "layers" in the becoming. This parallels the "layers" of the nested hierarchy of holographic surfaces in the brain itself. Brown is theory of microgenesis involves a progression in the genesis of the mental state in a developmental sequence that recapitulates the organism is evolutionary development (phylogeny), embryologic development (ontogeny), and history of personal and social experience and development, in a movement in an upward direction through structures of increasing phylogenetic and ontogenic recency (Brown, 1988, 1991, 1996, 1998, 2005). He developed a view that considered the evolution of mental states to be a continuum from depth to surface, with leaps or saltation between mental states at intervals that are temporally irreducible units of the duration of microgenetic process (Brown, 1996, 1998). The process of neoteny, through which earlier features of development and evolution are brought to the present, was described (Brown, 1996) as a change in the development of some aspect of the mental state, which is attenuated at some stage of elaboration. In neurological and psychiatric deficit syndromes, this attenuation corresponds with the locus of a lesion or functional deficiency. The signature of the attenuated stage is carried through the remaining process of microgenesis, leading to expression of the attenuated phase at the endpoint of the duration of microgenesis, coupled with full elaboration of other elements of microgenesis. The intellectual pedigree of the theory of microgenesis dates back to the principle of superposition of Sir Charles Lyell, according to which younger strata lie on top of older strata. Lyell, a geologist, was to have great influence on evolutionary theorists, including Darwin and Wallace. The principle of superposition was taken up by Herbert Spencer, a scholar of many fields, who was a contemporary of Charles Darwin, and who had developed an evolutionary theory that perhaps rivaled Darwin is in popularity during the late 1800ís and early 1900ís. An important element of Spencer is theory was his observation of the brain, which he fashioned to be layered, with the newer layers added on to older layer during the course of evolution. Looking at the deeper layers was, for Spencer, like peeling away the layers of an onion. Spencer is idea was taken up by the renowned British neurologist Hughlings Jackson (Kennard and Swash eds., 1989), who developed the notion of hierarchies of brain process and the concept of neurological deficit syndromes as disturbances in the hierarchical structure and function of the brain. The concepts of microgeny and microgenesis, derived from hierarchies of brain function, were later developed by Arieti (1962) and Werner (1956). Later, the concept of a hierarchy of adaptive ego mechanisms, including lower order defenses such as denial and reaction formation, and higher order defenses such as altruism, was developed by Vaillant (1971), with the more mature defense mechanisms bearing on higher order values. The process theory of the evolution of the brain is extraordinarily important to Brown is process theory of mind. We can place these theories in perspective with the process theory of Whitehead, into which the theory of Brown partially merged (Brown, 1996; 1998; 2005). The materialist perspective of evolution, was, according to Whitehead (1925/1953), inconsistent with the process by which organisms become: "...in truth, a thoroughgoing evolutionary philosophy is inconsistent with materialism. The aboriginal stuff, or material, from which a materialistic philosophy starts, is incapable of evolution. This material is in itself the ultimate substance. Evolution, in the materialistic theory, is reduced to the role of being another word for the description of the changes of the external relations between portions of matter. There is nothing to evolve, because one set of external relations is as good as any other set of external relations. There can merely be change, purposeless and unprogressive." Whitehead is (1929/1978) realm of eternal objects, apprehended in the unconscious, can be viewed as a basic element in the evolution of organisms and of mind. Whitehead (1929/1978), in his view of eternal objects, hearkens is back to Plato: "...eternal objects, as in Gods primordial nature, constitute the Platonic world of ideas." Whitehead is realm of eternal objects may include Carl Jung is (1934/1967) archetypes of the collective unconscious (Griffin, ed., 1989), connecting Whitehead is cosmology to the corpus of literature on depth psychology. Jung is archetypes were largely based on mythological, religious, and alchemical constructs. Mythology embodies the spiritual and

social history of the species, as well as a variety of metaphysical metaphors (Campbell, 1969/1976; Neumann, 1954/1970). The physical nature of the brain state, including the notion of binding of the mental state into a coherent whole, is also critical to Brown is theory of microgenesis. Recent evidence indicates that the brain is a chaotic or dynamical system (Freeman, 1987; Combs, 2004; Abraham, 2004). The self-organization of dynamical systems, which include all living organisms as well as the brain, has been described in detail by Ilya Prigogine (1980), who won the Nobel Prize in chemistry in 1977 for his work on dynamical systems. Prigogine defines an "internal time" or duration of the states of dynamical systems. Physical time, he states, is secondary, and internal time is primary (Prigogine, 1986). The dynamical interval of internal time, or duration, is governed by the baker transformation (Prigogine, 1980), which can be envisioned as the time it takes the baker to knead or fold two different colors of dough successively, until it seems to be uniform in color. The brain, as a dynamical system, thus "deposits" time in the sense of Brown (1996). Here we have a parallel with the "layering" of process time according to the Holographic Principle. We can also detect, remotely, the undertones of Lyellís principle of superposition, albeit in a different form, and in a sense that Lyell could never have imagined. The work of Jason Brown in process neurology and psychology stands in sharp contrast to the prevailing scientific and philosophical nihilism in these fields regarding the self, free will, and the process of experience. In the sense of Brown is theory of microgenetic process of the brain, and in the sense of the duration of process discontinuities of Whitehead as it relates to the mind, it is important for the purposes of this paper to explore the scientific basis of the brain state. Mental process involves successive durations of such states, each with a "temporal thickness" (Brown, 1996). In recent years the "brain electrical microstate" has been described in detail as patterns of electrical field potential which endure for a period a period of time, typically around 100 milliseconds or a tenth of a second, and are punctuated by rapid transitions to the next state (Fingelkurts and Fingelkurts, 2001; Koenig and others, 2002). Brain electrical microstates are prime candidates for the brain states of microgenetic process and for the dynamical interval of internal time. The application of the theory of discontinuous information states, with the complete repetition of former states, as applied to the Holographic Principle of Mind, requires an information theory that includes time and the duration of mental states (e.g. Germine, 1993).The process of microgenesis, as it applies to brain evolution, can be seen in purely classical way, related to the recapitulation theory of vertebrate brain evolution (Agoutis, Montiel, and Lopez, 2002), which implies that "sensory projection sites and sites and processing circuits have been conserved [recapitulated] in reptiles and mammals" in the course of evolution. Again, there is an element of predictability in evolution which seems to belie the purposelessness of random natural selection. In keeping with the Holographic Principle, the development of the organism is a product of its evolutionary history, and is in continuous intercourse with that history. Microgenesis is readily related to the Holographic Principle of Mind, whereby the past remains actual, and, in the course of each mental state or mental process, there is a literal recapitulation of the organisms' entire past. The depth to surface principle entails a direction of mental process which is essential to microgenesis, and which is explained naturally by the nested hierarchy of surfaces in the brain according to the Holographic Principle. In the Holographic Principle of Mind we have a universal principle of microgenesis, whereby the entire relevant history of the organism is repeated during the course of each timeless and eternal moment. In this way, microgenesis can extend back to the very inception of the Universe. MICROGENESIS AND THE HOLOGRAPHIC PRINCIPLE the Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho Science. Aristotle and others (including, especially, Leibniz) have argued that time does not exist independently of the events that occur in time. This view is typically called either "Reductionism with Respect to Time" or "Relationism with Respect to Time," since according to this view; all talk that appears to be about time can somehow be reduced to talk about temporal relations among things and events. The opposing view, normally referred to either as "Platonism with Respect to Time" or as "Substantivalism with Respect to Time" or as "Absolutism with Respect to Time," has been defended by Plato, Newton, and others. On this view, time is like an empty container into which things and events may be placed; but it is a container that exists independently of what (if anything) is placed in it. Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL =

<http://plato.stanford.edu/archives/spr2014/entries/time/>. The question of whether there could be time without change has traditionally been thought to be closely tied to the question of whether time exists independently of the events that occur in time. For, the thinking goes, if there could be a period of time without change, then it follows that time could exist without any events to fill it; but if, on the other hand, there could not be a period of time without change, then it must be that time exists only if there are some events to fill it. Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>."Whoever thou mayest be, beloved stranger, whom I meet here for the first time, avail thyself of this happy hour and of the stillness around us, and above us, and let me tell thee something of the thought which has suddenly risen before me like a star which would fain shed down its rays upon thee and every one, as befits the nature of light. - Fellow man! Your whole life, like a sandglass, will always be reversed and will ever run out again; - a long minute of time will elapse until all those conditions out of which you were evolved return in the wheel of the cosmic process. And then you will find every pain and every pleasure, every friend and every enemy, every hope and every error, every blade of grass and every ray of sunshine once more, and the whole fabric of things which make up your life. This ring in which you are but a grain will glitter afresh forever. And in every one of these cycles of human life there will be one hour where, for the first time one man, and then many, will perceive the mighty thought of the eternal recurrence of all things:- and for mankind this is always the hour of Noon". Notes on the Eternal Recurrence - Vol. 16 of Oscar Levy Edition of Nietzsche's Complete Works (in English). Physical time is public time, the time that clocks are designed to measure. Biological time, by contrast, is indicated by an organism's circadian rhythm or body clock, which is normally regulated by the pattern of sunlight and darkness. Psychological time is different from both physical time and biological time. Psychological time is private time. It is also called phenomenological time, and it is perhaps best understood as awareness of physical time. Psychological time passes relatively swiftly for us while we are enjoying an activity, but it slows dramatically if we are waiting anxiously for the pot of water to boil on the stove. The slowness is probably due to focusing our attention on short intervals of physical time. Meanwhile, the clock by the stove is measuring physical time and is not affected by any person's awareness or by any organism's biological time. When a physicist defines speed to be the rate of change of position with respect to time, the term "time" refers to physical time, not psychological time or biological time. Physical time is more basic or fundamental than psychological time for helping us understand our shared experiences in the world, and so it is more useful for doing physical science, but psychological time is vitally important for understanding many mental experiences. Psychological time is faster for older people than for children, as you notice when your grandmother says, "Oh, it's my birthday again." That is, an older person's psychological time is faster relative to physical time. Psychological time is slower or faster depending upon where we are in the spectrum of conscious experience: awake normally, involved in a daydream, sleeping normally, drugged with anesthetics, or in a coma. Some philosophers claim that psychological time is completely transcended in the mental state called nirvana because psychological time slows to a complete stop. There is general agreement among philosophers that, when we are awake normally, we do not experience time as stopping and starting Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>. But From The Profound Contemplation And Wisdom Of The Buddha And Mahabodhisattvas, We Know There Is No Such Reality. Instead, Egolessness (Non-Self) Is The Only Path To Understand The Reality Of The Deluded Life. All Existences Are Subject To The Law Of Causes And Conditions. These Include The Smallest Particles, The Relationship Between The Particles, The Planets, And The Relationship Between Them, Up To And Including The Whole Universe! From The Smallest Particles To The Biggest Matter, There Exists No Absolute Independent Identity. Egolessness (Non-Self) Implies The Void Characteristics Of All Existence. Egolessness (Non-Self) Signifies The Non-Existence Of Permanent Identity For Self And Existence (Dharma). Sunyata Stresses The Voidness Characteristic Of Self And Existence (Dharma). Sunyata And Egolessness Possess Similar Attributes. As We Have Discussed Before, We Can Observe The

Profound Significance Of Sunyata From The Perspective Of Inter-Dependent Relationships. Considering Dharma-Nature and the Condition of Nirvana, All Existences Are Immaterial and Of A Void-Nature. Then We See Each Existence As Independent Of Each Other. But Then We Cannot Find Any Material That Does Exist Independent Of Everything Else. So Egolessness Also Implies Void-Nature! From The Observation Of All Existences, We Can Infer The Theory Of Nirvana And The Complete Cessation Of All Phenomena. From The Viewpoint Of Phenomena, All Existences Are So Different From Each Other, That They May Contradict Each Other. They Are So Chaotic. In Reality, Their Existence Is Illusionary And Arises From Conditional Causation. They Seem To Exist On One Hand, And Yet Do Not Exist On The Other. They Seem To Be United, But Yet They Are So Different To One Another. They Seem To Exist And Yet They Do Cease! Ultimately Everything Will Return To Harmony And Complete Calmness. This Is The Nature Of All Existence. It Is The Final Resting Place For All. If We Can Understand This Reality And Remove Our Illusions, We Can Find This State Of Harmony And Complete Calmness. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia) for models please see one of the papers in the series. “The Difference Between Supermind And Big Mind (If We Take Big Mind To Mean The State Experience Of Nondual Suchness, Or Turiyatita) Is That Big Mind Can Be Experienced Or Recognized At Virtually Any Lower Level Or Rung. Magic to Integral. In Fact, One Can Be At, Say, The Pluralistic Stage, And Experience Several Core Characteristics Of The Entire Sequence Of State-Stages (Gross To Subtle To Causal To Witnessing To Nondual), Although, Of Course, The Entire Sequence, Including Nondual Suchness, Will Be Interpreted In Pluralistic Terms. This Is Unfortunate In Many Ways—Interpreting Dharma In Merely Pluralistic Terms (Or Mythic Terms, Or Rational, And So On)—Because It Is So Ultimately Reductionistic; But It Happens All The Time, Given The Relative Independence Of States And Structures At 1st And 2nd Tier. Supermind, On The Other Hand, As A Basic Structure-Rung (Conjoined With Nondual Suchness) Can Only Be Experienced Once All The Previous Junior Levels Have Emerged And Developed, And As In All Structure Development, Stages Cannot Be Skipped. Therefore, Unlike Big Mind, Supermind Can Only Be Experienced After All 1st-, 2nd-, and 3rd-Tier Junior Stages Have Been Passed Through. While, As Genpo Roshi Has Abundantly Demonstrated, Big Mind State Experience Is Available To Virtually Anybody At Almost Any Age (And Will Be Interpreted According To The View Of Their Current Stage), Supermind Is An Extremely Rare Recognition. Supermind, As The Highest Structure-Rung To Date, Has Access To All Previous Structures, All The Way Back To Archaic—And The Archaic Itself, Of Course, Has Transcended And Included, And Now Embraces, Every Major Structural Evolution Going All The Way Back To The Big Bang. (A Human Being Literally Enfolds And Embraces All The Major Transformative Unfoldings Of The Entire Kosmic History—Strings To Quarks To Subatomic Particles To Atoms To Molecules To Cells, All The Way Through The Tree Of Life Up To Its Latest Evolutionary Emergent, The Triune Brain, The Most Complex Structure In The Known Natural World.) Supermind, In Any Given Individual, Is Experienced As A Type Of “Omniscience”—The Supermind, Since It Transcends And Includes All Of The Previous Structure-Rungs, And Inherently Is Conjoined With The Highest Nondual Suchness State, Has A Full And Complete Knowledge Of All Of The Potentials In That Person. It Literally “Knows All,” At Least For The Individual.” ― Ken Wilber, The Fourth Turning: imagining the Evolution of an Integral Buddhism (fopr models please see one of the papers in the series). “The Movement Of Descent And Discovery Begins At The Moment You Consciously Become Dissatisfied With Life. Contrary To Most Professional Opinion, This Gnawing Dissatisfaction With Life Is Not A Sign Of "Mental Illness," or An Indication Of Poor Social Adjustment, or A Character Disorder. For Concealed Within This Basic Unhappiness With Life And Existence Is The Embryo Of A Growing Intelligence, A Special Intelligence Usually Buried Under The Immense Weight Of Social Shams. A Person Who Is Beginning To Sense The Suffering Of Life Is, At The Same Time, Beginning To Awaken To Deeper Realities, Truer Realities. For Suffering Smashes To Pieces The Complacency Of Our Normal Fictions About Reality, And Forces Us To Become Alive In A Special Sense—To See Carefully, To Feel Deeply, To Touch Ourselves And Our Worlds In Ways We Have Heretofore Avoided. It Has Been Said, And Truly I Think, That Suffering Is The First Grace. In A Special Sense, Suffering Is Almost A Time Of Rejoicing, For It Marks The Birth Of Creative Insight. But Only In a Special Sense. Some People Cling To Their Suffering As A Mother To Its Child, Carrying It As A Burden They Dare Not Set Down.

They Do Not Face Suffering With Awareness, But Rather Clutch At Their Suffering, Secretly Transfixed With The Spasms Of Martyrdom. Suffering Should Neither Be Denied Awareness, Avoided, Despised, Not Glorified, Clung To, And Dramatized. The Emergence Of Suffering Is Not So Much Good As It Is A Good Sign, An Indication That One Is Starting To Realize That Life Lived Outside Unity Consciousness Is Ultimately Painful, Distressing, And Sorrowful. The Life Of Boundaries Is A Life Of Battles—Of Fear, Anxiety, Pain, And Finally Death. It Is Only Through All Manner Of Numbing Compensations, Distractions, And Enchantments That We Agree Not To Question Our Illusory Boundaries, The Root Cause Of The Endless Wheel Of Agony. But Sooner Or Later, If We Are Not Rendered Totally Insensitive, Our Defensive Compensations Begin To Fail Their Soothing And Concealing Purpose. As A Consequence, We Begin To Suffer In One Way Or Another, Because Our Awareness Is Finally Directed Toward The Conflict-Ridden Nature Of Our False Boundaries And The Fragmented Life Supported By Them." ― Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth for models please see one of the papers in the series. "To The Extent That You Actually Realize That You Are Not, For Example, Your Anxieties, Then Your Anxieties No Longer Threaten You. Even If Anxiety Is Present, It No Longer Overwhelms You Because You Are No Longer Exclusively Tied To It. You Are No Longer Courting It, Fighting It, Resisting It, Or Running From It. In The Most Radical Fashion, Anxiety Is Thoroughly Accepted As It Is And Allowed To Move As It Will. You Have Nothing To Lose, Nothing To Gain, By Its Presence Or Absence, For You Are Simply Watching It Pass By. Thus, Any Emotion, Sensation, Thought, Memory, Or Experience That Disturbs You Is Simply One With Which You Have Exclusively Identified Yourself, And The Ultimate Resolution Of The Disturbance Is Simply To Dis-Identify With It. You Cleanly Let All Of Them Drop Away By Realizing That They Are Not You--Since You Can See Them, They Cannot Be The True Seer And Subject. Since They Are Not Your Real Self, There Is No Reason Whatsoever For You To Identify With Them, Hold On To Them, Or Allow Yourself To Be Bound By Them. Slowly, Gently, As You Pursue This Dis-Identification "Therapy," You May Find That You is Entire Individual Self (Persona, Ego, and Centaur), Which Heretofore You Have Fought to Defend and Protect, Begins to Go Transparent and Drop Away. Not That It Literally Falls Off And You Find Yourself Floating, Disembodied, Through Space. Rather, You Begin To Feel That What Happens To Your Personal Self—Your Wishes, Hopes, Desires, Hurts—Is Not A Matter Of Life-Or-Death Seriousness, Because There Is Within You A Deeper And More Basic Self Which Is Not Touched By These Peripheral Fluctuations, These Surface Waves Of Grand Commotion But Feeble Substance. Thus, Your Personal Mind-And-Body May Be In Pain, Or Humiliation, Or Fear, But As Long As You Abide As The Witness Of These Affairs, As If From On High, They No Longer Threaten You, And Thus You Are No Longer Moved To Manipulate Them, Wrestle With Them, Or Subdue Them. Because You Are Willing To Witness Them, To Look At Them Impartially, You Are Able To Transcend Them. As St. Thomas Put It, "Whatever Knows Certain Things Cannot Have Any Of Them In Its Own Nature." Thus, If The Eye Were Colored Red, It Wouldn't Be Able To Perceive Red Objects. It Can See Red Because It Is Clear, Or "Redless." Likewise, If We Can But Watch Or Witness Our Distresses, We Prove Ourselves Thereby To Be "Distress-Less," Free Of The Witnessed Turmoil. That Within Which Feels Pain Is Itself Pain-Less; That Which Feels Fear Is Fear-Less; That Which Perceives Tension Is Tensionless. To Witness These States Is To Transcend Them. They No Longer Seize You From Behind Because You Look At Them Up Front." ― Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth (for models please see one of the papers in the series). "The Difference Between Supermind And Big Mind (If We Take Big Mind To Mean The State Experience Of Nondual Suchness, Or Turiyatita) Is That Big Mind Can Be Experienced Or Recognized At Virtually Any Lower Level Or Rung. ……In Difference and Repetition (1968, English 1994), Deleuze Develops His Project in Multiple Directions. His Work, He Says, Stems From The Convergence Of Two Lines Of Research: The Concept Of Difference Without Negation, And The Concept Of Repetition, In Which Physical And Mechanical Repetitions Are Masks For A Hidden Differential That Is Disguised And Displaced. His Major Focus Is A Thoroughgoing Critique Of Representational Thinking, Including Identity, Opposition, Analogy, And Resemblance (Deleuze 1994, 132). For Deleuze, "Appearances Of" Are Not Representations, But Sensory Intensities Free Of Subjective Or Objective Identities (Deleuze 1994, 144). Without These Identities, Appearances Are Simulacra of A

Non-Apparent Differential He Calls the “Dark Precursor” Or “The In-Itself of Difference” (Deleuze 1994, 119). This Differential Is The Non-Sensible Being Of The Sensible, A Being Not Identical To The Sensible, Or To Itself, But Irreducibly Problematic Insofar As It Forces Us To Encounter The Sensible As “Given.” Furthermore, Any Move Against Representational Thinking Impinges Upon The Identity Of The Subject. Where Kant Founds The Representational Unity Of Space And Time Upon The Formal Unity Of Consciousness (Kant 1964, 135-137), Difference Re-Distributes Intuitions Of Past, Present, And Future, Fracturing Consciousness Into Multiple States Not Predicable Of A Single Subject. Intensive Qualities Are Individuating By Themselves, Says Deleuze, And Individuality Is Not Characteristic Of A Self Or An Ego, But Of A Differential Forever Dividing Itself And Changing Its Configuration (Deleuze 1994, 246, 254, 257). In Nietzschean Fashion, The “I” Refers Not To The Unity Of Consciousness, But To A Multitude Of Simulacra Without An Identical Subject For Whom This Multitude Appears. Instead, Subjects Arise And Multiply As “Effects” Of The Intensive Qualities Saturating Space And Time. This Leads Deleuze to Postulate Multiple Faculties for Subjectivity, Which Are Correlates of the Sensible Insofar As It Gives Rise to Feeling, Thought, and Action. “Each Faculty, Including Thought, Has Only Involuntary Adventures,” He Says, And “Involuntary Operation Remains Embedded In The Empirical” (Deleuze 1994, 145). Subjectively, The Paradox Of The Differential Breaks Up The Faculties' Common Function And Places Them Before Their Own Limits: Thought Before The Unthinkable, Memory Before The Immemorial, Sensibility Before The Imperceptible, Etc. (Deleuze 1994, 227). This Fracturing And Multiplying Of The Subject, He Notes, Leads To The Realization That “Schizophrenia Is Not Only A Human Fact But Also A Possibility For Thought” (Deleuze 1994, 148), Thus Expanding The Term Into A Philosophical Concept, Beyond Its Clinical Application. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative for models kindly see one of the papers in the series, procrastinated due to restrictions on space. The Dissolution Of The Subject And Its Implications For Society Is The Theme Of Anti-Oedipus: Capitalism And Schizophrenia, Which Deleuze Published With Félix Guattari In 1972 (English 1983). The Book, In Large Part, Is Written Against An Established Intellectual Orthodoxy Of The Political Left In France During The 1950s And 1960s, An Orthodoxy Consisting Of Marx, Freud, And Structuralist Concepts Applied To Them By Louis Althusser And Jacques Lacan. Deleuze And Guattari Argue That This Mixture Is Still Limited By Representational Thinking, Including Concepts Of Production Based Upon Lack, And Concepts Of Alienation Based Upon Identity And Negation. Furthermore, The Oedipus Concept In Psychoanalysis, They Say, Institutes A Theater Of Desire In Which The Psyche Is Embedded In A Family Drama Closed Off From The Extra-Familial And Extra-Psychic Forces At Work In Society. They Characterize These Forces As “Desiring Machines” Whose Function Is To Connect, Disconnect, And Reconnect With One Another Without Meaning Or Intention. The Authors Portray Society As A Series Of “Territorializations” Or Inscriptions Upon The “Body Without Organs, “Or The Free-Flowing Matter Of Intensive Qualities Filling Space In Their Varying Degrees. The First Inscriptions Are Relations Of Kinship And Filiation Structuring Primitive Societies, Often Involving The Marking And Scarring Of Human Bodies. As An Interruption and Encoding of “Flows,” The Primitive Inscriptions Constitute a Nexus of Desiring Machines, Both Technical and Social, Who’s Elements Are Humans and Their Organs. The Full Body Of Society Is The Sacred Earth, Which Appropriates To Itself All Social Products As Their Natural Or Divine Precondition, And To Whom All Members Of Society Are Bound By Direct Filiation (Deleuze 1983b, 141-42). These First Inscriptions Are Then De-Territorialized And Re-Coded By The “Despotic Machine,” Establishing New Relations Of Alliance And Filiation Through The Body Of The Ruler Or Emperor, Who Alone Stands In Direct Filiation To The Deity (Deleuze 1983b, 192) And Who Institutes The Mechanism Of The State Upon Pre-Existing Social Arrangements. Finally, Capitalism De-Territorializes The Inscriptions Of The Despotic Machine And Re-Codes All Relations Of Alliance And Filiation Into Flows Of Money (Deleuze 1983b, 224-27). The Organs Of Society And The State Are Appropriated Into The Functioning Of Capital, And Humans Become Secondary To The Filiation Of Money With Itself. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due

To Restrictions On Space. Deleuze And Guattari See In The Capitalist Money System "An Axiomatic Of Abstract Quantities That Keeps Moving Further And Further In The Direction Of The Deterritorialization Of The Socius" (Deleuze 1983a, 33), Which Is To Say That Capital Is Inherently Schizophrenic. However, Because Capital Also Re-Territorializes All Flows Into Money, Schizophrenia Remains Capitalism's External Limit. Nevertheless, It Is Precisely That Limit Against Which Thinking Can Subject Capitalism To Philosophical Critique. Psychoanalysis, They Say, Is Part Of The Reign Of Capital Because It Re-Territorializes The Subject As "Private" And "Individual," Instituting Psychic Identity Through Images Of The Oedipal Family. However, The Oedipal Triangle Is Merely A Representational Simulacrum Of Kinship And Filiation, Re-Coded Within A System Of Debt And Payment. In This System, They Insist, Flows Of Desire Have Become Mere Representations Of Desire, Cut Off From The Body Without Organs And The Extra-Familial Mechanisms Of Society. A Radical Critique Of Capital Cannot Therefore Be Accomplished By Psychoanalysis, But Requires A Schizoanalysis "To Overturn The Theater Of Representation Into The Order Of Desiring-Production" (Deleuze 1983b, 271). Here, The Authors See A Revolutionary Potential In Modern Art And Science, Where, In Bringing About The "New," They Circulate De-Coded And De-Territorialized Flows Within Society Without Automatically Re-Coding Them Into Money (Deleuze 1983a, 379). In This Revolutionary Aspect, Anti-Oedipus Reads As A Statement Of The Desire That Took To The Streets Of Paris In May Of 1968, And Which Continues, Even Now, To Make Itself Felt In Intellectual Life. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. The Term "Deconstruction," Like "Postmodernism," Has Taken On Many Meanings In The Popular Imagination. However, In Philosophy, It Signifies Certain Strategies For Reading And Writing Texts. The Term Was Introduced Into Philosophical Literature In 1967, With The Publication Of Three Texts By Jacques Derrida: Of Grammatology (English 1974), Writing And Difference (English 1978), And Speech And Phenomena (English 1973). This So-Called "Publication Blitz" Immediately Established Derrida As A Major Figure In The New Movement In Philosophy And The Human Sciences Centered In Paris, And Brought The Idiom "Deconstruction" Into Its Vocabulary. Derrida And Deconstruction Are Routinely Associated With Postmodernism, Although Like Deleuze And Foucault, He Does Not Use The Term And Would Resist Affiliation With "-Isms" Of Any Sort. Of The Three Books From 1967, Of Grammatology Is The More Comprehensive In Laying Out The Background For Deconstruction As A Way Of Reading Modern Theories Of Language, Especially Structuralism, And Heidegger's Meditations On The Non-Presence Of Being. It Also Sets Out Derrida's Difference With Heidegger Over Nietzsche. Where Heidegger Places Nietzsche Within The Metaphysics Of Presence, Derrida Insists That "Reading, And Therefore Writing, The Text Were For Nietzsche 'Originary' Operations," (Derrida 1974, 19), And This Puts Him At The Closure Of Metaphysics (Not The End), A Closure That Liberates Writing From The Traditional Logos, Which Takes Writing To Be A Sign (A Visible Mark) For Another Sign (Speech), Whose "Signified" Is A Fully Present Meaning. This Closure Has Emerged, Says Derrida, With The Latest Developments In Linguistics, The Human Sciences, Mathematics, And Cybernetics, Where The Written Mark Or Signifier Is Purely Technical, That Is, A Matter Of Function Rather Than Meaning. Precisely The Liberation Of Function Over Meaning Indicates That The Epoch Of What Heidegger Calls The Metaphysics Of Presence Has Come To Closure, Although This Closure Does Not Mean Its Termination. Just As In The Essay "On The Question Of Being" (Heidegger 1998, 291-322) Heidegger Sees Fit To Cross Out The Word "Being," Leaving It Visible, Nevertheless, Under The Mark, Derrida Takes The Closure Of Metaphysics To Be Its "Erasure," Where It Does Not Entirely Disappear, But Remains Inscribed As One Side Of A Difference, And Where The Mark Of Deletion Is Itself A Trace Of The Difference That Joins And Separates This Mark And What It Crosses Out. Derrida Calls This Joining And Separating Of Signs Différance (Derrida 1974, 23), A Device That Can Only Be Read And Not Heard When Différance And Différence Are Pronounced In French. The "A" Is A Written Mark That Differentiates Independently Of The Voice, The Privileged Medium Of Metaphysics. In This Sense, Différance As The Spacing Of Difference, As Archi-Writing, Would Be The Gram Of Grammatology. However, As Derrida Remarks: "There Cannot Be A Science Of Difference Itself In Its Operation, As It Is Impossible To Have A Science Of The Origin Of

Presence Itself, That Is To Say Of A Certain Non-Origin" (Derrida 1974, 63). Instead, There Is Only The Marking Of The Trace Of Difference, That Is, Deconstruction. Because At Its Functional Level All Language Is A System Of Differences, Says Derrida, All Language, Even When Spoken, Is Writing, And This Truth Is Suppressed When Meaning Is Taken As An Origin, Present And Complete Unto Itself. Texts That Take Meaning Or Being As Their Theme Are Therefore Particularly Susceptible To Deconstruction, As Are All Other Texts Insofar As They Are Conjoined With These. For Derrida, Written Marks Or Signifiers Do Not Arrange Themselves Within Natural Limits, But Form Chains Of Signification That Radiate In All Directions. As Derrida Famously Remarks, "There Is No Outside-Text" (Derrida 1974, 158), That Is, The Text Includes The Difference Between Any "Inside" Or "Outside." A Text, Then, Is Not A Book, And Does Not, Strictly Speaking, Have An Author. On The Contrary, The Name Of The Author Is A Signifier Linked With Others, And There Is No Master Signifier (Such As The Phallus In Lacan) Present Or Even Absent In A Text. This Goes For The Term "Différance" As Well, Which Can Only Serve As A Supplement For The Productive Spacing Between Signs. Therefore, Derrida Insists That "Différance Is Literally Neither A Word Nor A Concept" (Derrida 1982, 3). Instead, It Can Only Be Marked As A Wandering Play Of Differences That Is Both A Spacing Of Signifiers In Relation To One Another And A Deferral Of Meaning Or Presence When They Are Read. How, Then, Can Différance Be Characterized? Derrida Refuses To Answer Questions As To "Who" Or "What" Differs, Because To Do So Would Suggest There Is A Proper Name For Difference Instead Of Endless Supplements, Of Which "Différance" Is But One. Structurally, This Supplemental Displacement Functions Just As, For Heidegger, All Names For Being Reduce Being To The Presence Of Beings, Thus Ignoring The "Ontological Difference" Between Them. However, Derrida Takes The Ontological Difference As One Difference Among Others, As A Product Of What The Idiom "Différance" Supplements. As He Remarks: "Différance, In A Certain And Very Strange Way, (Is) 'Older' Than The Ontological Difference Or Than The Truth Of Being" (Derrida 1982, 22). Deconstruction, Then, Traces The Repetitions Of The Supplement. It Is Not So Much A Theory About Texts As A Practice Of Reading And Transforming Texts, Where Tracing The Movements Of Différance Produces Other Texts Interwoven With The First. While There Is Certain Arbitrariness In The Play Of Differences That Result, It Is Not The Arbitrariness Of A Reader Getting The Text To Mean Whatever He Or She Wants. It Is A Question Of Function Rather Than Meaning, If Meaning Is Understood As A Terminal Presence, And The Signifying Connections Traced In Deconstruction Are First Offered By The Text Itself. A Deconstructive Reading, Then, Does Not Assert Or Impose Meaning, But Marks Out Places Where The Function Of The Text Works Against Its Apparent Meaning, Or Against The History Of Its Interpretation. Hyperreality is closely related to the concept of the simulacrum: a copy or image without reference to an original. In postmodernism, hyperreality is the result of the technological mediation of experience, where what passes for reality is a network of images and signs without an external referent, such that what is represented is representation itself. In Symbolic Exchange and Death (1976) (English 1993), Jean Baudrillard uses Lacan's concepts of the symbolic, the imaginary, and the real to develop this concept while attacking orthodoxies of the political Left, beginning with the assumed reality of power, production, desire, society, and political legitimacy. Baudrillard argues that all of these realities have become simulations, that is, signs without any referent, because the real and the imaginary have been absorbed into the symbolic Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. Magic to Integral. In Fact, One Can Be At, Say, The Pluralistic Stage, And Experience Several Core Characteristics Of The Entire Sequence Of State-Stages (Gross To Subtle To Causal To Witnessing To Nondual), Although, Of Course, The Entire Sequence, Including Nondual Suchness, Will Be Interpreted In Pluralistic Terms. This Is Unfortunate In Many Ways—Interpreting Dharma In Merely Pluralistic Terms (Or Mythic Terms, Or Rational, And So On)—Because It Is So Ultimately Reductionistic; But It Happens All The Time, Given The Relative Independence Of States And Structures At 1st And 2nd Tier. Supermind, On The Other Hand, As A Basic Structure-Rung (Conjoined With Nondual Suchness) Can Only Be Experienced Once All The Previous Junior Levels Have Emerged And Developed, And As In All Structure Development, Stages Cannot Be Skipped. Therefore, Unlike Big Mind,

Supermind Can Only Be Experienced After All 1st-, 2nd-, and 3rd-Tier Junior Stages Have Been Passed Through. While, As Genpo Roshi Has Abundantly Demonstrated, Big Mind State Experience Is Available To Virtually Anybody At Almost Any Age (And Will Be Interpreted According To The View Of Their Current Stage), Supermind Is An Extremely Rare Recognition. Supermind, As The Highest Structure-Rung To Date, Has Access To All Previous Structures, All The Way Back To Archaic—And The Archaic Itself, Of Course, Has Transcended And Included, And Now Embraces, Every Major Structural Evolution Going All The Way Back To The Big Bang. (A Human Being Literally Enfolds And Embraces All The Major Transformative Unfoldings Of The Entire Kosmic History—Strings To Quarks To Subatomic Particles To Atoms To Molecules To Cells, All The Way Through The Tree Of Life Up To Its Latest Evolutionary Emergent, The Triune Brain, The Most Complex Structure In The Known Natural World.) Supermind, In Any Given Individual, Is Experienced As A Type Of "Omniscience"—The Supermind, Since It Transcends And Includes All Of The Previous Structure-Rungs, And Inherently Is Conjoined With The Highest Nondual Suchness State, Has A Full And Complete Knowledge Of All Of The Potentials In That Person. It Literally "Knows All," At Least For The Individual." ― Ken Wilber, The Fourth Turning: imagining the Evolution of an Integral Buddhism (fopr models please see one of the papers in the series). As Lyotard Argues, Aesthetic Judgment Is The Appropriate Model For The Problem Of Justice In Postmodern Experience Because We Are Confronted With A Plurality Of Games And Rules Without A Concept Under Which To Unify Them. Judgment Must Therefore Be Reflective Rather Than Determining. Furthermore, Judgment Must Be Aesthetic Insofar As It Does Not Produce Denotative Knowledge About A Determinable State Of Affairs, But Refers To The Way Our Faculties Interact With Each Other As We Move From One Mode Of Phrasing To Another, I.E. The Denotative, the Prescriptive, the Performative, the Political, the Cognitive, the Artistic, Etc. In Kantian Terms, This Interaction Registers As An Aesthetic Feeling. Where Kant Emphasizes The Feeling Of The Beautiful As A Harmonious Interaction Between Imagination And Understanding, Lyotard Stresses The Mode In Which Faculties (Imagination And Reason,) Are In Disharmony, I.E. The Feeling of the Sublime. For Kant, The Sublime Occurs When Our Faculties Of Sensible Presentation Are Overwhelmed By Impressions Of Absolute Power And Magnitude, And Reason Is Thrown Back Upon Its Own Power To Conceive Ideas (Such As The Moral Law) Which Surpass The Sensible World. For Lyotard, However, The Postmodern Sublime Occurs When We Are Affected By A Multitude Of Unpresentables Without Reference To Reason As Their Unifying Origin. Justice, Then, Would Not Be A Definable Rule, But Ability To Move And Judge Among Rules In Their Heterogeneity And Multiplicity. In This Respect, It Would Be More Akin To The Production Of Art Than A Moral Judgment In Kant's Sense. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative. The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars of Xian Shou Say, "It Is Arising From Primal Nature", And the Scholars of Chan (Zen) Say, "It Is Nature That Causes the Rising of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). In Other Words, All Dharma Arises From Causes and Conditions, and All Dharma Is Empty In Nature. The Law of Dependent Origination (Existence) and the Nature of Emptiness Is Neither the Same nor Different. They Exist Mutually. The Truth Of "Sunyata" And "Existence", And "Nature" And "Phenomena" Are Not In Conflict With Each Other. Unlike The Scholars Of The Dharmalaksana Sect Who Explain The Dharma Only From The Aspect Of Dependent Origination, Or The Scholars Of Dharma-Nature That Explain The Existence Of Dharma Only From The Aspect Of Dharma-Nature, The Scholars Of Madhyamika Explain The Truth Of The Dharma From Both Aspects. Hence This Is Called The Middle Path Which Does Not Incline To Either Side. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). For Stoics, Causes Involved Reference To The Depths, But Effects At The Surface Could Also Have Relations Among Themselves. This Permitted The Distinction To Be Drawn Between Destiny And Necessity—The Stoics Wanted To Affirm Destiny And Deny Necessity. In The First Place, Effects

Express Causes, But Expressions Of Relations [Rather Than Necessity] Connects Those Effects. Those Relations May Be Described As Compatibility Or Incompatibility, Conjunction Or Disjunction. These Are Not Causal Relations Themselves, But Represent 'An Aggregate Of Noncausal Correspondences Which Form A System Of Echoes, Of Presumptions And Resonances, A System Of Signs—In Short, An Expressive Quasi Causality And Not At All A Necessitating Causality' (170). This Need Not Involve Contradiction, Which Is Applying To Events Rules That Really Only Applied To Logic And Argument. There Can Be Incompatability Without Contradiction, A Noncausal Correspondence. Leibniz Described Incompossible Worlds [Delanda Is Very Clear On This Too]. Only Impossible Events Contradict Possible Ones. Events Can Be Compossible [Roughly, They Have Predictable and Predicative Future and Past Events]. For Deleuze, It Is A Matter Of 'The Convergence Of Series Which Singularities Of Events Form As They Stretch Themselves Out Over Lines Of Ordinary Points. Incompossibility Must Be Defined By the Divergence of Such Series' (171). Such A Notion Is Essential To Any Theory of Sense. We Should Not See Divergence As A Matter Of Exclusion, As Leibniz Did [Since God Chose Actual Events]. Divergent And Disjunction Can Both Be Positive, While Preserving Differences. In Fact, Differences Are Crucial, Preserving The Distance Between Objects While Affirming That They Are Related. This 'Permits The Measuring Of Contraries Through Their Finite Difference Instead Of Equating Difference With A Measureless Contrariety' (173). It Is Contradiction Which Is The Special Case. Difference Here Is A Topological Term Relating To Distance On Surfaces Rather Than Depths. It Is Not Just A Matter Of Suggesting 'Some Unknown Identity Of Contraries (As In Commonplace In Spiritualist And Colorist Philosophy)' (173) [Take That St Pierre!]. An Example Is Nietzsche Arguing That Health And Sickness Can Both Inform Each Other; Act As Points Of View, Remembering That 'Things, Beings, Are Themselves Points Of View' (173). Divergence Does Not Mean Exclusion, And Disjunction Does Not Mean Separation. Connective Syntheses '(If…, Then)' Construct A Single Series; Conjunctive Series '(And)' Produces Convergent Series, But Disjunctive Series '(Or)' Produces A Divergence Series. Normally, Disjunction Helps Us To Criticise Synthesis, But Disjunction Can Still Be A Synthesis Itself, Despite Its Use In Logical Analysis [I Think What Is Going On Here Is Arguing That There Is A Difference Between 'Either/Or' In A Logical Sense, And 'One Or Two' In The Real Sense—The Latter Can Mean That Both Are Compossible. This Is The 'Communication Of Events' Rather Than The Logical Business Of Analysing Predicates (174)]. The Synthetic Disjunction Expresses The Paradox, With Divergence At The Centre. The Discussion of Paradoxes and Esoteric Words Above Are Examples: They Contract 'The Multitude of Divergent Series in the Successive Appearance of A Single One' (175). There Is A Difference Again Between the Subversion of the Present and Simple Identity by Depths, And Operations at the Surface. By Considering The Depths, We Encounter Infinite Identities [As Events Become Examples Of Deeper Categories, Wholes?]. At The Surface Events Communicate With Each Other Directly Through Maintaining Distance And By Affirming Disjunctions. Disjunction Threatens The Identity Of The Self, And Helps Us To See The Self As 'So Many Impersonal And Pre-Individual Singularities' [Connected Through Disjunctive Synthesis. Hence the Importance of Heterogeneity] (175). The Normal Concept Of The Self Implies Some Connected Series, 'But When Disjunction Accedes To The Principle Which Gives To It A Synthetic And Affirmative Value, The Self, The World, And God Share In A Common Death' (176). [There Is Also A Point That Divergent Series Explain And Also Exceed Normal Conjunctive And Connective Series]. In The Usual Conception, 'The Self Is the Principle of Manifestation, In Relation To the Proposition, the World Is the Principle of Denotation, And God the Principle of Signification' (176). But The Theory Of Sense Here Says That It Emanates From Nonsense, From Paradox, And From The 'Eternally Decentred Ex-Centric Centre' (176). This Position 'Does Not Tolerate the Subsistence of God as an Original Individuality, Nor The Self As The Person, Nor The World As An Element Of The Self And As God's Product. The Divergence Of The Affirmed Series Forms A "Chaosmos" And No Longer A World; The Aleatory Point Which Traverses Them Forms A Counter self, And No Longer A Self' (176). There Is No Centre But Only 'Pure Events Which The Instant, Displaced Over The Line [Of Aion], Goes On Dividing Into Already Past And Yet To Come. Nothing Other Than The Event Subsists… This Communicates With Itself through Its Own Distance and Resonates Across All of Its Disjuncts' (176). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark

Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis Twenty-fourth series of the communication of events. The Buddha Always Used The Terms Void, No Rising And Falling, Calmness And Extinction To Explain The Profound Meaning Of Sunyata And Cessation. For Example, Sunyata And The State Of Nirvana Where There Is No Rising Nor Falling, Are Interpreted By Most People As A State Of Non-Existence And Gloom. They Fail To Realise That Quite The Opposite, Sunyata Is Of Substantial And Positive Significance. The Sutras Often Use The Word "Great Void" To Explain The Significance Of Sunyata. In General, We Understand The "Great Void" As Something That Contains Absolutely Nothing. However, From A Buddhist Perspective, The Nature Of The "Great Void" Implies Something Which Does Not Obstruct Other Things, In Which All Matters Perform Their Own Functions. Materials Are Form, Which By Their Nature, Imply Obstruction. The Special Characteristic Of The "Great Void" Is Non-Obstruction. The "Great Void" Therefore, Does Not Serve As An Obstacle To Them. Since The "Great Void" Exhibits No Obstructive Tendencies, It Serves As The Foundation For Matter To Function. In Other Words, If There Was Neither “Great Void” Nor Characteristic Of Non-Obstruction, It Would Be Impossible For The Material World To Exist And Function. The "Great Void" Is Not Separated From The Material World. The Latter Depends On The Former. We Can State That the Profound Significance of Sunyata and the Nature of Sunyata in Buddhism highlight The "Great Void’s" Non-Obstructive Nature. Sunyata Does Not Imply The "Great Void". Instead, It Is The Foundation Of All Phenomena (Form And Mind). It Is The True Nature Of All Phenomena, And It Is The Basic Principle Of All Existence. In Other Words, If The Universe’s Existence Was Not Empty Nor Impermanent, Then All Resulting Phenomena Could Not Have Arisen Due To The Co-Existence Of Various Causes And There Would Be No Rising Nor Falling. The Nature Of Sunyata Is Of Positive Significance. Calmness And Extinction Are The Opposite Of Rising And Falling. They Are Another Way To Express That There Is No Rising And Falling. Rising And Falling Are The Common Characteristics Of Worldly Existence. All Phenomena Are Always In The Cycle Of Rising And Falling. However, Most People Concentrate On Living (Rising). They Think That The Universe And Life Are The Reality Of A Continuous Existence. Buddhism On The Other Hand, Promotes The Value Of A Continuous Cessation (Falling). This Cessation Does Not Imply That It Ceases To Exist Altogether. Instead, It Is Just A State In The Continuous Process Of Phenomena. In This Material World, Or What We May Call This "State Of Existence", Everything Eventually Ceases To Exist. Cessation Is Definitely The Home Of All Existences. Since Cessation Is The Calm State Of Existence And The Eventual Refuge Of All Phenomena, It Is Also The Foundation For All Activities And Functions. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). You Are In Your Own Film A Subjective Experience Where You Are Both Actor And Director Writing Your Script As You Go Along. You Are Both The Audience And The Actor A Near State Of Pure Consciousness. You Are An Individual General Ledger With A Place In Collective General Ledger And Cosmic General Ledger With Celestial Brahman Anti Brahman Simulating The Events Which Appear To You On The Screen Of Consciousness. Sensations Are Disruptions Within Actual Identifications; They Are Signs Of The Expression Of Ideas And Intensities In Actual Structures. There Is No Denying the Difficulty of Deleuze’s Account of Reality – Necessary for an Understanding of the Role Played by Kantian Arguments. It May Therefore Be Helpful To Think In Terms Of An Example. You Have Waited Years To See A Loved One Again, Tarrying With Dimming Memories And Shifts In Longings And Needs. A Meeting Has Been Arranged. The Opening Gestures, Words And Images Bring Together Fixed Memories And Preconceptions (Actual Identifications) With Feelings That Destroy those (Actual Sensations Of Disappointment, Excitement, Bemusement, And Renewed Passion). The Power Of These Sensations Cannot Come Solely From The Disrupted Identities; Rather, They Are Expressions Of Deeper Charges (Virtual Intensities). But, Beyond Local Identifications And Their More And More Distant Actual Effects, Imagined And Even Undreamed Worlds Of Ideas Are Changed Through The Meeting. According To Deleuze, These Are Not Possible Worlds, But Virtual Ones That Ground Actual Identifications. The Actual Meeting Is Accompanied By A Virtual One, Where Ideas Take Shape And Acquire Significance Through Shifts In Relations

Between Intensities. These Shifts Light Up Actual Situations In New Ways And Determine Different Ideas To Come Into Play. The Meeting Is A Failure. Intensities Push Feelings of Disappointment to the Fore, Destroying One World of Ideas and Perhaps Determining a Colder One with Greater Clarity. The Transversal Thought of Gilles Deleuze: Encounters and Influences James Williams. For models please see one of the papers in the series. Models are procrastinated due to spatial restrictions. You Can Stir The Conscious State And Consequentially Connect All The Three States. As Lyotard Argues, Aesthetic Judgment Is The Appropriate Model For The Problem Of Justice In Postmodern Experience Because We Are Confronted With A Plurality Of Games And Rules Without A Concept Under Which To Unify Them. Judgment Must Therefore Be Reflective Rather Than Determining. Furthermore, Judgment Must Be Aesthetic Insofar As It Does Not Produce Denotative Knowledge About A Determinable State Of Affairs, But Refers To The Way Our Faculties Interact With Each Other As We Move From One Mode Of Phrasing To Another, I.E. The Denotative, the Prescriptive, the Performative, the Political, the Cognitive, the Artistic, Etc. In Kantian Terms, This Interaction Registers As An Aesthetic Feeling. Where Kant Emphasizes The Feeling Of The Beautiful As A Harmonious Interaction Between Imagination And Understanding, Lyotard Stresses The Mode In Which Faculties (Imagination And Reason,) Are In Disharmony, I.E. The Feeling of the Sublime. For Kant, The Sublime Occurs When Our Faculties Of Sensible Presentation Are Overwhelmed By Impressions Of Absolute Power And Magnitude, And Reason Is Thrown Back Upon Its Own Power To Conceive Ideas (Such As The Moral Law) Which Surpass The Sensible World. For Lyotard, However, The Postmodern Sublime Occurs When We Are Affected By A Multitude Of Unpresentables Without Reference To Reason As Their Unifying Origin. Justice, Then, Would Not Be A Definable Rule, But Ability To Move And Judge Among Rules In Their Heterogeneity And Multiplicity. In This Respect, It Would Be More Akin To The Production Of Art Than A Moral Judgment In Kant's Sense. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative. In Dissolving All Social Phenomena In The Acid Bath Of Power And Domination, Foucault Prevents Critical Theory From Drawing Crucial Distinctions, Such As Those "Between Just And Unjust Social Arrangements, Legitimate And Illegitimate Uses Of Political Power, Strategic And Cooperative Interpersonal Relations, Coercive And Consensual Measures" (McCarthy 1991: 54). One Cannot Say, For Example, That One Regime Of Power Is Any Better Or Worse Than Another, Only That They Are Different -- "Another Power, Another Knowledge" (Foucault 1979: 226). Since Ruling Powers Attempt To Erase Such Distinctions, Or To Present Injustice As Justice, Falsehood As Truth, And Domination As Freedom, Foucault's Position Unwittingly Supports The Mystifications Of Orwellian Doublespeak, Now More Rife Than Ever (See Kellner 2001), And Blocks The Discriminations Necessary For Social Critique. If There Are No Standards Or Right, Then, With Thrasymacus And Hobbes, We Can Conclude Might Is As Right As Anything. There Can Be No Ideology Critique Where There Is No Distinction Between True And False, And any Social Or Moral Critique Without A Distinction Between Right And Wrong. The Evaluative Character Of Foucault's Own Work Is Not Any Less Normative For His Refusal To Explicitly Confront It. The Problem Becomes Glaring In His Later Work Where He Employs Normative Terms Such As Liberty And Autonomy, But Fails To State What We Should Be Free For. Foucault's Anti-Normative Stance Therefore Forces Him into Self-Defeating Value Neutrality. Richard Rorty, The Attack on Theory, And Renunciation of Radical Politics (for models see one of the papers in the series). Our Environment Can Either Open Up or Constrict Possibilities for Spiritual and Intellectual Growth. In The Politics Of Experience Laing Offers A Fine Passage From Erving Goffman’s Studies Encounters: There Seems To Be No Agent More Effective Than Another Person In Bringing A World For Oneself Alive, Or, By A Glance, A Gesture, Or A Remark, Shriveling Up The Reality In Which One Is Lodged. "The Anxiety Of Meaninglessness Is Anxiety About The Loss Of An Ultimate Concern, Of A Meaning Which Gives Meaning To All Meanings. This Anxiety Is Aroused By The Loss Of A Spiritual Center, Of An Answer, However Symbolic And Indirect, To The Question Of The Meaning Of Existence." Philosophy And Depression By Tim Ruggiero -- Paul Tillich, The Courage To Be (1952) As Quoted In Depression And The Threat Of Nonbeing Tim Ruggiero, September 4, 2001 (Philosophical Society.Com).The Dissolution Of The Subject And Its Implications For Society Is The Theme Of Anti-Oedipus: Capitalism And Schizophrenia, Which Deleuze

Published With Félix Guattari In 1972 (English 1983). The Book, In Large Part, Is Written Against An Established Intellectual Orthodoxy Of The Political Left In France During The 1950s And 1960s, An Orthodoxy Consisting Of Marx, Freud, And Structuralist Concepts Applied To Them By Louis Althusser And Jacques Lacan. Deleuze And Guattari Argue That This Mixture Is Still Limited By Representational Thinking, Including Concepts Of Production Based Upon Lack, And Concepts Of Alienation Based Upon Identity And Negation. Furthermore, The Oedipus Concept In Psychoanalysis, They Say, Institutes A Theater Of Desire In Which The Psyche Is Embedded In A Family Drama Closed Off From The Extra-Familial And Extra-Psychic Forces At Work In Society. They Characterize These Forces As "Desiring Machines" Whose Function Is To Connect, Disconnect, And Reconnect With One Another Without Meaning Or Intention. The Authors Portray Society As A Series Of "Territorializations" Or Inscriptions Upon The "Body Without Organs, "Or The Free-Flowing Matter Of Intensive Qualities Filling Space In Their Varying Degrees. The First Inscriptions Are Relations Of Kinship And Filiation Structuring Primitive Societies, Often Involving The Marking And Scarring Of Human Bodies. As An Interruption and Encoding of "Flows," The Primitive Inscriptions Constitute a Nexus of Desiring Machines, Both Technical and Social, Who's Elements Are Humans and Their Organs. The Full Body Of Society Is The Sacred Earth, Which Appropriates To Itself All Social Products As Their Natural Or Divine Precondition, And To Whom All Members Of Society Are Bound By Direct Filiation (Deleuze 1983b, 141-42). These First Inscriptions Are Then De-Territorialized And Re-Coded By The "Despotic Machine," Establishing New Relations Of Alliance And Filiation Through The Body Of The Ruler Or Emperor, Who Alone Stands In Direct Filiation To The Deity (Deleuze 1983b, 192) And Who Institutes The Mechanism Of The State Upon Pre-Existing Social Arrangements. Finally, Capitalism De-Territorializes The Inscriptions Of The Despotic Machine And Re-Codes All Relations Of Alliance And Filiation Into Flows Of Money (Deleuze 1983b, 224-27). The Organs Of Society And The State Are Appropriated Into The Functioning Of Capital, And Humans Become Secondary To The Filiation Of Money With Itself. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. "The Vanity Of Existence Is Revealed In The Whole Form Existence Assumes: In The Infiniteness Of Time And Space Contrasted With The Finiteness Of The Individual In Both; In The Fleeting Present As The Sole Form In Which Actuality Exists; In The Contingency And Relativity Of All Things; In Continual Becoming Without Being; In Continual Desire Without Satisfaction; In The Continual Frustration Of Striving Of Which Life Consists." (For Models See One of the Papers in the Series, Which Are Procrastinated Due to Spatial Constraints) -- Arthur Schopenhauer, "On the Vanity of Existence" as quoted in Philosophy and Depression by Tim Ruggiero. "There Is Little Conjunction Of Truth And Social 'Reality'. Around Us Are Pseudo-Events, To Which We Adjust With A False Consciousness Adapted To See These Events As True And Real, And Even As Beautiful. In The Society Of Men The Truth Resides Now Less In What Things Are Than In What They Are Not. Our Social Realities Are So Ugly If Seen In The Light Of Exiled Truth, And Beauty Is Almost No Longer Possible If It Is Not A Lie." (For Models See One of the Papers in the Series, Which Are Procrastinated Due to Spatial Constraints) -- R.D. Laing, the Politics of Experience. [We're going to Use the Idea of a Game to Further Illustrate How the Virtual Is Imminent in the Actual. Why This Example? To Have A Go At The Metaphor Of Games In Philosophy? To Instate Lewis Carroll As Some Great Philosopher?] The Actual Games Have A Series Of Rules About Playing And Outcomes That Limit The Influence Of Chance. It's Possible to Invert These Rules to arrive at an Ideal Game, With No Rules, And with the Full Operation of Chance [Which Describes the Virtual, I Suppose]. Carroll Describe Some. We Also Learn About Time And How It Operates. In Each Throw Of The Dice, Singularities Are Distributed As A 'Nomadic And Non Sedentary Distribution, Wherein Each System Of Singularities Communicate And Resonates With The Others, Being At Once Implicated By The Others And Implicating Them In The Most Important Cast' (60). This Game Can Only Be Played In Thought [Since Any Empirical Reality Limits Chance. Chance Is Fixed By The Mechanisms Of Say The Roulette Wheel]. Infinite Time Can Be Compared With Finite [Or Metric] Time. They Are Linked Together, Although It Is Useful To Consider Both. In Finite Time, The Present Exists And Features Mixtures Of Matter Which Are Temporally

Realized And Can Take On A [Empirical] Depth. Any Other Kind of Time, Only the Past and Future Exist, and Colonise Moments in the Presence. It Offers An Unlimited Kind Of Time, Since The Gap Between Past And Future Is Endlessly Subdivided And Never Closed. It Is Neutral And Incorporeal. Here, the Present Is A 'Pure Mathematical Instant' (62). It Is The Time Of Events Which Are Always Things Which Have Just Happened And Things Which Are About To Happen 'Never Something Which Is Happening' (63). As With The Operations Of Chance In A Game, Specific Events Or Throws Occupy A Time Which Is Smaller Than That Which Can Be Thought [That Is The Unfolding Of Singularities Which Happen Instantly?], And Also The Maximum Notion Of Time [I Can Only Grasped This By Thinking Of Specific Chances Being Understood By Their Location In Whole Sequences Of The Infinite Throws Of The Dice]. [There Is Some Stuff About The Eternal Return Here Too Which I Don't Understand—63 – 64]. 'The Aion Is The Straight Line Traced By The Aleatory [Unpredictable, Accidental] Point' [Search Me] Which Means That Events The Can Communicate With All The Others, As In Events Of The Aion Itself. This Sort Of Unity Is Not The Same Is An Empirical Corporeal Unity. The Aion Underpins or Circulates Within the Series of Events. This Also Helps Explain The Connection Between The Sense Of Propositions And The Sense Of States Of Affairs. [Weird. Making The Strongest Possible Case For Chance, I Assume. Denying Essence or Subjective Intent. Makes Empirical {Including Social} Science As Limited To The Finite And Metric, But Requiring Thought Experiments To Move To The 'Pure' Level? So Says Delanda Anyway, I Think]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Please See One Of The papers In The Series Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable and Ascribable To Constraints on Space Accomodation Model Is Provided In Some Paper, Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. [Particularly Dense And Baffling Stuff, Attempting To Show How The Subjective World Is A Derivative Of The Objective Play Of Singularities. Makes More Sense With Concrete And Familiar Examples As In Delanda's Stuff On Biological Membranes And Their Importance In Embryology, Instead Of The Abstract 'Surfaces' Of This Account]] Individuals Are Capable Of Infinite Description, Limited Only By Their Bodies To Express. Persons [Human Beings Based the More Abstract Notion of an Individual?] Can Only Produce Propositions To Describe The World To A Limited Extent. Both, However, Are 'Ontological Propositions' [That Is They Create A Kind Of Reality?]. Multiple Classes And Systems Of Categories Are Not Produced By Propositions. Instead We Have To Look At Something That Now Produces Human Propositions Themselves, As 'Material Instances' (118). [So Human Ontological Activity Only Realises Sense Making In The Form Of Denotations, Manifestations, And Signification As Above?]. Denotations And The Others Are Interconnected As We Saw Above. There Is Also No Simple Connection Between, Say, 'The Individual And Denotation, The Person And Manifestation, Multiple Classes Or Variable Properties And Signification' (119) [For Similar Reasons—For Example, Signification Depends On The Good Sense Already Established By Individuation]. The Whole Complex Structure Is Produced By Both Ontological And Logical Genesis. Sense Operates On The Whole Structure. [We Can See This Both In The Fragility Of Sense And Its Tendency To Be Threatened By Nonsense, And Because The Alternative Is Unpalatable—Language And Sense-Making Would Be Based On Nothing But A 'Undifferentiated Abyss' (120)]. So Sense, 'In Its Organization of Aleatory and Singular Points, Problems and Questions, Series and Displacement' (120) Generates both Logical Propositions and Also 'the Objective Correlates' Of Propositions… The Denoted, the Manifested, and the Signified' (120). The Notion Of An Error Suggests This Although In A Confused Way. Normally We Think Of Error As A Matter Of Truth Or Falsity, When Propositions Are Formed And Tested. However, When We Consider Problems Instead Of Propositions That Offer Solutions, The Category Of Sense Emerges Strongly. [We Have Seen Above the Problem Is an Objective Matter of Structured Possibilities]. We Can See How Both Knowledge And The Known Are Produced By This Structure. [Problematics Are Further Discussed As Involving Particular Distributions Of Singularities In Space And In Time. As Problems Condense Out, So Do Solutions—'The Synthesis Of The Problem With Its Conditions

Engenders Propositions, Their Dimensions, And Their Correlates' (121)]. Sense Is Produced Or Expressed When Solutions, Expressed As Propositions, Correspond To Problems [Act As 'Instances Of A General Solution' (121)] It Is Common To Express Sense In An Interrogative Form [Although The Interrogative Also Includes A More Limited Operation As In The Closed Question]. Specific Questions And Solutions Are Already Determined By The Problematic, However—'The Problem In Itself Is The Reality Of A Genetic Element, The Complex Theme Which Does Not Allow It To Be Reduced To Any Propositional Thesis' (122). It Is A Mistake To Define A Problem In Terms Of Possible Solutions [Which Arise From Human Consciousness]. This Would Mean Us 'Confuse Sense with Signification, and… Conceive Of the Condition Only In the Image of the Conditioned' (122). [This Autonomous Constitution Of Problems Shows The Inadequacy Of Subjective Conceptions. Seeing Problems As Derived From Propositions Expressing Solutions Would Also Infringe The Neutrality Of Sense]. Problems Are Neutral Insofar As [Modes Of] Propositions Are Concerned. For Example 'A Circle Qua Circle Is Neither A Particular Circle, Nor A Concept Represented In An Equation… It Is Rather A Differential System To Which An Emission Of Singularities Corresponds' (123). Problems In This Sense Exist In Propositions But Also Allude To None Being, As Above. As A Result, Problems Are Neutral, That Is 'Independent of both the Negative and the Affirmative' (123). The Neutrality of Sense Means It Is Never Just the Echo ['the Double'] Of Propositions. Working With Propositions Can Only Lead To A Partial Understanding Of Sense. We Have To Develop Another Conception, Not Based On Propositions Or On Images Of Conventional Logical Thinking. Philosophy Must 'Purge The Transcendental Field Of All Resemblance' In Order To Avoid Trap Of Consciousness As The Origin Of The Transcendental (123). However, The Earlier Discussions Defined The Neutrality Of Sense As An Effect Produced By Corporeal Causes. Here, We Are Implying That It Arises From Its Genetic Power [To Produce Problematics] And This Relates To A Quasi Cause. Sense Is Produced By Bodies In A Way Which Presupposes This More General Kind Of Sense. The More General Kind Operates In A Different Way, Not Through Concepts Or Describing Mixtures. This Time It Is a Matter of Depth [Of Bodies] and the Effects of a Surface. The Pulsations Of Bodies Produce Surfaces In Particular Ways, Sometimes As A Minimum Energy Conserving Form [Delanda's Soap Bubble], Sometimes As A More Complex Structure Of Multiplied Differentiated Surfaces [Stretched, Emulsified, Absorbing Etc Are The Examples Given, P. 124]. Surfaces Are Produced By the 'Actions and Passions of Mixed Bodies' (124). Surfaces Have No Thickness Of Their Own, Which Permits Contact Between The Internal And The External. The Quasi Causes Play On These Surfaces, As A Kind Of 'Fictitious Surface Tension… A Force Exerting Itself On The Plane Of The Surface' (125). Singularities Are Condensed Extended And Reshuffled. These Surfaces Have An Existence In Actual Physics, And Also In Metaphysics—The Surface Becomes The Transcendental Field, The Border Between Bodies And Propositions. As Such, The Surface Becomes The 'Locus Of Sense And Expression' (125). Propositions And Bodies Are Actually Articulated On Surfaces, 'So That Sense Is Presented Both As That Which Happens To Bodies And That To Which Insists [Sic] In Propositions' (125). It Is In This Process Of 'Doubling Up' That Neutrality Arises [I Think The Argument Here Is That Neutrality Is Not Some Disembodied Quality But A Function Of Surfaces Which Enable 'The Continuity Of Reverse And Right Sides' (125). It Is The Indifference Of The Objective, Immune To Appeals From Human Subjectivity?]. The Surface Enables Sense To Be Distributed, As Both The Expressed In Propositions, And The Event In Bodies [Depends On This Very General Definition Of 'Expression' Again - More General Than The Use Above Where It Means Human Expression As In Manifestation Etc]. If The Surface Is Destroyed, Bodies' Fall Back Into The Depth, The 'Primary Order [Some Sort Of Natural Being Which Cannot Be Named Or Expressed?] Which Grumbles Beneath the Secondary Organisation of Sense' (125). But On The Surface, Sense Is Unfolded And Is Also Affected By Quasi Causes. This Sense In Turn Individuates and Determines Bodies, and Signification, and All the Propositions 'The Entire Tertiary Arrangement or the Object of the Static Genesis' (126). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth

Series Of The Static Ontological Genesis. Chronos And Aion Are Compared In Terms Of The Importance Of The Present As Opposed To The Past And The Future. For Chronos, The Present Represents The Main Interest, Although There Is 'The Relativity Of Presents Themselves In Relation To Each Other. God Experiences As Present That Which for Me Is Future or Past, Since I Live Inside More Limited Presents' (162). The Present Is Corporeal, A Matter of Mixtures [Imperfect for Humans and Perfect in the Divine Present for Stoics as Above]. The Present Limits The Action Of Bodies [Compare This With Bergsonian Stuff About The Present As A Cone—Here We Are Talking About Contraction And Dilation To Connect The Empirical With A Cosmic Present]. The Partial Mixtures Of Bodies In The Present Threaten To Subvert The Sufficiency Of The Notion. Stoics Had To Distinguish Between Good Or Bad Mixtures, For Example, Which Lead To Their Notion Of Cosmic Perfection, And To Aion. This Implies That Bodies Are Actually Nothing But Simulacra, And The Present Less Important Than The Future Or The Past. However, Chronos Represents the Only Kind Of Empirical Understanding [?] (164). For Aion, The Present Is A Mere Instant, Always Divided Into Past And Future. This Is A Different Way Of Subverting The Present As In Depth Metaphors [Some Of Which Appear To Linger In The Argument About Partial Mixtures Of Bodies In The Present?]. [Somehow], Aion Operates At The Surface, Since The Present Is Evaded In Favour Of The Instant Rather Than The Fathomless Depths. Aion Operates With Incorporeal Events, And Their Effects, Which Are Limitless And Infinite. Thus Aion Is 'The Eternal Truth Of Time: Pure Empty Form Of Time, Which Is Freed Itself Of Its Present Corporeal Content' (165) [I Think Delanda Is Much Clearer On The Differences Between Metric And Intensive Time]. Aion Is An Essential Element In The Development Of Language, Allowing Language To Escape Corporeal Determinations, Alluding To An Existence Outside Of The Present, Allowing Signification And Manifestation. [Once Having Escaped, Language Can Persist In Itself]. The Instant Demonstrates The Aleatory Point, Nonsense, And Quasi-Cause. It Is A Pure Abstraction And It 'Extracts Singularities From The Present, And From Individuals And Persons Which Occupy This Present' (166) [Because It Alludes To Future And Past, And Thereby Constitutes The Pure Event?]. Without The Notion Of Aion, We Would Only Be Left With Bodies And States Of Affairs, Not Language And Propositions. [The Argument Here Seems To Be That Language Necessarily Has A Future Element?]. The Surface Divides The Two Series Of States Of Affairs And Propositions, And Sense Can Now Relate To Propositions And Events [In The Form Of The Commentary On Events Mentioned Wayback At The Beginning?]. [There Is Some Sort Of Topological Connection Between Points, Lines And Surfaces, Presumably In Terms Of The Way In Which One Can Be 'Cut' From The One Above]. Actualizations Occur When Bodies, States Of Affairs And Mixtures Intersect At The Surface. This Is A Matter Of 'Imprisoning First Their Singularities Within The Limits Of Worlds, Individuals, And Persons' (167). However, There Is Always Something In Excess Of Actualizations, Which Alludes To The Quasi- Cause [Which Apparently Can Be Identified By Sages]. This Involves Analysing the 'Pure Perverse "Moment"', the Pure Operation—It Is Graspable At and As A Moment of Counter Actualization. Twenty-Third Series Of The Aionalexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Wild And Wacky Stuff Here Relating To Artaud's Work. I Would Normally Chop It Out Altogether, Except It Provides Some Context For The Emergence Of The Phrase 'Body Without Organs' That Crops Up So Prominently In Anti Oedipus Sense Is Fragile And Is Threatened By Nonsense. Sometimes Such Nonsense Can Destroy Everything, And We See This If We Switch From The Playful Portmanteaux Of Carroll To The Schizophrenic Writings Of Artaud. These Are Real Examples Of Nonsense, Instead Of The Artificial Ones Discussed By Philosophers. Artaud Apparently Disliked Carroll, and Rendered Jabberwocky in A Much More Challenging Schizophrenic Language. Carroll Is Too Superficial, Whereas Schizophrenia Reveals The Real Problems With Language. Another Work Is Discussed, By Wolfson, To Focus On The Duality Between Things And Words. This Takes The Form Of Someone [Schizophrenic?] Experiencing A Problem In Translating From One Language To The Other, Which Somehow Becomes Transposed Into An Anxiety

About Eating. A Schizophrenic Interlude Ensues Involving Associations Between Consonants As The Basis For Translation, And The Paradoxes That Emerge (85). The Same Basic 'Oral Duality (To Eat/To Speak)' Is Also Found In Carroll's Work, And In Artaud 's. However, In Caroll, Some Sense Is Retained Since This Duality Is Explored 'At The Surface' (86) [This Also Indicates, Apparently That The Operation Of Sense Of The Surface Shows Only A Quasi-Causal Relation Between Its Elements, Since There Are Incorporeal Elements Driving It]. For Artaud, The Classic Schizophrenic Symptoms Included The Absence Of Surface, Especially With Bodies. Apparently Freud Also Noticed This Tendency For Schizophrenics To See Their Body As 'Punctured By An Infinite Number Of Little Holes' (87). The Body Therefore Incorporates Everything Into Its Depths, Everything Becomes Corporeal And Physical. The Surface No Longer Limits The Extension Of The Body. 'Hence The Schizophrenic Manner Of Living The Contradiction: Either In The Deep Fissure Which Traverses The Body, Or In The Fragmented Parts Which Encase One Another And Spin About' (87). The World Loses Its Meaning And Sense [Because It Can No Longer Split Sensation Into A Signifying And Signified Separated By A Surface?] Words Become Physical And Affect Bodies, Or They Burst Into Components [Which Relates Back To The Wolfson Example]. Schizophrenics Experience 'A Pure Language–Affect' (88) [Sic --Affect Not Effect]. Schizophrenics Manage This By Overcoming The Effects Of Language, As In The Strange Translation Activity In The Wolfson Example. In Artaud's Case The Solution Was To Create Special Words Expressing 'Values Which Are Exclusively Tonic And Not Written' (88). 'To These Values A Glorious Body Corresponds, Being A New Dimension Of The Schizophrenic Body, An Organism About Parts Which Operates Entirely By Insufflations, Respiration, Evaporation, And Fluid Transmission (The Superior Body Or Body Without Organs Of Antonin Artaud)' (88). This Solution Can Never Be Complete Because There Can Never Be A Total Separation Between Sufferings ['Passion'] And [Remedial] Action, And Passion Can Be Reintroduced, And The Body Corrupted-- A Schizophrenic Body Is Therefore A Constant Mixture Of Two Actions Or Principles. Artaud Tries To Invent A New Language Which Cannot Be Decomposed And Thus Cannot Be Colonised, A Language Of 'Consonantal Guttural And Aspirated Overloads' (89). The Words Are Joined By Some Invented Principle, In This Case A 'Palatalized' One (89) Which Blurs The Consonants Together And Prevents Them Being Written Down. The Result Is 'So Many Active Howls In One Continuous Breath' (89) [Sounds Very Much Like Tara's Dadaist Tone Poems --Artaud's Example On P.83]]. These Words Are Often The Equivalent To Portmanteaux [Some Examples Are Given On Page 90]. Using These Words Can 'Enact A Chain Of Associations… In A Region Of Infra Sense, According To A Fluid And Burning Principle Which Absorbs And Reabsorbs Effectively The Sense As Soon As It Is Produced' (90). So Two Sorts Of Words Related To Two Sorts Of Bodies, One Fragmented And One Without Organs. There Are Also Two Theatres Or Two Types Of Nonsense Implied Here: One Where Ordinary Words Are Decomposed Into Nonsense, And One Where Tonic Elements Alone Form Nonsensical Words. They Are Produced By Things Happening Beneath The Surface, Unlike Carroll's Playful Superficiality. The Two Signifying and Signified Series Disappear, and Non Sense Engulfs Signifiers and Signified. There Is No Surface Division To Separate The Expressivity Of Words And The Attributes Of Actual Bodies [Which Regulates Ordinary Language]. In Schizophrenic Language There Is No Grammar Or Syntax Either, Although Both Are Preserved In Carroll. Nevertheless, It Is Artaud Who Has 'Discovered A Vital Body And The Prodigious Language Of This Body… He Explored The Infra Sense Which Is Still Unknown Today' (93). However, Caroll Has Explored Those Important Surfaces, On Which 'The Entire Logic Of Sense Is Located' (93). We Can Still Find Schizoid Fragments In Ordinary Speech, But These Are Normally Reorganized. Similarly, Carroll Can Be Retranslated As A Schizophrenic Piece (92). But It Is Wrong To Generalise Here, 'Believing To Have Discovered Analogous Forms Which Create False Differences' (92). Psychoanalysis Should Operate With A Surface/Depth Structure Rather Than With Analogies—'It Is Geographical Before It Is Historical' (93). [While We Are Here, Note That 'It Is Hardly Acceptable… To Run Together a Child's Nursery Rhymes, Poetic Experimentations, And Experiences of Madness… [And] Justify the Grotesque Trinity of Child, Poet, and Madmen' (82-83). This Must Be A Problem For Those Who Think That Deleuze Is Arguing That Children Are Philosophers?] Thirteenth Series Of The Schizophrenic And The Little Girl Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark

Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable and Ascribable To Constraints on Space Accomodation Model Is Provided In Some Paper, Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. Singularities Constitute The Transcendental Field. The First Stage Of Actualization Involves The Derivation Of Individuals. Singularities Spread Out 'In a Determined Direction over a Line of Ordinary Points', Up To the Vicinity of another Singularity (109). When Series Converge, A World Is Constituted [And Other Worlds Arise From Diverging Series]. The Infinite System Of Singularities Are Selected And Rendered Finite By Individuals [Does He Mean Human Individuals Throughout? I Think So] Who Combine Them, 'Spread Them Out Over Their Ordinary Lines' (109) And Even Form Them Again On Various Membranes. 'An Individual Is Therefore Always In A World As A Circle Of Convergence, And A World May Be Formed And Thought Only In The Vicinity Of The Individuals Which Occupy Or Fill It' (110). [Deleuze Is Suggesting That This Role For Individuals Is The Only Way For Worlds To Persist, Since Entropy Prevents The Endless Renewal Of Singularities At The Virtual Level—'The Power Of Renewal Is Conceded Only To Individuals In The World, And Only For A Time—The Time Of Their Living Present' (110). This Is Static Genesis, The First Level Of Actualization. Singularities Are Actualized In The World And In Individuals. Actualization Means Being Extended, Selected, Incarnated In A Body, And Renewed Locally. These Qualities Require Individuals. Actualization Also Means Being Expressed, But It Would Be Wrong To Assume That Only The Expressed World Exists. There Are, For Example Incompossible Worlds, Produced By Diverging Series [As Real Events, And Not Just Matters Of Contradictory Consciousness]. This Implies That There Is A Continuum Of Singularities Outside Of [Conscious] Individuals, Worlds In Which Other Possibilities Arise. [So Grasping These Other Possibilities Again Distinguishes Sense From Mere Logic, Based On The Truth And Falsity Of Propositions Expressed By Individuals]. Because Individuals Express Worlds, However, The World Looks As If It Is Merely Subjective, And Objective Events Appear To Be 'The Analytic Predicate Of A Subject' (112). [These Analytic Predicates Appear To Be Properties Of People Or Objects]. [Much Of This Argument Is Referring To Leibniz]. However, The Problem Is To Explain Logical Hierarchies Of Properties And General Categories. Subjective Analytic Predicates Have Immediacy, But Do Not Have Any Kind of Order; They Get Elaborated But Only As Mixtures. They Are Descriptive Of Actual Structures And Diversities. They Are 'Intuitions... Immediate Representations' (113). The Second Level Of Actualization Involves The Transcendental Again, Originally Conceived As The Development Of Transcendency In The Individual [See The Critique Of Husserl Above]. The Transcendental Ego Is Constituted Just As Other Individuals Are, As A Circle Of Convergences, So The Problem Is To Escape This And Develop Knowledge Of The Whole Continuum, Of Incompossible Worlds And Divergent Series, A New Sense Of World [Deleuze Uses The German Terms Welt For This New World And Umwelt For The Subjective World. I Am Grateful To Wikipedia For Explaining That: In The Semiotic Theories Of Jakob Von Uexküll And Thomas A. Sebeok, Umwelt (Plural: Umwelten; The German Word Umwelt Means "Environment" Or "Surrounding World") Is The "Biological Foundations That Lie At The Very Epicenter Of The Study Of Both Communication And Signification In The Human [And Non-Human] Animal."[Citation Needed] The Term Is Usually Translated As "Self-Centered World". [1] Uexküll Theorised That Organisms Can Have Different Umwelten, Even Though They Share The Same Environment']. Proper Transcendence Involves Transcending Individuated Worlds. Singularities Come With Their Own 'Perfectly Objective Determination Which Is The Open Space Of Its Nomadic Distribution' (113), And This Is How Deleuze Wants To Define A Problematic [Above]. Leibniz Apparently Half Grasped This Argument, Suggesting That Conic Sections, For Example, All Referred Back To The Same Event, Subdivided By 'Ambiguous Signs' (114). Similarly, Incompossible Worlds Also Have 'Something Objectively In Common', And 'Several Worlds Appear As Instances Of Solutions For One And The Same Problem... Variants of the Same Story... There Is Thus A"Vague Adam", That Is A Vagabond, A Nomad… Common to Several Worlds' (114). In Other Words, All The Objective And Subjective Worlds Can Be Explained By A Series Of Singularities. It Is Not Just The Analytic Predicates Of Individuals That Actualize Worlds. 'On The Contrary, [There] Are Predicates Which Define Persons Synthetically And Open Different Worlds And Individualities To Them As So Many Variables Or

Possibilities' (115). It Is Incompossible Worlds Which Synthesise 'Primary Possibilities Or Categories' (115). These Possibilities or Categories 'Necessarily Signify Classes and Properties', And These Are Distinct From Individual Categories at the First Level. They Still Seen Grounded In Persons, But That Is 'Because Persons Themselves Are Primarily Classes Having One Single Member, And Their Predicates Are Properties Having One Constant' (115). There Is Thus A System Involving Persons, Classes With One Single Member, Extensive Classes And Variable Properties '-- That Is The General Concepts Which Derive From Them' (115). If There Is A Universal Ego, It Is Something That Corresponds To Common Elements In All Worlds. So, Firstly, Sense Gets Actualized In A Field Of Singularities, Then The Umwelt Develops Around Individuals Which Express Or Describe This World, Then, Secondly, A Whole Objective World Develops From Common Elements, A Welt. Persons Can Then Define This Common Element And Develop Classes And Properties Derived From It. At The First Level We Find Good Sense, 'An Already Fixed And Sedentary Organization Of Differences'. At The Second Level, Commonsense Serves The Function Of Identification. Neither Term Explains How These Activities Are Derived [In Fact, Deleuze Wants To Suggest That The Second Level 'Is The Work Of Non Sense Which Is Always Copresent To Sense (Aleatory Point Or Ambiguous Sign)... Productive Nonsense Which Animates The Ideal Game And The Impersonal Transcendental Field' (116). There Is No Conventional Transcendental Movement, Especially One Driven By A Version Of The Ego. The Person Is A 'Produced Form, Derived From This Impersonal Transcendental Field… Always an Individual in General, Born… From The Singularity Which Extends Itself Over A Line of Ordinary Points and Starts from the Preindividual Transcendental Field' (116). Persons And Their Varieties Of Sense Are All Produced 'On The Basis Of Sense And Nonsense Which Do Not Resemble Them' (117). This Explains The Paradoxes And Limits Of Good Sense And Common Sense. [So The Existence Of Nonsense -- Paradox And Impossible Sentences Etc -- In The Ordinary Subjective World Shows The Existence Of Non Sense -- Something That Produces Ordinary Sense In The First Place, Something Objective? Individuals Can Only Describe But As Soon As They Generalise And Order Concepts They Necessarily Assume Some Non-Subjective World? The Man oeuvre seems Rather like the Illicit Transcendental Ego Move by Husserl, Generalising from an Ordinary Ego: Here, Explaining Ordinary Experience Is Used to Explain Some Deeper Virtual World as an Extension?] Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. The Issue Is Of The Validity Or Truth Of Propositions And How This Might Be Grounded. It Is As Difficult To Root Propositions In The Idea As It Is Actual Events. Significations Cannot Be Just Seen As Exhausted By Their Denoted Examples. A List Of Denoted Examples Would Be Endless And Groundless Anyway. Classic Philosophers Often Refuse To Answer Questions Denoted Examples Any Way, And Replied With A Blow From The Staff, Or A Mute Demonstration. After All, 'There Is No Resemblance (Nor Should There Be One) Between What One Points Out And What One Has Been Asked' (135) [Typical Philosophical Elitism]. The Issue Is Often Covered By Humour, Especially Irony or the Absurd. We Must Refuse Ascent and Descent and Occupy the Surface 'Where Pure Sense Is Produced' (136). At The Surface, 'One Finds Pure Singularities, An Emission Of Singularities Considered From This Perspective Of The Aleatory Element, Independent Of The Individuals And Persons Which Embody Or Actualise Them' (136). Getting To The Surface Involves Humour And Adventure, Seen In The Routines Of Zen Masters, Who Can Allude To The Void Which Is Present In Events [Events Are 'Not The Object As Denoted, But The Object As Expressed Or Expressible, Never Present, But Always Already In The Past And Yet To Come' (136), And The Abolition Of The Object Alludes To The Void]. Negating Objects Indicates The Expressible. [Examples From Zen Follow, Page 137. What They Seem To Be Suggesting Is That Sense Combines Actual Objects With The Notion Of The Void, Non Sense—An Actual Brushstroke And Also The White Space—'Language Becomes Possible And, By Becoming Possible, It Inspires Only A Silent And Immediate Communication' Somehow Outside All The Significations And Denotations]. The Same Reasoning Affects The Question Of Who Speaks—Individuals Or Language Itself In The

Form Of Ideal Forms [And We Are Back With Greek Philosophy Examples. Socratic irony apparently Refers to the Fact That Individuals Both Initiate Speech and Are the Products of Speech—138. This In Turn Permits The Idea Of A 'Pure Rational Language… [Enabling]… Natural Communication between A Supremely Individuated God and the Derived Individuals Which He Created' (138)]. Further Irony Awaits The Romantic Notion Of The Person, 'The Finite Synthetic Unity Of The Person... [Not Just]... The Analytic Identity of the Individual' (138). Here, The Person And Representations Are Linked Ironically, Implying Both A Universal Idea And 'Sensible Particularity' (138), Both Universal And Particular Characteristics Of The Person—Hence The Earlier Argument That Persons Should Really Be Seen As A Class Which Has Only One Member. All These Formulations Assume That Singularities Are Located In The Individual Or The Person, Influenced Only By Some Groundless Abyss, Or Chaos, Threatening Both Classical And Romantic Discourse With The Lack Of Articulation. There Is Also A Threefold Division Of Language—The Ordinary Or Real; The Ideal Language, Like The Purely Rational One; Esoteric Language Which Subverts The Ideal Language And The Individuality Of The Speaker Of Ordinary Language [For Example Social Scientific Explanations, Or Structural Linguistic Ones?] [Weird Examples of Esoteric Languages Which Mean Nothing to Me on Page 140, With a Reference Back To Portmanteau Words]. So Sometimes Individual Speaks, Sometimes The Person, And Sometimes 'The Ground Which Dissolves Both' (140). The Only Way Out Is To See That Singularities Are Not Coterminous With Individuals, And There Is Not Just A Groundless Abyss Beneath Them. There Are Other Impersonal And Pre-Individual Singularities. Nonsense And Sense Collaborate At The Surface. Irony Gives Way To Humour—'The Co extensiveness Off Sense with Nonsense', A Matter Of 'Surfaces… Doubles… Nomad Singularities and Of the Always Displaced Aleatory Point' (141). Normal Significations, Denotations and Manifestations Are Suspended, and 'All Height and Depth Abolished' (141). Nineteenth Series of Humour. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis.Doctrines Come From 'Wounds And Vital Aphorisms' (148), And Some Writers See Themselves As Embodying Events [Hints Of The Stuff On Nietzsche And Illness Earlier]. Our Will Can Act As A Quasi Cause Of Bodily Events, As We Live Them. This Is An Ethical Stance—'Not To Be Unworthy Of What Happens To Us' (149). The Alternative Is Ressentiment. The Normal Moral Notions Such As Just Or Unjust Are Themselves Immoral. It Is Not Just A Matter Of Resignation To Events, Which Can Still Be Ressentiment, More An 'Apotheosis Of The Will' (149) [Citing Bousquet]. The Organic Is Exchanged For The Spiritual Will [Isn't This Just Making The Best Of Things, Finding Some Deep Meaning In Personal Tragedy?]. In This Sense, Freedom Is The Same As Submitting To Fate, Actualizing Events, Making Sense Of Events, Seeing Events As Something Expressed. One Becomes 'The Offspring of One's Events Are Not of One's Actions, For the Action Is Itself produced By the Offspring Of The Event' (150). In This Way, Actors Somehow Communicate With The Aion, Instead Of Being Dominated By Chronos. [Lots Of Implicit Christianity Here, Surely?]. All The Components Of An Event, Future As Well As Past, Are United, So That One Realises The Impact Of Singularities, Including Preindividual Components. [In This Way, Some Sort Of Agency Seems To Remain? One Can Become 'The Actor of One's Own Events – A Counter Actualization' (150)]. Ressentiment Arises From Not Realizing That Our Particular Part Of Experience, Which May Well Be Unjust, Is Part Of A More Perfect Whole. It Is Incorrect To Judge Everything From The Perspective Of The Present. Some Humourous Recognition of the Futility of It All Is Valuable—In the Great Scheme of Things, States of Affairs Are Always 'Impersonal and Pre-Individual, Neutral, neither General nor Particular' (151). In These Circumstances, Either Life Can Seem 'Too Weak For Me' [Not Vivid Enough, Not Enough Focused In The Present?], Or 'It Is I Who Am Too Weak For Life, It Is Life Which Overwhelms Me, Scattering Its Singularities All About, In No Relation To Me, Nor To A Moment Determinable As The Present Except An Impersonal Instant' (151). This Is Apparent When One Considers Death Or The Mortal Wound, An Event Which Is Indifferent To Me, 'Incorporeal And Infinitive, Impersonal, Grounded Only In It' (151),

Although It Is I Who Has To Actualize The Event. However I Can Also Counter actualizes In the Sense Above. It Follows That The Events Of Reality Are Quite Different From Personal Experience, Shown In The 'Splendour Of The "They"… The Splendour of the Event Itself Order Of the Fourth Person' (152) [This Fourth Person Is the 'It' As In Phrases Like 'It Is Raining']. The Old Distinctions Between Private And Collective Refer Only To Personal Experience, Whereas 'Everything Is Singular, And Thus Both Collective And Private… Which Private Event Does Not Have All Its Coordinates, That Is, All Its Impersonal Social Singularities?' (152). Freedom Exists Only In Recognising The Impersonal Nature Of The Event, Seeing All Events As 'A Single Event Which No Longer Makes Room For The Accident, And Which Denounces And Removes The Power Of Ressentiment Within The Individual As Well As The Power Of Oppression Within Society' (152) [Ridiculous Philosophical Notion Of Freedom Overcoming Oppression By An Act Of Philosophy]. Ressentiment Ties One to an Oppressive Order. [Then A Really Sentimental And Pathos Ridden Bit: 'It Is At This Mobile And Precise Point, Where All Events Gather Together In One That Transmutation Happens: This Is The Point At Which Death Turns Against Death; Where Dying Is The Negation Of Death And The Impersonal At He Of Dying No Longer Indicates Only The Moment When I Disappear Outside Of Myself, But Rather The Moment When Death Loses Itself In Itself, And Also The Figure Which The Most Singular Life Takes On In Order To Substitute Itself For Me' (153)] Ethics Links Logic and the Body. The Stoic Notion Of Bodies Included The Passions, And Good And Evil Intents: Particular Bodies Might Have Evil Mixtures, But The Aggregate Of Bodies Is Perfect Or Good, The Unity Of Causes Themselves. In Principle, Each Event Can Be Linked To A Particular Cause And Thus The Unity Of Causes, And This Could Lead To The Activity Of Divination [Grasping The Divine Unity Of Causes] As A Basis For Ethics. The Effects, Lines Have To Be Traced Back From Events To Pure Events And Then To Actions And Passions [With A Lot Of Poetic Stuff As Examples, 143]. Stoics Took Another Route To Ethics Through Logic [Kind Of Working Out Which Events Will Actualize]. Stoic Philosophy Then Saw In Representations Of The Limited Event A Connection With Pure Events [And Ethical Conduct Seems To Be To Work Towards Actualizing Such Events?]. [Stoic Accounts Of Representations Ensue, 144-5.] There Is A Difference [For Stoics] Between Representations Of Sense And Of Logic, Denotations And Significations, Representations And Expressions. Representation Is Not Just Resemblance, But Includes A Notion of Adequate Expression. Concepts Have To Be Actualised In Representation, And Also Expressed If They Are To Be Comprehensive. Thus Our Knowledge Of Death Is Abstract, Despite The Number Of Deaths We Witness, Until It Becomes Personal, Not Indifferent, But Concrete. This Is Where Expression Is Needed. In Stoic Philosophy, Moral Conduct Involves A Relation with Pure Events Unites Those Events with One's Self. [The Examples Again Come From Zen: 'The Bowman Must Reach The Point Where The Aim Is Also Not The Aim, That Is To Say, The Bowman Himself' (146).] An Understanding Of The Pure Event Is Required, As Something Which Is 'Eternally Yet To Come And Always Already Past', But Which Has To Be Willed Into Actualization (146). The Stoic Embodies Incorporeal Effects, Aligning Themselves With The Quasi Cause. It Is Necessary Because Quasi Causes Cannot Embody Themselves, Except To The Immediate Instant, The Present. [There Is An Equally Baffling Aside About The Difference Between Actors And Characters—Actors Represent By Occupying The Instant, While The Character Also 'Portrays Hopes Or Fears In The Future And Remembers Or Repents In The Past' (147). Stoics Therefore See Themselves As Uniting The Instant With The Unlimited Future And Past, Willing The Event And Also Representing It. [Largely Incomprehensible, Although I Am Starting To See How The Normal Notion Of Stoicism Might Fit—The Patient Resignation Of One's Self To One's Fate—And Also Seeing How Deleuze Thinks That Human Beings Should Reconcile Themselves To Solving The Problems That Reality Creates In The Form Of A Problematic]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Deleuze And Guattari See In The Capitalist Money System "An Axiomatic Of Abstract Quantities That Keeps Moving Further And Further In The Direction Of The Deterritorialization Of The Socius"

(Deleuze 1983a, 33), Which Is To Say That Capital Is Inherently Schizophrenic. However, Because Capital Also Re-Territorializes All Flows Into Money, Schizophrenia Remains Capitalism's External Limit. Nevertheless, It Is Precisely That Limit Against Which Thinking Can Subject Capitalism To Philosophical Critique. Psychoanalysis, They Say, Is Part Of The Reign Of Capital Because It Re-Territorializes The Subject As "Private" And "Individual," Instituting Psychic Identity Through Images Of The Oedipal Family. However, The Oedipal Triangle Is Merely A Representational Simulacrum Of Kinship And Filiation, Re-Coded Within A System Of Debt And Payment. In This System, They Insist, Flows Of Desire Have Become Mere Representations Of Desire, Cut Off From The Body Without Organs And The Extra-Familial Mechanisms Of Society. A Radical Critique Of Capital Cannot Therefore Be Accomplished By Psychoanalysis, But Requires A Schizoanalysis "To Overturn The Theater Of Representation Into The Order Of Desiring-Production" (Deleuze 1983b, 271). Here, The Authors See A Revolutionary Potential In Modern Art And Science, Where, In Bringing About The "New," They Circulate De-Coded And De-Territorialized Flows Within Society Without Automatically Re-Coding Them Into Money (Deleuze 1983a, 379). In This Revolutionary Aspect, Anti-Oedipus Reads As A Statement Of The Desire That Took To The Streets Of Paris In May Of 1968, And Which Continues, Even Now, To Make Itself Felt In Intellectual Life. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. The Term "Deconstruction," Like "Postmodernism," Has Taken On Many Meanings In The Popular Imagination. However, In Philosophy, It Signifies Certain Strategies For Reading And Writing Texts. The Term Was Introduced Into Philosophical Literature In 1967, With The Publication Of Three Texts By Jacques Derrida: Of Grammatology (English 1974), Writing And Difference (English 1978), And Speech And Phenomena (English 1973). This So-Called "Publication Blitz" Immediately Established Derrida As A Major Figure In The New Movement In Philosophy And The Human Sciences Centered In Paris, And Brought The Idiom "Deconstruction" Into Its Vocabulary. Derrida And Deconstruction Are Routinely Associated With Postmodernism, Although Like Deleuze And Foucault, He Does Not Use The Term And Would Resist Affiliation With "-Isms" Of Any Sort. Of The Three Books From 1967, Of Grammatology Is The More Comprehensive In Laying Out The Background For Deconstruction As A Way Of Reading Modern Theories Of Language, Especially Structuralism, And Heidegger's Meditations On The Non-Presence Of Being. It Also Sets Out Derrida's Difference With Heidegger Over Nietzsche. Where Heidegger Places Nietzsche Within The Metaphysics Of Presence, Derrida Insists That "Reading, And Therefore Writing, The Text Were For Nietzsche 'Originary' Operations," (Derrida 1974, 19), And This Puts Him At The Closure Of Metaphysics (Not The End), A Closure That Liberates Writing From The Traditional Logos, Which Takes Writing To Be A Sign (A Visible Mark) For Another Sign (Speech), Whose "Signified" Is A Fully Present Meaning. This Closure Has Emerged, Says Derrida, With The Latest Developments In Linguistics, The Human Sciences, Mathematics, And Cybernetics, Where The Written Mark Or Signifier Is Purely Technical, That Is, A Matter Of Function Rather Than Meaning. Precisely The Liberation Of Function Over Meaning Indicates That The Epoch Of What Heidegger Calls The Metaphysics Of Presence Has Come To Closure, Although This Closure Does Not Mean Its Termination. Just As In The Essay "On The Question Of Being" (Heidegger 1998, 291-322) Heidegger Sees Fit To Cross Out The Word "Being," Leaving It Visible, Nevertheless, Under The Mark, Derrida Takes The Closure Of Metaphysics To Be Its "Erasure," Where It Does Not Entirely Disappear, But Remains Inscribed As One Side Of A Difference, And Where The Mark Of Deletion Is Itself A Trace Of The Difference That Joins And Separates This Mark And What It Crosses Out. Derrida Calls This Joining And Separating Of Signs Différance (Derrida 1974, 23), A Device That Can Only Be Read And Not Heard When Différance And Différence Are Pronounced In French. The "A" Is A Written Mark That Differentiates Independently Of The Voice, The Privileged Medium Of Metaphysics. In This Sense, Différance As The Spacing Of Difference, As Archi-Writing, Would Be The Gram Of Grammatology. However, As Derrida Remarks: "There Cannot Be A Science Of Difference Itself In Its Operation, As It Is Impossible To Have A Science Of The Origin Of Presence Itself, That Is To Say Of A Certain Non-Origin" (Derrida 1974, 63). Instead, There Is Only The Marking Of The Trace Of Difference, That Is, Deconstruction. Because At Its Functional Level All Language Is A System Of

Differences, Says Derrida, All Language, Even When Spoken, Is Writing, And This Truth Is Suppressed When Meaning Is Taken As An Origin, Present And Complete Unto Itself. Texts That Take Meaning Or Being As Their Theme Are Therefore Particularly Susceptible To Deconstruction, As Are All Other Texts Insofar As They Are Conjoined With These. For Derrida, Written Marks Or Signifiers Do Not Arrange Themselves Within Natural Limits, But Form Chains Of Signification That Radiate In All Directions. As Derrida Famously Remarks, "There Is No Outside-Text" (Derrida 1974, 158), That Is, The Text Includes The Difference Between Any "Inside" Or "Outside." A Text, Then, Is Not A Book, And Does Not, Strictly Speaking, Have An Author. On The Contrary, The Name Of The Author Is A Signifier Linked With Others, And There Is No Master Signifier (Such As The Phallus In Lacan) Present Or Even Absent In A Text. This Goes For The Term "Différance" As Well, Which Can Only Serve As A Supplement For The Productive Spacing Between Signs. Therefore, Derrida Insists That "Différance Is Literally Neither A Word Nor A Concept" (Derrida 1982, 3). Instead, It Can Only Be Marked As A Wandering Play Of Differences That Is Both A Spacing Of Signifiers In Relation To One Another And A Deferral Of Meaning Or Presence When They Are Read. How, Then, Can Différance Be Characterized? Derrida Refuses To Answer Questions As To "Who" Or "What" Differs, Because To Do So Would Suggest There Is A Proper Name For Difference Instead Of Endless Supplements, Of Which "Différance" Is But One. Structurally, This Supplemental Displacement Functions Just As, For Heidegger, All Names For Being Reduce Being To The Presence Of Beings, Thus Ignoring The "Ontological Difference" Between Them. However, Derrida Takes The Ontological Difference As One Difference Among Others, As A Product Of What The Idiom "Différance" Supplements. As He Remarks: "Différance, In A Certain And Very Strange Way, (Is) 'Older' Than The Ontological Difference Or Than The Truth Of Being" (Derrida 1982, 22). Deconstruction, Then, Traces The Repetitions Of The Supplement. It Is Not So Much A Theory About Texts As A Practice Of Reading And Transforming Texts, Where Tracing The Movements Of Différance Produces Other Texts Interwoven With The First. While There Is A Certain Arbitrariness In The Play Of Differences That Result, It Is Not The Arbitrariness Of A Reader Getting The Text To Mean Whatever He Or She Wants. It Is A Question Of Function Rather Than Meaning, If Meaning Is Understood As A Terminal Presence, And The Signifying Connections Traced In Deconstruction Are First Offered By The Text Itself. A Deconstructive Reading, Then, Does Not Assert Or Impose Meaning, But Marks Out Places Where The Function Of The Text Works Against Its Apparent Meaning, Or Against The History Of Its Interpretation. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. "Anything Which Is Just Born, Which Has Just Come Into Existence, Has No Past Behind It. Birth, In Other Words, Is The Condition Of Having No Past. And Likewise, Anything Which Now Dies, Which Has Just Ceased To Be, Has No Future Left In Front Of It. Death Is The Condition Of Having No Future. But We Have Already Seen That This Present Moment Has Both No Past And No Future Simultaneously. That Is, Birth And Death Are One In This Present Moment. This Moment Is Just Now Being Born—You Can Never Find A Past To This Present Moment, You Can Never Find Something Before It. Yet Also, This Moment Is Just Now Dying — You Can Never Find A Future To This Moment, Never Find Something After It. This Present, Then, Is A Coincidence Of Opposites, A Unity Of Birth And Death, Being And Non-Being, Living And Dying. As Ippen Put It, "Every Moment Is The Last Moment And Every Moment Is A Rebirth." Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth for models please see one of the papers in the series. "Because We Demand A Future, We Live Each Moment In Expectation And Unfulfillment. We Live Each Moment In Passing. In Just This Way The Real Nunc Stans, The Timeless Present, Is Reduced To The Nunc Fluens, The Fleeting Present, The Passing Present Of A Mere One Or Two Seconds. We Expect Each Moment To Pass On To A Future Moment, For In This Fashion We Pretend To Avoid Death By Always Rushing Toward An Imagined Future. We Want To Meet Ourselves In The Future. We Don't Want Just Now—We Want Another Now, And Another, And Another, Tomorrow And Tomorrow And Tomorrow. And Thus, Paradoxically, Our Impoverished Present Is Fleeting Precisely Because We Demand That It End! We Want It To End So That It Can Thereby Pass On To Yet Another Moment, A Future Moment, Which Will In Turn Live Only To Pass." — Ken Wilber, No

Boundary: Eastern And Western Approaches To Personal Growth. For Stoics, Causes Involved Reference To The Depths, But Effects At The Surface Could Also Have Relations Among Themselves. This Permitted The Distinction To Be Drawn Between Destiny And Necessity—The Stoics Wanted To Affirm Destiny And Deny Necessity. In The First Place, Effects Express Causes, But Expressions Of Relations [Rather Than Necessity] Connects Those Effects. Those Relations May Be Described As Compatibility Or Incompatibility, Conjunction Or Disjunction. These Are Not Causal Relations Themselves, But Represent 'An Aggregate Of Noncausal Correspondences Which Form A System Of Echoes, Of Presumptions And Resonances, A System Of Signs—In Short, An Expressive Quasi Causality And Not At All A Necessitating Causality' (170). This Need Not Involve Contradiction, Which Is Applying To Events Rules That Really Only Applied To Logic And Argument. There Can Be Incompatability Without Contradiction, A Noncausal Correspondence. Leibniz Described Incompossible Worlds [Delanda Is Very Clear On This Too]. Only Impossible Events Contradict Possible Ones. Events Can Be Compossible [Roughly, They Have Predictable And Predicative Future And Past Events]. For Deleuze, It Is A Matter Of 'The Convergence Of Series Which Singularities Of Events Form As They Stretch Themselves Out Over Lines Of Ordinary Points. Incompossibility Must Be Defined By The Divergence Of Such Series' (171). Such A Notion Is Essential To Any Theory Of Sense. We Should Not See Divergence As A Matter Of Exclusion, As Leibniz Did [Since God Chose Actual Events]. Divergent And Disjunction Can Both Be Positive, While Preserving Differences. In Fact, Differences Are Crucial, Preserving The Distance Between Objects While Affirming That They Are Related. This 'Permits The Measuring Of Contraries Through Their Finite Difference Instead Of Equating Difference With A Measureless Contrariety' (173). It Is Contradiction Which Is The Special Case. Difference Here Is A Topological Term Relating To Distance On Surfaces Rather Than Depths. It Is Not Just A Matter Of Suggesting 'Some Unknown Identity Of Contraries (As In Commonplace In Spiritualist And Dolorist Philosophy)' (173) [Take That St Pierre!]. An Example Is Nietzsche Arguing That Health And Sickness Can Both Inform Each Other, Act As Points Of View, Remembering That 'Things, Beings, Are Themselves Points Of View' (173). Divergence Does Not Mean Exclusion, And Disjunction Does Not Mean Separation. Connective Syntheses '(If…, Then)' Construct A Single Series; Conjunctive Series '(And)' Produces Convergent Series, But Disjunctive Series '(Or)' Produces A Divergence Series. Normally, Disjunction Helps Us To Criticise Synthesis, But Disjunction Can Still Be A Synthesis Itself, Despite Its Use In Logical Analysis [I Think What Is Going On Here Is Arguing That There Is A Difference Between 'Either/Or' In A Logical Sense, And 'One Or Two' In The Real Sense—The Latter Can Mean That Both Are Compossible. This Is The 'Communication Of Events' Rather Than The Logical Business Of Analysing Predicates (174)]. The Synthetic Disjunction Expresses The Paradox, With Divergence At The Centre. The Discussion Of Paradoxes And Esoteric Words Above Are Examples: They Contract 'The Multitude Of Divergent Series In The Successive Appearance Of A Single One' (175). There Is A Difference Again Between The Subversion Of The Present And Simple Identity By Depths, And Operations At The Surface. By Considering The Depths, We Encounter Infinite Identities [As Events Become Examples Of Deeper Categories, Wholes?]. At The Surface Events Communicate With Each Other Directly Through Maintaining Distance And By Affirming Disjunctions. Disjunction Threatens The Identity Of The Self, And Helps Us To See The Self As 'So Many Impersonal And Pre-Individual Singularities' [Connected Through Disjunctive Synthesis. Hence The Importance Of Heterogeneity] (175). The Normal Concept Of The Self Implies Some Connected Series, 'But When Disjunction Accedes To The Principle Which Gives To It A Synthetic And Affirmative Value, The Self, The World, And God Share In A Common Death' (176). [There Is Also A Point That Divergent Series Explain And Also Exceed Normal Conjunctive And Connective Series]. In The Usual Conception, 'The Self Is The Principle Of Manifestation, In Relation To The Proposition, The World Is The Principle Of Denotation, And God The Principle Of Signification' (176). But The Theory Of Sense Here Says That It Emanates From Nonsense, From Paradox, And From The 'Eternally Decentred Ex-Centric Centre' (176). This Position 'Does Not Tolerate The Subsistence Of God As An Original Individuality, Nor The Self As The Person, Nor The World As An Element Of The Self And As God's Product. The Divergence Of The Affirmed Series Forms A "Chaosmos" And No Longer A World; The Aleatory Point Which Traverses Them Forms A Counterself, And No Longer A Self' (176). There Is No Centre But

Only ‘Pure Events Which The Instant, Displaced Over The Line[Of Aion] , Goes On Dividing Into Already Past And Yet To Come. Nothing Other Than The Event Subsists… This Communicates With Itself Through Its Own Distance And Resonates Across All Of Its Disjuncts’ (176). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper of the series, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis Twenty-fourth series of the communication of events . “As Simple As That Sounds, It Is Nevertheless Extremely Difficult To Adequately Discuss No-Boundary Awareness Or Nondual Consciousness. This Is Because Our Language — The Medium In Which All Verbal Discussion Must Float — Is A Language Of Boundaries. As We Have Seen, Words And Symbols And Thoughts Themselves Are Actually Nothing But Boundaries, For Whenever You Think Or Use A Word Or Name, You Are Already Creating Boundaries. Even To Say "Reality Is No-Boundary Awareness" Is Still To Create A Distinction Between Boundaries And No-Boundary! So We Have To Keep In Mind The Great Difficulty Involved With Dualistic Language. That "Reality Is No-Boundary" Is True Enough, Provided We Remember That No-Boundary Awareness Is A Direct, Immediate, And Nonverbal Awareness, And Not A Mere Philosophical Theory. It Is For These Reasons That The Mystic-Sages Stress That Reality Lies Beyond Names And Forms, Words And Thoughts, Divisions And Boundaries. Beyond All Boundaries Lies The Real World Of Suchness, The Void, The Dharmakaya, Tao, Brahman, The Godhead. And In The World Of Suchness, There Is Neither Good Nor Bad, Saint Nor Sinner, Birth Nor Death, For In The World Of Suchness There Are No Boundaries.” ― Ken Wilber, No Boundary: Eastern And Western Approaches To Personal Growth. It Has Just Been Argued That Divergence Can Produce A Positive Synthesis, And That Ultimately, Events And States Of Affairs Are Compatible. Incompatibility Arises Only With Actualizations. It Is Individuals, For Example, Who Actualize Divergent Events. Even These Divergences Are Not Just Logical Contradictions But ‘Alogical ‘Incompatibilities. Persons Can Enjoy The Paradoxes That Ensue, But The Point Is How To Get To The Universal Communication Of Events, The Disjunctive Syntheses. Individuals Must Grasp Themselves As Events, And See What They Normally Regard As Themselves As An Actualization. Normal Individuality Is Only ‘Fortuitous’ (178). It Follows That The Same Thinking Must Be Extended To All Other Events And Individuals. ‘Each Individual Would Be Like A Mirror For The Condensation Of Singularities… The Ultimate Sense Of Counteractualization’ (178) [Undertaken For Cognitive Reasons Here Not Ethical Ones, Although Deleuze Says That This Is A Route To ‘The Universal Freedom’ (178)]. The Normal Sense Of The Individual As Having A Coherent Identity Must Be Replaced By The Notion Of A Series Of Individualities [With Nomadic Lines Connecting Them?]. We Then Get To The Realm Of Pure Events, And The Universal Connections Between Them, Seen As Disjunctive Syntheses Of Series, And The Knowledge That Actualized Events Are Fortuitous. [The Example Here Is That A Friend In One World Could Be An Enemy In Another Equally Possible World]. ‘Philosophy Merges With Ontology, But Ontology Merges With The Univocity Of Being (Analogy Has Always Been A Theological Vision, Not A Philosophical One, Adapted To The Forms Of God, The World, And The Self)’ (179). This Does Not Mean That There Is Only One And The Same Being [And Certainly Not Identical Beings, Because These Are Always Heterogeneous]. Instead, ‘Being Is Voice That Is Said, And It Is Said In One And The Same “Sense” Of Everything About Which It Is Said’ (179). It Is The Ultimate Form. All The Other Forms Remain ‘Disjointed In It’, But Being Joins Them Into Series And Disjunctions. ‘The Positive Use Of The Disjunctive Synthesis… Is The Highest Affirmation... A Single Voice For Every Hum Of Voices And Every Drop Of Water In The Sea’ (180). It Is Not Just That There Is A Connection At The Level Of Language—‘Being Cannot Be Said Without Also Occurring’ (180). In This Way, Event And Sense Are Identical. ‘Univocal Being Is Neutral. It Is Extra Being, That Is The Minimum Of Being Common To The Real, The Possible And The Impossible… The Pure Form Of The Aion… One Single Event For All Events; One And The Same Aliquid [Something] For That Which Happens And That Which Is Said; And One And The Same Being For The Impossible, The Possible, And The Real’ (180). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The

Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. Sāṅkhya (Often Spelled Sāṁkhya) Is One Of The Major "Orthodox" (Or Hindu) Indian Philosophies. Two Millennia Ago It Was The Representative Hindu Philosophy. Its Classical Formulation Is Found In Īśvarakṛṣṇa's Sāṅkhya-Kārikā (Ca. 350 C.E.), A Condensed Account In Seventy-Two Verses. It Is A Strong Indian Example Of Metaphysical Dualism, But Unlike Many Western Counterparts It Is Atheistic. The Two Types Of Entities Of Sāṅkhya Are Prakṛti And Puruṣa-S, Namely Nature And Persons. Nature Is Singular, And Persons Are Numerous. Both Are Eternal And Independent Of Each Other. Persons (Puruṣa-S) Are Essentially Unchangeable, Inactive, Conscious Entities, Who Nonetheless Gain Something From Contact With Nature. Creation As We Know It Comes About By A Conjunction Of Nature And Persons. Prakṛti, Or Nature, Is Comprised Of Three Guṇa-S Or Qualities. The Highest Of The Three Is Sattva (Essence), The Principle Of Light, Goodness And Intelligence. Rajas (Dust) Is The Principle Of Change, Energy And Passion, While Tamas (Darkness) Appears As Inactivity, Dullness, Heaviness And Despair. Nature, Though Unconscious, Is Purposeful And Is Said To Function For The Purpose Of The Individual Puruṣa-S. Aside From Comprising The Physical Universe, It Comprises The Gross Body And "Sign-Body" Of A Puruṣa. The Latter Contains Among Other Things The Epistemological Apparati Of Embodied Beings (Such As The Mind, Intellect, And Senses). The Sign Body Of A Puruṣa Transmigrates: After The Death Of The Gross Body, The Sign-Body Is Reborn Into Another Gross Body According To Past Merit, And The Puruṣa Continues To Be A Witness Through Its Various Bodies. An Escape From This Endless Circle Is Possible Only Through The Realization Of The Fundamental Difference Between Nature And Persons, Whereby An Individual Puruṣa Loses Interest In Nature And Is Thereby Liberated Forever From All Bodies, Subtle And Gross. Much Of The Sāṅkhya System Became Widely Accepted In India: Especially The Theory Of The Three Guṇa-S; And It Was Incorporated Into Much Latter Indian Philosophy, Especially Vedānta. Internet Encyclopedia Of Philosophy Sankhya Author Information Ferenc Ruzsa Email: Ferenc.Ruzsa@Elte.Hu Eötvös Loránd University Hungary. The Amitabha Buddha Who Was, And Is, Revered And Praised By Buddhists Around The World, Radiates Indefinite Light And Life From This "State Of Cessation". This State Is A Continuous Process Of Calmness. It Will Be The Eventual Refuge For Us All. If We Think Carefully About The Definitions Of Calmness And Extinction, Then We Can Deduce That They Are The True Natural End-Points Of Rising And Falling. The True Nature Of The Cycle Of Rising And Falling Is Calmness And Extinction. Because Of This Nature, All Chaos And Conflicts In The State Of Rising And Falling Will Eventually Cease. This Is Attainable By The Realisation Of Prajna. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia). Like Psychic DNA, The Collective Unconscious Contains "Inherited" Psychic Material That Links Us Not Only To Other Humans In The Present But Also To Our Ancestors From The Past. According To Jung's Theory, Though Each Of Us Appears To Function Independently, In Actuality We're All Tapped Into The Same Global Mind. Symbols Interact With And Condition Our Biohologram. The Collective Unconscious Is Essentially A Hologram. Symbols Arise From And Are Embedded In The Environment As Holographic Fields Of Energy. They Are Morphogenic Veils Of Primal Forces. If Your Brain Acts Like A Self-Contained Hologram, It Is Possible Your Consciousness Is Actually A Piece Of A Much Larger Hologram Of Overall Human Consciousness. Jung Noticed That Patterns Spontaneously Appear Over And Over Around The World. They Also Appear As Our Ancestral Memories Or Holographic Wisdom Field. In The Archetypal World, Everyone Is The Same, All Around The World... We Are All Gods, And Our Emotional Addictions To Pain And Suffering, Contempt, Insecurities, Doubt, Failure, Is Holographically-Recorded And Can Be Holographically Healed. All Archetypes Are A Form Of Human Expression That Is Both Holographic And Physical. Physical Formations Of Archetypal Sequences Cause Humans To Behave In Parallel Manners To Each Ancestor That Is Associated. Integration Is A Function Of Intentionality -- Conscious And Unconsciously Maintained, Or Incorporated. Integration Occurs Both Without Effort, As A Redesign Of The Central Processor Of Our Minds, And Voluntarily As A Deliberate Effort To Understand, Find Meaning, And As Rectification Of Our Behavior Towards

Others And Towards Ourselves. Imagination Is Structured By The Archetypal Potentials Of The Unconscious. Archetypes Structure The Possibility To Generate And Entertain Such Ideas. The Archetype Itself Cannot Be Known But Structures Everything We Come To Know. Their Totality Functions As A Psychic Organ. Universal Themes Appear In Distinct Cultural Garb. Over Millennia, All The Archetypes Have Emerged In Stories, Deities And Cultural Forms. One Of The Striking Points Of Religious Faith Is That They Aren't True. In Early History People Didn't Know The 'Real' Reasons Things Happened, So It's No Surprise That Their Explanations Were Wrong. But Then, Why Would We Want To Retrieve Such Superstitions? Kuhn Reminds Us That Even The Most Absurd Or Confused Explanation Of A Phenomenon Can Find Acceptance In The Absence Of A Competing Idea. Once Any Explanation Is Offered For A Phenomenon, It Has An Explanation From That Time On. Succeeding Speculations Might Be Able To Explain The Relevant Phenomena Better Than Its Predecessors. Deities, As Archetypal Role Models, Are The Opening Gambit In The Drama Of Understanding Human Subjectivity. Folk Tales Function The Same Way. Deities Or Archetypes May Have Evolved To Smooth Interpersonal Relations By Including An Understanding Of Human Types, Along With Rules For Helping The Different Types Relate With One Another. They Are Reflections Of Ancient Evolutionary Pressures, With A Dash Of Neuroanatomy. We Still Have Yjr Same Pressures Today So The Ancient Archetypes Still "Work", Regardless Of Objective Existence Of Gods And Goddesses. Burke (2003) Describes How The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology. Such Processes Include Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. Identifying The Similar Patterns Of These Processes Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole. The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. Cognition, Itself Is A Holographic Archetype. Many Essential Qualities Of The Whole Are Reflected Or Contained In Each Of The Parts That Make Up That Whole. It Is A Subtle Net Of Metaphor, Analogy And Simile. Holographic Archetypes Effectively Echo Their Resonant Patterns Through Literal And Symbolic Reflectaphors. The Passing Forms Of The Holographic Archetype Include The Hologram, Psychic Structure, Synchronicity, Wisdom Traditions, Memory, And The Process Of Scientific Discovery, Chaos In Physics, Nonlocality And Virtuality In Physics. As Unconscious Structuring Principle, The Archetype Lies Beyond Normal Consciousness And Emerges Suddenly And Dynamically From The (Holographic) Psychoid Field, With A Powerful Emotional Charge That Invests It With Significance. Everything That Happens Is Conditioned By The Moment In Which It Happens. The Universal Field Imposes The Conditions. Matter Is Not Inert But Receptive To Holistic Patterning. If The Mythic World Taught Our Ancestors How To Manipulate The Empirical World, It Also Taught Them To Manipulate That Mythic Narrative Itself For Control Purposes. Socioeconomic Power Enforces Its Mythic Narrative. The Psychoid Field Imposes Holistic Function. Autonomous Inner Forces Arouse Compelling Opportunities To Enact Archetypal Behaviors. They Guide Our Perceptions And Behavior, Usually Without Our Awareness. Limbic Action Of Complexes Is A Big Part Of The Holistic Dynamics Of The Psyche. Dreams Report What Goes On Beneath The Veil Of Conscious Awareness. To The Consciousness Of The 'Thinker', Knowledge Is Thought. Period. Without Thought, The Consciousness Of The 'Thinker' Collapses Into Psychosis. With Archetypes Come The Potential For Wisdom, Relatedness, Sociality, Ambiguity, Paranoia, Projection, Identification, Denial, Inflation, Sub Personalities (Fragmentation), Defensiveness, Obsession, Hypnotic Dissociation, The Contagion Of Participation Mystique, Mythologizing, Complexes, Compensation, And Self-Delusion. Compensation May Calm Or Disturb Consciousness. There Is No Imperative For The Ego To Integrate These Alternative Perspectives, Private And Public Myths. The Unconscious Can Produce Deep Wisdom And Utter Nonsense. It Is Up To The Ego To Discriminate. The Value Of Myths Is Purely Heuristic, Not Pragmatic. Jung In The 21st Century: Synchronicity And Science By John Ryan Haule Mind Control Countermeasures. Forms Of Experience, Such As Madness, Violence, Or Sexuality That Break From The Prison Of Rationality. Where Modern Societies "Problematize" Forms Of Experience Such As Madness, Illness, And Sexuality, That Is, Turn Them Into Governmental Problems, Into Areas Of Life In Need Of Control And Regulation,

Foucault In Turn Queries The Social Construction Of "Problems" By Uncovering Their Political Motivations And Effects And By Challenging Their Character As Natural, Necessary, Or Timeless. In What He Calls A "Diagnostic Critique" That Combines Philosophy And History (1989: 38-39, 73), Foucault Attempts To Clarify The Nature Of The Present Historical Era, To Underline Its Radical Difference From Preceding Eras, And To Show That Contemporary Forms Of Knowledge, Rationality, Social Institutions, And Subjectivity Are Contingent Socio-Historical Constructs Of Power And Domination, And Therefore Are Subject To Change And Modification. Foucault's Ultimate Task, Therefore, Is "To Produce A Shift In Thought So That Things Can Really Change" (Quoted In O'Farrell 1989: 39). The Goal Of Foucault's Historico-Philosophical Studies, As He Later Came To Define It, Is To Show How Different Domains Of Modern Knowledge And Practice Constrain Human Action And How They Can Be Transformed By Alternative Forms Of Knowledge And Practice In The Service Of Human Freedom. Foucault Is Concerned To Analyze Various Forms Of The "Limit Experience" Whereby Society Attempts To Define And Circumscribe The Boundaries Of Legitimate Thought And Action. The Political Vision Informing Foucault's Work Foresees Individuals Liberated From Coercive Social Norms, Transgressing All Limits To Experience, And Transvaluing Values, Going Beyond Good And Evil, To Promote Their Own Creative Lifestyles And Affirm Their Bodies And Pleasures, Endlessly Creating And Recreating Themselves. Foucault Denies There Can Be Any Basis For Objective Descriptive Statements Of Social Reality Or Universal Normative Statements That Are Not Socially Conditioned And Locally Bound. He Tries To Show That All Norms, Values, Beliefs, And Truth Claims Are Relative To The Discursive Framework Within Which They Originate. Any Attempt To Write Or Speak About The Nature Of Things Is Made From Within A Rule-Governed Linguistic Framework, An "Episteme," That Predetermines What Kinds Of Statements Are True Or Meaningful. All Forms Of Consciousness, Therefore, Are Socio-Historically Determined And Relative To Specific Discursive Conditions. There Is No Absolute, Unconditioned, Transcendental Stance From Which To Grasp What Is Good, Right, Or True. Foucault Refuses To Specify What Is True Because There Are No Objective Grounds Of Knowledge; He Does Not State What Is Good Or Right Because He Believes There Is No Universal Standpoint From Which To Speak. Universal Statements Merely Disguise The Will To Power Of Specific Interests; All Knowledge Is Perspectival In Character. For Postmodern Theorists Like Foucault, The Appeal To Foundations Is Necessarily Metaphysical And Assumes The Fiction Of An Archimedean Point Outside Of Language And Social Conditioning. Habermas (1987) Rightly Finds Perplexing An Approach That Raises Truth Claims While Destroying A Basis For Belief In Truth, Which Takes Normative Positions While Suppressing The Values To Which They Are Committed. For Critique To Be Justified And Effective, It Should Preserve Standards By Which To Judge And Evaluate, But Foucault's Total Critique Turns Against Itself And Calls All Rational Standards Into Question. **Richard Rorty, The Attack On Theory, And Renunciation Of Radical Politics.** Deleuze's Analysis Begins by Coming to New Understandings of the Concepts of the Image and Movement. The Image, Above All, Is Not A Representation Of Something, That Is, A Linguistic Sign. This Definition Relies Upon The Age-Old Platonic Distinction Between Form And Matter, In Its Modern Saussurean Form Of Signifier-Signified. Rather, Deleuze Wants To Collapse These Two Orders Into One, And The Image Thus Becomes Expressive And Affective: Not An Image Of A Body, But The Body As Image (C1 58). This Collapse Comes About With Reference To Two Philosophers, Henri Bergson And Charles Sanders Pierce. Deleuze Dedicated A Book-Length Study To The Former Entitled Bergsonism (1968), And His Use Of His Notions Of Movement And Time In The Cinema Texts Is Already Presaged By This Text. Movement For Bergson, Deleuze Argues, Is Not Separable From The Object Which Moves: (I Am Not Movement: Italics Mine) They Are Literally The Same Thing. Thus, No Representative Relationship Can Be Established Without Artificially Halting The Flow Of Movement And Thus Misconstruing The Frozen 'Element' As Self-Sufficient. (See the befuddling similarity of holographic principle of Modern Quantum Mechanics and the Deleuzean idea; italics mine) There Is Only The Flow Of Movement Which Expresses Itself In Different Ways. Among Other Things, This Is One Of Deleuze's Critiques Of Phenomenology (C1 56, 60). Thus The Early Cinema Is Characterised For Deleuze By The Reign Of What He Calls The Sensory-Motor Schema. This Schema Is The Unity Of The Viewed And The Eye That Views In Dynamic Movement. This Model Of The Movement-Image Is Precisely The Nature Of Cinema, For Deleuze. It Does Not

Falsify Movement By Extracting Segments And Stringing Them Together In A Representative Fashion, But Creates A Wide Range Of Expressive Images. It Is In Order To Come To Terms With The Varieties Of Movement-Images That Deleuze Turns To Pierce, Who Developed, "The Most Extraordinary Classifications Of Images And Signs . . ." (C2 30). ……. With Some Alterations, Pierce's Semiotic Classifications To Describe The Use Of Movement-Images In Cinema, And Their Centrality Before The Second World War The Movement From The First Text To Cinema 2: The Time-Image Has A Significance Closely Related To Kant's So-Called Copernican Revolution In Philosophy. Up Until Kant, Time Was Subject To The Events That Took Place Within It, Time Was A Time Of Seasons And Habitual Repetition (See (3) (C) Above); It Was Not Able To Be Considered On Its Own, But As A Measure Of Movement (C2 34-5; KCP Iv.). One Element Of Kant's Achievement For Deleuze, As We Have Seen, Is His Reversal Of The Time-Movement Relationship: He Establishes Time Itself As An Element To Which Movement Must Be Subordinated, A Pure Time. In The Cinema, Deleuze Argues, A Similar Reversal Takes Place. The Historico-Cultural Reason Behind This Reversal Is The Event Of World War Two Itself. With The Great Truths Of Western Culture Put So Deeply In Question By The Before Unimaginable Methods Employed And Their Forthcoming Results, The Sensory-Motor Apparatus Of The Movement-Image Are Made To Tremble Before The Unbearable, The Too-Much Of Life's Possibilities, The Potential Of The Present (C2 35). No Longer Could The Dogmatic Truths That Had Guided Society, And Cinema To An Extent, Allow The Apparently 'Natural' Movement From One Thing To The Next In An Habitual Fashion: 'Natural' Links Precisely Lost Their Efficacy. And With The Use Of Unnatural Or False Links, Which Do Not Follow The Sequence Or Narrative Affect Of The Movement-Image, Time Itself, The Time-Image, Is Manifested In Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137)). Rather Than Finding Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest In Order To Gain Any Sort Of Movement Whatsoever. Along With This 'External' Reason, There Is Also For Deleuze A Motivation Within Cinema Itself To Go From The Movement-Image To The Time-Image. The Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation To Truth: As Deleuze Argues With Regard To The Dogmatic Image Of Thought (See (3)(D) Above), There Is The Presupposition That Thought Naturally Moves Towards Truth. Of Course, Deleuze Suggests, Cinema, When Truly Creative, Never Relied Upon This Presupposition, And Yet, "The Movement-Image, In Its Very Essence, Is Answerable To The Effect Of Truth Which It Invokes While Movement Preserves Its Centres." (C2 142). In Questioning Its Own Presuppositions, Deleuze Argues, Cinema Moved Towards A New, Different, Way Of Understanding Movement Itself, As Subordinate To Time. (Wikipedia: Gilles Deleuze). From His Early Writings On, Nietzsche, Like Kierkegaard, Rails Against A Life-Denying Rationalism And Idealist Philosophy Which Champions' Reason Over The Passions. Nietzsche Interprets The "Subject" As A Mere Construct, An Idealized Sublimation Of Bodily Drives, Experiences, And A Multiplicity Of Thoughts And Impulses. This "Little Changeling," On Nietzsche's View, This Subject, "Is Believed in More Firmly than Anything Else on Earth," But Is For Him A Simple Illusion Created Out Of Modern Desperation To Have A Well-Grounded Identity. Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A Subject/Predicate Form, Giving Rise To The Fallacy That The "I" Is A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38). For Nietzsche, "The Doer" Is "Merely A Fiction Added To The Deed -- The Deed Is Everything" (1968b: 45). "The Subject," He Concludes, Is Thus But A Shorthand Expression For A Multiplicity Of Drives, Experiences, And Ideas. In The Spirit Of Enlightenment, Nietzsche Also Polemicizes Against Metaphysics, Arguing That It Illicitly Generalizes From Ideas In One Historical Epoch To The Entirety Of History. Against This Form Of Philosophical Universalism, Nietzsche Argues "There Are No Eternal Facts, Just As There Are No Absolute Truths. Consequently, What Is Needed From Now On Is Historical Philosophizing, And With It The Virtue Of Modesty" (Nietzsche 1986: 13). Castigating Traditional Philosophy And Values From A Critical Enlightenment Perspective, Nietzsche Anticipates Later Postmodern Critiques Of Metaphysics, Assailing The Concept Of Enduring Knowledge, The Notion Of A Transcendental World, And Presenting Metaphysical Thought As A Thoroughly Obsolete Mode Of Thinking. He Attributes The "Metaphysical Need," At The Heart Of

Philosophies Such As Schopenhauer's, To Primitive Yearnings For Religious Consolation For The Sufferings Of Life And He Urges "Free Spirits" To Liberate Them And Pursue Thinking And Living Experimentally (1986: 8). Nietzsche's Attack On Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy Through A Radical Deconstruction Of Modern Theory. But While Deconstructionist Philosophies Typically Terminate In The No, Merely Seeking To Unravel A Positive Modern Value System Into A Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To Reconstruct. Moving Far Away From Schopenhauerian Pessimism, Back Toward A Greek View Of Tragedy, Toward A Dionysian View Of Existence, Nietzsche Seeks "A Justification Of Life, Even At Its Most Terrible, Ambiguous, And Mendacious" (1968a: 521), A Justification Found In Art, Creativity, Independence, And The Emergence Of "Higher Types" Of Humanity. Yet Nietzsche's Perspectivism Denies The Possibility Of Affirming Any Absolute Or Universal Values: All Ideas, Values, Positions, And So On Are Posits Of Individual Constructs Of A Will To Power, Which Are To Be Judged According To The Extent To Which They Do Or Do Not Serve The Values Of Life, Creativity, And Strong Individuality. For Nietzsche There Are No Facts, Only Interpretations, And He Argues That All Interpretation Is Constituted By The Individual's Perspectives And Is Thus Inevitably Laden With Presuppositions, Biases, And Limitations. For Nietzsche, A Perspective Is Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, The More One Can See, And The Better One Can Understand And Grasp Specific Phenomena. To Avoid Limited And Partial Vision One Should Learn "How To Employ A Variety Of Perspectives And Interpretations In The Service Of Knowledge" (Nietzsche 1968a: 119). THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best And Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html. And Just As During Much Of The Twentieth Century, Any Plea For Greater Social Justice Could Be Successfully Damned As Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned In Similar Reactionary Terms: Either Wire Head Hedonism Or Revamped Brave New World. This Response Is Not Just Facile And Simplistic. If It Gains Currency, The Result Is Morally Catastrophic. Of Course, The Abolitionist Issue Rarely Arises. Typically, Universal Bliss Is Still More-Or-Less Unthinkingly Dismissed As Technically Impossible. Insofar As The Prospect Is Even Contemplated - Grudgingly - It Is Usually Assumed That The New Regime Would Be Underwritten Day-By-Day With Drugs Or, More Crudely, Electrodes In The Pleasure-Centres. These Techniques Have Their Uses. Yet In The Medium-To-Long-Term, Stopgaps Won't Be Enough. All Use Of Psychoactive Drugs May Be Conceived As An Attempt To Correct Something Pathological With One's State Of Consciousness. There's Something Deeply Wrong With Our Brains. If What We Had Now Was OK, We Wouldn't Try To Change It. But It Isn't, So We Do. Mature Biological Psychiatry Will Recognise Inadequate Innate Bliss As A Pandemic Form Of Mental Ill-Health: Good For Selfish DNA In The Ancestral Environment Where The Adaptation Arose, But Bad For Its Throwaway Vehicles, Notably Us. The Whole Gamuts Of Behavioural Conditioning, Socio-Economic Reform, Talk-Therapies - And Even Euphoriant Superdrugs - Are Just Palliatives, Not Cures, For A Festering Global Illness. Its Existence Demands A Global Eradication Program, Not Idle Philosophical Manifestos And Scientific Belle's Letters. But One Does One's Best. The Ideological Obstacles To Genetically Pre-Programmed Mental Super-Health Are Actually More Daunting Than The Technical Challenges. To Be Cured, Hypo-Hedonia Must Be Recognised As A Primarily Genetic Deficiency-Disorder. Designer Mood-Brighteners And Anti-Anxiety Agents To Alleviate It Are Sometimes Branded "Lifestyle-Drugs"; But This Is To Trivialize A Serious Medical Condition Which Must Be Corrected At Source. Happily, Our Hereditary Neuropsychiatric Disorder Is Likely To Become Extinct Within A Few Generations As The Reproductive Revolution Unfolds. Aversive Experience, And The Poisonous Metabolic Pathways That Mediate Its Textures, Will Become Physiologically Impossible Once The Genes Coding Its Neural Substrates Have Been Eliminated. We Won't Miss Its Corrupting Effect When It's Gone. In The Medium-Term, The Functional Equivalent Of Aversive Experience Can Help Animate Us Instead. Late In The Third Millennium And Beyond, Its Functional Successors May Be Expressed As Gradients Of Majestic Well-Being. On This Scenario, Our Descendants Will Enjoy A Civilisation Based On Information-Bearing Pleasure-Gradients: Whether Steep Or

Shallow, We Simply Don't Know. Such A Global Species-Project Does Not Have The Desperate Moral Urgency Of Eliminating The Phenomenon Of Darwinian Pain - Both "Mental" And "Physical", Human And Non-Human Alike. Abolishing Raw Nastiness - Sometimes Vile Beyond Belief - Remains The Over-Riding Ethical Priority. One Doesn't Have To Be An Outright Negative Utilitarian To Acknowledge That Getting Rid Of Agony Takes Moral Precedence Over Maximizing Pleasure. But Both Genetic Fundamentalists And Gung-Ho Advocates Of Better Living Through Chemistry Today Agree On One Crucial Issue. There Is No Sense In Sustaining A Legacy Of Mood-Darkening Metabolic Pathways Out Of Superstitious Deference To Our Savage Past. When Bernard Marx Tells The Savage He Will Try To Secure Permission For Him And His Mother To Visit The Other Place, John Is Initially Pleased And Excited. Echoing Miranda In The Tempest, He Exclaims: "O Brave New World That Has Such People In It." Heavy Irony. Like Innocent Miranda, He Is Eager To Embrace A Way Of Life He Neither Knows Nor Understands. And Of Course He Comes Unstuck. Yet If We Swallow Such Fancy Literary Conceits, Then Ultimately The Joke Is On Us. It Is Only Funny In The Sense There Are "Jokes" About Auschwitz. For It Is Huxley Who Neither Knows Nor Understands The Glory Of What Lies Ahead. A Utopian Society In Which We Are Sublimely Happy Will Be Far Better Than We Can Presently Imagine, Not Worse. And It Is We, Trapped In The Emotional Squalor Of Late-Darwinian Antiquity, Which Neither Know Nor Understand The Lives Of The God-Like Super-Beings We Are Destined To Become. **A Defence Of Paradise-Engineering Brave New World By Aldous Huxley Brave New World (1932) Is One Of The Most Bewitching And Insidious Works Of Literature Ever Written.** .

Deleuze attacks Hegel and others in what we can call—though Deleuze did not—the “identitarian” tradition first of all by means of (e) a radicalized reading of Kant, whose genius, as Deleuze explains in Kant's Critical Philosophy (1963), was to have conceived of a purely immanent critique of reason—a critique that did not seek “errors” of reason produced by external causes, but rather “illusions” that arise (eb) from within reason itself by the illegitimate (transcendent) uses of the syntheses of (e) consciousness. Deleuze characterized his own work as a philosophy of immanence, arguing that Kant himself had failed to realize fully the ambitions of his critique, for at least two reasons: first, the failure to pursue a fully immanent critique, and second, the failure to propose a genetic account of (e) real experience, resting content with the account of the conditions of possible experience. First, Kant made the field of consciousness immanent to (e) a transcendental subject, thereby reintroducing an element of identity that is (=) transcendent (that is, external) to (e) the field itself, and reserving all power of synthesis (that is, identity-formation) in the field to the activity of (e) the always already unified and transcendent subject. (Deleuze was influenced in this regard by his reading of Sartre's 1937 essay “The Transcendence of the Ego.”) Already in his Hume book, Empiricism and Subjectivity (1953), Deleuze had pointed to an empiricist reversal of Kant. Where Kant's question had been “How can the given be given to (e) a subject?” Hume's question had been “How is the subject (human nature) constituted within (eb) the given?” Metz asserting a notion of langue with the formal elements associated with narrative. Moreover he narrowed this gap further by claiming a version of langue's syntagmatic and paradigmatic in the operation of cinematic narrative. Deleuze notes the significance of this move in The Time-Image: [L]anguage features which necessarily apply to utterances will be found in the cinema, as rules of use, in the language system and outside of it: the syntagm (conjunction of present relative units) and the paradigm (disjunction of present units with comparable absent units). The semiology of the cinema will be the discipline that applies linguistic model, especially syntagmatic ones, to images as constituting one of their principle "codes" (25-26). (As quoted in Roger Dawkin’s making matter our of Deleuzean concept of cinema) An exemplary and illustrious account of unbounded consciousness has been studied by CORRINA BONSHEK Deleuzian sensation and unbounded consciousness in Anna & Corrina Bonshek’s Reverie I (2002). Entire article has been modelled in one of the paper in the series. It is true that Deleuze sometimes suggests follow of the Model and Shadow theme instead of getting into the trap of consciousness and cogito. This should not be confused with the unbounded consciousness Deleuze mentioned in his studies on Cinema. In his mature work, Deleuze argues for an “impersonal and pre-individual” transcendental field in which the subject as identity pole which produces (eb) empirical

identities by active synthesis is itself the result or product of (e) differential passive syntheses (for instance, in what Deleuze calls the syntheses of habit, we find bodily, desiring, and unconscious "contractions" which unify (e&eb) a series of experiences, extracting (eb) that which it to be retained in the habit and allowing (eb) the rest to be "forgotten"). The passive syntheses responsible for subject formation must be qualified as (=) "differential," for three reasons. Each passive synthesis is (=) serial, never singular (there is never one synthesis by itself, but always a series of (e) "contractions," that is to say, experience is (=) ongoing and so our habits require (e) constant "updating"); each series is related to (e&eb) other series in the same body (at the most basic level, for instance, the series of taste contractions is related to (e&eb) those of smell, sight, touch, hearing and proprioception); and each body is related to (e&eb) other bodies, which are themselves similarly differential (the series of syntheses of bodies can resonate or clash). Together the passive syntheses at all these levels form (eb) a differential field within which subject formation takes place as (=) an integration or resolution of that field; in other words, subjects are (=) roughly speaking the patterns of (e) these multiple and serial syntheses which fold in on themselves producing (eb) a site of self-awareness. Of course, Deleuze never simply proclaims this as a bald thesis, but develops a genetic account of subjectivity in many of his books. Taking all this into account, Deleuze summarized his differential, immanent and genetic position by the at first glance odd phrase of "transcendental empiricism." This is cashed out in terms of two characteristics: (1) the abstract (e.g., "subject," "object," "State," the "whole," and so on) does not explain, but must itself be (=) explained; and (2) the aim of philosophy is not to rediscover (eb) the eternal or the universal, but to find the singular conditions under which (e&eb) something new is produced. In other words—and this is a pragmatic perspective from which Deleuze never deviated—philosophy aims not at stating the conditions of knowledge qua representation, but at finding and fostering (eb) the conditions of creative production. Deleuze's second criticism of Kant claims that he had simply presumed (eb) the existence of knowledge and morality as "facts" and then sought their conditions of possibility in the transcendental But already in 1789, Salomon Maimon, whose early critiques of Kant helped generate the post-Kantian tradition, had argued that Kant's critical project required (e) a method of genesis—and not (e) merely a method of conditioning—that would account for the (eb) production of knowledge, morality, and indeed reason itself. In other words, Maimon called for a genetic method that would be able to reach (eb) the conditions of real and not merely possible experience. Maimon found a solution to this problem in a principle of difference: whereas identity is (=) the condition of possibility of thought in general, it is difference that constitutes (e) the genetic and productive principle of real thought. These two Maimonian exigencies—the search for the genetic conditions of real experience and (e&eb) the positing of a principle of difference—appear in almost every one of Deleuze's early monographs. Nietzsche and Philosophy (1962), for instance, suggests that Nietzsche completed and inverted Kantianism by bringing critique to bear, not simply on false claims to knowledge or morality, but on (eb) true knowledge and true morality, and indeed on truth itself: "genealogy" constituted (e) Nietzsche's genetic method, and the will-to-power was (=) his principle of difference. Deleuze's anti-Hegelianism is shown in his focus on (eb) the productivity of the non-dialectical ("affirmative") differential forces termed by Nietzsche "noble." These forces affirm themselves, and thereby differentiate (e&eb) themselves first, and only secondarily consider that from which they have (e) differentiated themselves. In Bergsonism (1966), Deleuze develops the ideas of virtuality and (e&eb) multiplicity that will serve as (=) the backbone of his later work. From Maimon's reading of Kant, we know that Deleuze needs to substitute the notion of the condition of the genesis of the real for (e&eb) the notion of conditions of possibility of representational knowledge. The positive name for that genetic condition is the virtual, which Deleuze adopts from the following Bergsonian argument. The notion of the, possible Bergson holds in Creative Evolution, is derived from (e) a false problem that confuses the "more" with (e&eb) the "less" and ignores (e) differences in kind; there is not less but more in (eb) the idea of the possible than in the real, just as there is (=) more in the idea of nonbeing than in that of being, or more in the idea of disorder than (e) in that of order. When we think of the possible as (=) somehow "pre-existing" the real, we think of (e) the real, then we add to (e) it the negation of its existence, and then we project (e&eb) the "image" of the possible into the past. We then reverse the procedure and think of the real as (=) something more than possible, that is, as the possible with existence added to (e) it. We then say that the possible has been "realized" in (eb) the real. By

contrast, Deleuze will reject the notion of the possible in favor of (e) that of the virtual. Rather than awaiting realization, the virtual is (=) fully real; what happens in genesis is (=) that the virtual is (=) actualized. The fundamental characteristic of the virtual, that which means (eb) it must be actualized rather than realized, is its differential makeup. Here Deleuze used terms actualisation and realisation as if they are (=) differential in nature Actualisation of the dreams could be tantamount to (=) the realization of the dreams, but the reciprocal is not true. (My interpretation) Deleuze always held the critical axiom that the ground cannot (e) resemble that which it grounds; he constantly critiques the "tracing" operation by which identities in real experience are said to be conditioned (e&eb) by identities in the transcendental. Mention may be made of the Deleuzian statement that the relationship between problem and its conditions bears ample testimony and infallible observatory to the truth of the problem as such. Deleuze writes: …….as for the determination of the conditions it implies (eb) on one hand, a space of nomadic distribution, in which singularities are (=) distributed, on the other hand, it implies, (eb) a time of decomposition whereby a space is (=) subdivided in to subspaces. (See Seventeenth series of Logical genesis: Gilles Deleuze: Translation by Constantin Boundas) **It is to be noted that Deleuze writes in a progressive vein; and the subspaces he mentions are those which incorporate adjunct and corresponding points, which in fact defines progressively a determined domain under consideration.** This also brings us to the question whether there is any space which has singularities in its domain under consideration, and if so effects of such singularities incorporated thereof the space itself. Modem quantum mechanics proves beyond doubt that the happenings in space affect the space and vice versa. Deleuze goes on to state that verily there is a space that incorporates these singularities. What does this mean in the most general sense and to us who are used to quantum mechanical world? It is clear that there exists a spatio temporal self determination of the problem in which (eb) the problem advances, making up for (e) its deficiencies and thwarting (e) its own conditionalities. This can be made possible only by a transcendental power or complete computer simulation ad dissimulation. Mention may be made that many studies of the Universe being a computer has been put forward. And it had its many ramifications in the positive direction, albeit it did not received the much needed accentuation and augmentation when the theory was proposed. There is a mistaken notion that once the solution is obtained, problem is solved. Infact solutions are engendered precisely at (eb) the same moment problem determines itself. Thus sense in its organsiation of problems and solutions, aleatory and (e&eb) singular points, series and displacements are (=) doubly generative. Solutions does not attribute a subjective status to the problem retrospectively as is thought of by the widest commonalty spread when the problem is determined it also determines the solutions under which it persist. Argument also can be put forth from Stoic Philosophy that in order for the thing to be bad, it should have been good. But it has its own limitations as one could discern with circumspection and jurisprudence. For instance, Deleuze criticizes Kant for copying the transcendental field in the image of (e) the empirical field. In fact Kant himself clarifies that the figment of imagination lies dormant like in the mind. If cinnabar were now red, now black, now light, now heavy, if a human being were now changed into this animal shape, now into that one, if on the longest day the land were covered now with fruits, now with ice and snow, then my empirical imagination would never even get the opportunity to think of (e) heavy cinnabar on the occasion of the representation of the color red; or if a certain word were now attributed now to this thing, now to that, or if one and the same thing were sometimes called this, sometimes that, without the governance of (e&eb) a certain rule to which the appearances are already subjected (e&eb) in themselves, then no empirical synthesis of reproduction could take place (CPR, A100-101) as quoted in Real Experience and Possible Experience: Deleuze's Ontology of Experience – Conclusion. This is very much true,. In fact associations are created because of the nonchangeabilityof of the object, notwithstanding certain change in the subject. Such continuous associations are what form the basis of our understanding or misunderstanding (see earlier papers). That is, empirical experience is (=) personal, identitarian and centripetal; there is (=) a central focus, the subject, in which all our experiences are tagged as belonging (e&eb) to us. Mention may be made that Satre established that pre reflexive cogito is primary consciousness notwithstanding the fact that he makes in later work, he makes this his original point of departure. Kant says this empirical identity is only possible if (e) we can posit the Transcendental Unity of Apperception, that is, the possibility of adding "I think" to (e&eb) all our judgments. Instead of this smuggled-in or "traced" identity, Deleuze

will want to have the transcendental field be (=) differential. Deleuze still wants to work back from experience, but since the condition cannot (e) resemble the conditioned, and since the empirical is (=) personal and individuated, the transcendental must be (=) impersonal and pre-individual. Deleuze is of the opinion that the interrogative problem cannot bear resemblance to propositions it subsumes(e) under it; it rather engenders (eb) them as it determines its own conditions and assigns (eb) individual order of individual order of permutation of (e) the propositions within (eb) the framework of generalised signification (See seventeenth series of static genesis) The virtual is the condition for (e) real experience, but it has (e) no identity; identities of the subject and the object are products of (e) processes that resolve, integrate, or actualize (the three terms are synonymous for Deleuze) a differential field. Satre was furious about Descartes for having confused spontaneous doubt which is consciousness with methodological doubt. I may add the later might arise from the non emotional feeling, and the question of consciousness itself would remain slightly confused with Satre as far as the author is concerned. Satre holds that all consciousness is consciousness of something. In other words consciousness is (=) pre reflexive and directive. It is (=) certainly (consciousness) intentional or so it would seem. Author's view is that consciousness is not utterly translucent but is to major extent cultivated through reference groups and peer groups which follow the principle of making a choice of 'coming up in life". "I know" is a tendency that grows in an individual and this is a relation with his objects or information (as we have taken) in mind which is essentially anagrammatic. The Deleuzean virtual is (=) thus not the condition of possibility of (e) any rational experience, but the condition of (e) genesis of real experience. Interrogation itself is (=) the shadow of the problem projected, or rather reconstructed based on (e) empirical propositions. This is true of every parameter like language, region, religion, science. As we have seen, the virtual, as genetic ground of the actual cannot (e) resemble that which it grounds; thus, if we are confronted with actual identities in experience, then (eb) the virtual ground of those identities must be purely differential. Deleuze adopts "multiplicity" from Bergson as the name for such a purely differential field. In this usage, Deleuze later clarifies; "multiplicity" designates the multiple as (=) a substantive, rather than as (=) a predicate. Deleuze has been candid about the fact that there exists no parallelism between logical genesis and (e&eb) ontological genesis. There is rather a relay between logical genesis and ontological genesis which permits every sort of shifting (e&eb) and jamming. It is therefore too simple to argue for the correspondence between the individual and (e&eb) the denotation the person and (e&eb) manifestation, multiple properties or variable properties and (e&eb) signification. (Seventeenth series of logical genesis) The multiple as predicate generates (eb) a set of philosophical problems under (e&eb) the rubric of "the one and the many" (things is one or multiple, one and multiple, and so on). With multiplicity, or the multiple as substantive, the question of the relation between the predicates one/multiple is replaced by (e&eb) the question of distinguishing types of multiplicities (as with Bergson's distinction of qualitative and (er&eb) quantitative multiplicities in Time and Free Will). A typological difference between substantive multiplicities, in short, is substituted for (e&eb) the dialectical opposition of the one and the multiple. In sum, then, against the "major" post-Kantian tradition of Fichte, Schelling, and Hegel, Deleuze in effect posited his (el&eb) own "minor" post-Kantian trio of Maimon, Nietzsche, and Bergson. To these he added a trio of pre-Kantians, Spinoza, Leibniz and Hume, but read through a post-Kantian lens. We have already touched on Deleuze's reading of Hume. Let us now turn to Spinoza, for whom Deleuze's admiration was seemingly limitless; for Deleuze, Spinoza was the "prince" or even the "Christ" of philosophers. There are many Spinozist inheritances in Deleuze, but one of the most important is certainly the notion of univocity in ontology. Univocity—as opposed to its great rivals, equivocity and analogy—is (=) the key to developing (eb) a "philosophy of difference" (Deleuze's term for his project in Difference and Repetition), in which difference would no longer be (=) subordinated to identity. The result is a Spinozism minus (-MD) substance, (=) a purely modal or (=) differential universe. In univocity, as Deleuze reads Spinoza, the single sense of Being frees (e) a charge of difference throughout all that is. In univocal ontology being is said in a single sense of (e) all of which it is said, but it is said of difference itself. What is that difference? Difference is (=) difference in degrees of "power"; in interpreting this term we must distinguish the two French words puissance and pouvoir. In social terms, puissance is (=) immanent power, power to act (e&eb) rather than power to (e) dominate another; we could say that puissance is (=) praxis (in which equals clash or act together) rather than poiesis (in

which others are matter to be formed by the command of a superior, a sense of transcendent power that matches (e&eb) what pouvoir indicates for Deleuze). In the most general terms Deleuze develops throughout his career, puissance is (=) the ability to affect and to be affected, to form (eb) assemblages or consistencies, that is, to form (eb) emergent unities that nonetheless respect the heterogeneity of their components. (Here we see the empiricist theme of the "externality of relations": in an assemblage or consistency, the "becoming" or relation of the terms attains (eb) its own independent ontological status. In Deleuze's favorite example, the wasp and orchid create (eb) a "becoming" or symbiotic emergent unit.). Although Deleuze wrote a touching and certainly important book in tribute to his friend Foucault after the latter's death in 1984, the final important figure in Deleuze's readings of other philosophers is Leibniz, to whom, it must be recalled, Maimon appealed in his criticism of Kant. In 1988, Deleuze published a book on Leibniz entitled The Fold: Leibniz and the Baroque, which added new elements to the reading of Leibniz that appeared in Deleuze's earlier books: an interpretation centered on the concept of the fold, a development of a concept of the Baroque, and a attempt to define a neo-Leibnizianism in terms of (e&eb) contemporary artistic and scientific practices. While The Fold is a fascinating work, we will concentrate here on Deleuze's early reading of Leibniz, which plays an important role in Difference and Repetition. Deleuze pushes Leibniz's thought to a point where Leibniz could never have taken it, given his (Leibniz's) theological presuppositions. This is the point where one begins to consider the virtual domain on its own account, freed from (e) its actualization in a world and its individuals. Never has the enterprise and industry of "demystification" been further to the horizon where a person stands on the threshold of infinity trying to ponder what lies beyond its viel which separates the seen from unseen. Myth is always the expression (e) false infinite and thé disturbance of spirit. One of the most profound constants of Naturalism is to denounce everything that is (=) sadness, everything that is (=) cause of sadness, and everything that needs (e) sadness to exercise its power. Naturalism makes of (e) thought and sensibility (=)an affirmation .Naturalism directs its attack against (e&eb) the prestige of the negative (negativity is not bad and it has its own cognizance and prestige) Naturalism deprives (e) negative of all its power. Naturalism also disallows (e) negativity to speak in the name of philosophy which is unfair. Afterall, the spirit of negative made an appearance out of (e) the sensible and linked (e&eb) the intelligible to (e&eb) One or the Whole. But this one and One Whole as we have stressed many a time is that which cancels each projections, actions, cognitions, perceptions, concurrences, rotations, foldings, motions and shocks in the universe atleast holistically and is conservative. This bears ample testimony and infallible observatory and impeccable demonstration to the fact that One and the Whole is (=) nothingness of thought. On a similar veing appearance might also be (=) nothing ness of sensation. Naturalism is (=) the thought of an infinite sum all of the elements which are not composed at once. Naturalism is (=) also the sensation of finite compounds which are not (e) added up as such with one another. Thus multiplicity is (=) affirmed Multiple of multiple is (=) the object of affirmation .Diverse of diverse is (=) an object of joy. Notwithstanding the fact that gender is no longer widely considered to be a property of individuals, the alternative of viewing it in terms of (e&eb) performativity, where gender is (=) the outcome of linguistic and social performances, unnecessarily limits (e) the possibilities of thinking of gender as(=) a form of multiplicity that is both internally and externally differentiated. Any attempt to move beyond binary thinking in gender relations initiates (eb) a consideration of multiplicity, and the way in which multiplicity is conceptualized exerts a critical influence on (e&eb) the possibilities that are opened up. This article interrogates existing understandings of multiplicity and finds (eb) three actual or possible types - multiplicities of the same, characteristic of feminist approaches which authors critique through (e&eb) a reconceptualization of desire; multiplicities of (e) the third, characterized by anthropological, transgender and queer theory approaches; and multiplicities of difference and dispersion, typified by the rhizomatics and fluid theorizing of Deleuze and Guattari, Grosz and Olkowski. They propose ontology of gender as (=) a creative and productive form of desire, realized as (=) proliferation in Deleuze and Guattari's model of the rhizome. Gender identity is accordingly rethought as (=) immanence, intensity and consistency. **Gender as multiplicity: Desire, displacement, difference and dispersion Stephen Linstead Alison Pullen.** On this score, Deleuze often likes to cite Jorge Luis Borges's famous story, "The Garden of the Forking Paths," in which such a virtual world is described in (eb) the labyrinthine book of a Chinese philosopher named Ts'ui Pên: "In all fiction, when a man is

faced with (e&eb) alternatives, he chooses one at the expense of (e) others. In the almost unfathomable Ts'ui Pên, he chooses—simultaneously—all of (e) them… In Ts'ui Pên's work, all the possible solutions occur, each one being the point of departure for (e) other bifurcations." Leibniz had in fact given a similar presentation of the world at the conclusion of the Theodicy. In Deleuze's transformation of the Leibnizian / Borgesian image, the three Kantian transcendent Ideas of God, World, and Self all take on (eb) a completely different demeanor. . First, God is no longer a Being who compares and chooses (e&eb) the richest compossible world; he has now become (eb) a pure Process that affirms incompossibilities and passes through (e&eb) them. (As the notion of "process" here attests, Deleuze's relation to Whitehead is one of the most important contemporary issues for students of his thought; although the points of comparison are many, Deleuze himself rarely discussed Whitehead, save for several important pages in The Fold.) Second, the world is no longer a continuous world defined by (e) its pre-established harmony; instead, divergences, bifurcations, and incompossibles must now be seen to belong to (e) one and the same universe, a chaotic universe in which divergent series trace (e&eb) endlessly bifurcating paths, and give rise to (eb) violent discords and dissonances that are never resolved into (eb) a harmonic tonality: a "chaosmos," as Deleuze puts it (borrowing a word from Joyce) and no longer a world. In contrast, Leibniz could only save the "harmony" of this world by relegating (e) discordances and disharmonies to other possible worlds—this was his theological sleight of hand. Compare this with the present parallel universe philosophy which has intearctionability as its main thematic and discursive form. Third, selves or individuals, rather than being closed upon the compossible and convergent world they express from within, are now torn open, and kept open through (e&eb)the divergent series and incompossible ensembles that continually pull them outside themselves. The "monadic" subject, as Deleuze puts it, becomes (=) the "nomadic" subject. Pluralism is always linked to (e&eb) multiple affirmation. The interrogation problem does not bear resemblance to (e) the proposition it subsumes under it; it rather engenders them as it determines (eb) its own conditions and assigns (eb) individual order of permutation of the propositions within (eb) the framework of generalised significations and functionalized manifestations. Sensuality is connected with (e&eb) the joy of the diverse (one gentleman here says variety is the spice of life; so diversity is spice and savoury to be palatable). In other words, if Deleuze is Leibnizian, it is only by eliminating the idea of a God who chooses (e&eb) the best of all possible worlds, with (e&eb) its pre-established harmony and well-established selves; in Deleuze, incompossibilities and dissonances belong to (eb) one and the same world, the only world, our world. But they belong to our world as (=) its virtual register; developing the thought of the virtual is one of the great challenges of Deleuze's masterpiece, Difference and Repetition, to which we now turn. **The Philosophy of Difference Difference and Repetition:** Deleuze's historical monographs were, in a sense, preliminary sketches for the great canvas of Difference and Repetition (1968), which marshaled these resources from the history of philosophy in an ambitious project to construct a "philosophy of difference." Following Maimon's critique, Difference and Repetition produces a two-fold shift from the Kantian project of providing (eb) the universal and necessary conditions for (e) possible experience. First, rather than seeking the conditions for possible experience, Deleuze wants to provide an account of the genesis of (e) real experience, that is (=) the experience of this concretely existing individual here and now. Second, the genetic principle to respect the demands of the philosophy of difference must itself be (=) a differential principle. However, despite these departures, Deleuze maintains a crucial alignment with Kant; Difference and Repetition is (=) still a transcendental approach. Here we should remind ourselves that the terms "transcendent" and "transcendental" have (e) opposing significations. Transcendental philosophy in fact critiques (eb) the pretensions of other philosophies to transcend (e) experience by providing (eb) strict criteria for the use of (e) syntheses immanent to experience. On this score, at least, Deleuze aligns himself with Kant's critical philosophy. Three further preliminary notes are in order here. First, as we will discuss in section 4 below, the Capitalism and Schizophrenia project of Deleuze and Guattari will bring to the fore naturalist tendencies that are only implicitly present in the still-Kantian framework of Difference and Repetition. So, although there is some risk of reading backwards in this formulation, we can say that the "of" in the phrase "the experience of (e) this concretely existing individual here and now" is (=) both subjective and objective. It is the experience by (e) human subjects of this individual object in front of it, and it is (=) the experience enjoyed by (e) the concretely existing individual itself, even when that individual is

(=) non-human or even non-living. (Deleuze's panpsychism is treated briefly in Protevi 2011.) Many Physicists' have taken objection to any "quantum information" being expended after life, for Einstein's Field Equations does not characterise such a situation, notwithstanding the fact that the equations provide travel backwards in time and the referential integrity and concomitant relational veracity of the equations are unquestionably true. (Please see earlier papers) It is very consistent to bring in to the incorporation the expositional quality Deleuzian remarks about the quantitative and qualitative aspects of life as is studied in LOS (page 310): Then suddenly there is a click. The subject breaks away (e) from the object divesting (e&eb) it of a part of its colour and substance. There is (=) a rift in the scheme of things and the whole range of objects crumbles in becoming (eb) me; each object transferring (e&eb)its quality to an appropriate subject. The light becomes (=) the eye, and such no longer exists; it is (=) simply the stimulation of the retina. Te smell becomes (eb) the nostril- and the world declares itself (eb) odorless. The song of the wind in the trees is (=) disavowed: it was (=) nothing but the quivering of the timpani……The subject is (=) the disqualified object. My eye is (=) the corpse of the light and color. My nose is (=) all that remains of odors when their unreality becomes manifested and demonstrated. My hand refutes (e) the thing it holds. Thus the problem of awareness is born of (e) anachronism. It implies (eb) the simultaneous existence of the subject and the object whose mysterious relationship to (E&eb) himself he seeks to define. Subject and object cannot (e) exist apart from one another since they are one and the same at first integrated (e&eb) in to the real world and then cast out by (e) it. (I have written from small note made by me years ago; there might be some variations. Kindly pardon me on that count. Essence however remains the same). Second, then, in the demand for genetic principles to account for the real experience of (e) concrete individuals, Deleuze is working in the tradition of the Principle of Sufficient Reason. Third, the notion of "genesis" is (=) itself double; in Chapter 3, Deleuze lays out a dynamic genesis that moves from an encounter with intensity in sensation to (e&eb) the thinking of virtual Ideas, while Chapters 4 and 5 lay out a static genesis that moves from the virtual Idea through (e&eb) an intensive individuation process to (e&eb) an actual entity. We are now ready to discuss the book itself. Murphy 1992 suggests that the first part of the book (the Introduction and Chapters 1 and 2) constitutes Deleuze's treatment of the history of philosophy, while in the second part of the book (Chapters 4 and 5) Deleuze is doing philosophy in his own name. From this point of view, Chapter 3, on the "image of thought," plays a pivotal role, leading us into Deleuze's own philosophy. This transitional role of Chapter 3 is confirmed elsewhere when Deleuze says that the study of the image of thought is (=) the "prolegomena to philosophy" (Negotiations, 149). In Chapters 1 and 2, to find a differential genetic principle, Deleuze works through the history of philosophy to isolate the concepts of "difference in itself" and (e&eb) "repetition for itself" that the assumptions of previous philosophies had prevented from (e) being formulated. "Difference in itself" is (=) difference that is freed from (e) identities seen as metaphysically primary. Normally, difference is conceived of as (=) an empirical relation between two terms which each has (e) a prior identity of its own ("x is different from y"). Deleuze inverts this priority: identity persists, but identity is now a something produced by (e) a prior relation between (e&eb) differentials (dx rather than not-x). Difference is no longer an empirical relation but becomes (=) a transcendental principle that constitutes (e&eb) the sufficient reason of (e) empirical diversity (for example, it is the difference of electrical potential between cloud and ground that constitutes (eb) the sufficient reason of the phenomenon of lightning). In Chapter 2, the concept of "repetition for itself" is produced as (=) repetition that is freed from (e) being repetition of an original self-identical thing so that it can be (=) the repetition of difference. Following the formula of Deleuze's reading of Nietzsche's eternal return, repetition is (=) the return of the differential genetic condition of (e) real experience each time there is (=) an individuation of a concrete entity. Ultimately, then, Difference and Repetition will show (eb) that the individuation of entities is produced by (e) the actualization, integration, or resolution (the terms are synonymous for Deleuze) of a differentiated virtual field of (e) Ideas or "multiplicities" that are themselves changed, via (e&eb) "counter-effectuation," in (eb) each individuating event. Chapter 3 lays out 8 postulates of the "dogmatic image of thought." Between the first four and last four postulates we find a theory of the faculties, which is thus at the crossroads of both the chapter and the book. Let us take up the first four postulates. The first postulate concerns our supposed natural disposition to (e) think; the denial of this (natural disposition to think) is (=) what necessitates our

being forced to think. The second and third postulates concern subjective and (e&eb) objective unity. Subjective unity is captured by (e) the notion of "common sense" such that our faculties of sensation, memory, imagination, and thought work in harmony, while objective unity is captured by (e) the notion of "recognition" such that it is the same object that is sensed, remembered, imagined, and thought. The fourth postulate concerns "representation", a key target of Deleuze's critique. Here difference is submitted to a fourfold structure that renders (eb) difference subordinate to (e) identity: 1) identity in (eb) the concept; 2) opposition of (e) predicates; 3) analogy in (eb) judgment; and 4) resemblance in (eb) perception. A good way to approach Deleuze's notion of representation is via (e&eb) Aristotle and Porphyry. Specific differences are the opposed predicates that function on (eb) a horizon of identity in the concept under division; thus animal is the genus that is divided into rational and irrational as specific differences that enable (eb) the isolation of the species "human." Then, we find that the difference between individuals of the same species is (=) infra-conceptual and can only be made via (e&eb) the perception of resemblances; Theaetetus looks like Socrates but not so much that they cannot be distinguished. Finally, the relation of substance to (e&eb) the other categories is (=) analogical, such that being is (=) said in many ways, but with substance as the primary way in (eb) which it is said. After the first four postulates, we find the theory of the faculties, which will be Deleuze's account of what it means to be (=) "forced" to think in differential rather than identitarian terms. To free a notion of "difference in itself" such that difference need not be thought on the basis of (e) a prior horizon of identity, Deleuze looks for an "encounter," a sensation that cannot (e) be thought, that cannot find (eb) the empirical category under which an object can be recognized, and thus forces (eb) the "transcendent exercise" of the faculty of sensibility, when something can only be sensed. Here we see the dynamic genesis from intensity in sensation to (e&eb) the thinking of virtual Ideas. Each step here has (e) a distinct Kantian echo. The faculties are (=) linked in order; here Deleuze as well as Kant looks to the privilege of sensibility as (=) the origin of knowledge—the "truth of empiricism." With sensibility, pure difference in intensity is grasped immediately in the encounter as (=) thé sentiendum, that which can only be sensed. In the differential theory of the faculties, sensibility, imagination, memory, and thought all "communicate violence" from one to (e&eb) the other—here Deleuze works with the Kantian notion of the sublime as (=) discordant accord of the faculties. The "free form of difference" in intensity moves each faculty and communicates its violence to (e&eb) the next, though in this case there is no supernatural vocation that will redeem (e&eb) the conflict of imagination and reason, as there is in the resolution (e) to the discussion of the sublime in the Critique of Judgment. Rather than a reconciliation of the faculties, with (e&eb) thought, a "fractured self"—here Deleuze takes up Kant's notion of the split (e&eb) between the empirical ego and the transcendental subject—is constrained (e) to think "difference in itself" in Ideas. For now, let us note that two of Deleuze's technical terms, intensity and (e&eb) virtuality, occupy two different places on (eb) this line of dynamic genesis. Intensity is (=) the characteristic of the encounter, and sets off (eb) the process of thinking, while virtuality is (=) the characteristic of the Idea. Extensive differences, such as length, area or volume, are (=) intrinsically divisible. A volume of matter divided into two equal halves produces (eb) two volumes, each having half the extent of the original one. Intensive differences, by contrast, refer to (e) properties such as temperature or pressure that cannot (e) be so divided. Recently scientists have been able to separate the characterstics from the object and this has been a thoroughbred break in quantum mechanics. In this connection, advert to the definition of "error' by Deleuze which we discuss elsewhere in the following. If a volume of water whose temperature is 90º is divided in half, the result is (=) two volumes at the original temperature, not two volumes at 45º. However, the important property of intensity is not that it is (=) indivisible, but that it is (=) a property that cannot (e) be divided without (e) involving a change in kind. The temperature of a volume of water, for instance, can be "divided" by (e) heating the container from below, causing (eb) a temperature difference between the top and the bottom. In so doing, however, we change (e&eb) the system qualitatively; moreover, if the temperature differences reach a certain threshold (if they attain a certain "intensity" in Deleuze's terms), the system will undergo (e&eb) a "phase transition," losing (e) symmetry and changing (e&eb) its dynamics, entering into (eb) a periodic pattern of motion—convection—which displays (eb) extensive properties of size: X centimeters of length and breadth. Drawing on these kinds of analyses, Deleuze will assign (eb) a **transcendental status** to the intensive**:**

intensity, he argues, constitutes (e) the genetic condition of (e) extensive space. Intensive processes are themselves in turn structured by (e&eb) Ideas or multiplicities. An Idea or multiplicity is (=) really a process of progressive determination of differential elements, differential relations, and singularities. Let us take these step-by-step. "Elements" must have no independent existence from (e) the system in which they inhere; phonemes as the elements of the virtual linguistic Idea are (=) an example Deleuze uses in Difference and Repetition. Here it might be said the conjecture is too disconcerting for Deleuze seems to contradict himself from the propositions of logical genesis and ontological genesis as he had so assiduously alluded to before. When phonemes are actualized they enter into differential relations that determine (eb) the patterns of individual languages; thus the English phoneme /p/ is reciprocally determined by (e) its differences from /t/, /b/, /d/, and so on. Finally, these differential relations of an individual language determine (eb) singularities or remarkable points at which the pattern of that language can shift: the Great Vowel Shift of Middle English being an example, or more prosaically, dialect pronunciation shifts. Deleuze also holds that events have such points of inflexion, boiling points et al., For another example—and here, in the applicability of his schema to (e&eb) widely divergent registers, is one of the aspects of Deleuze as metaphysician—let us try to construct the Idea of hurricanes. The differential elements would be material "flows" driven by (e) intensive differences in temperature and pressure but undetermined in form (neither smooth nor turbulent, neither big nor small) and function (neither forming nor destroying of (e) weather events). A suggestion can be made to create indeterminate spaces and modules in respect of such variables like 0/0; infinity/ infinity…….It is not as if such existence of indeterminate constituents are not palatable. An example is that of violation of general relativity at remotest parts of the universe as in case of a bill in transit that remains unsecure in its obligation. These flows qua differential elements enter into relations of reciprocal determination linking changes in any one element to (e&eb) changes in the others; Thus temperature and pressure differences will link changes in air and (e&eb) water currents to each other. Updrafts are related to (e&eb) downdrafts even if the exact relations (the tightness of the links (e&eb) the velocity of the flows) are not yet determined. Finally, at singular points in these relations singularities are determined that mark (eb) qualitative shifts in the system, such as the formation of thunderstorm cells (e&eb) the eye wall, and so on. But this is still the virtual Idea of hurricanes; real existent hurricanes will have (e) measurable values of these variables so that we can move from (e) the philosophical realm of sufficient reason to that of scientific causation. A hurricane is explained by (e) it's Idea, but it is caused by (e) real wind currents driven by (e) real temperature supplied by (e) the sun to (e&eb) tropical waters.. To see how Ideas are transcendental and immanent, we have to appreciate that an Idea is (=) a concrete universal. There is a statement in Bhagavat Geeta that creation is attributed and ascribed to "Yoga maya" of Brahman itself. In fact as had been stated earlier both Brahman and Anti Brahman form two sides of the same coin what with Vishwaroopa of Vishnu standing out for both benediction and belligerence. Advert various papers on the indeterminate and latent thought and manifest action (cf Merton) in various papers on Holographic principle, where in it is clearly stated that the information at the boundary is what is being produced at the universe virtually like a film. Let us for the present state that it is the mind that projects this Universal Mind's thought on the screen of consciousness of individual which is mistaken for objective reality. By Universal Mind we mean cosmic consciousness or Nature's General ledger which has been proposed and defined many a time earlier in the deliberation on various topics like Holographic principle, Quantum Holography, and its concomitance to string theory. This aspect is further discussed and expatiated upon in further papers exclusively. In an early article on Bergson ("The Conception of Difference in Bergson" [1956]), Deleuze gave a particularly helpful example of this notion. In La Pensée ET le Mouvant, Bergson had shown that there are two ways of determining what the spectrum of "colors" has in (eb) common. (1) You can extract from (e) particular colors an abstract and general idea of color ("by removing from the red that which makes (eb) it red, from the blue what makes it blue, from the green what makes it (eb) green"). Or, (2) you can make all these colors "pass through (e&eb) a convergent lens, bringing them to a single point," in which case a "pure white light" is obtained that "makes the (eb) differences between the shades stand out." Please see earlier papers colours are befuddling you" The former case defines a single generic "concept" with (e&eb) a plurality of objects The relation between concept and (e&eb) object is (=) one of subsumption; and the state of difference remains exterior to

(e) the thing. Here Deleuze makes a profound statement. It is virtual statement that concept itself is object and both cannot coexist like subject and object. Startling both in its thematic and discursive form genius of Deleuze finds its pristine glory in Eastern primordial thought. The second case (see item numbered 156) on the contrary, defines (eb) a differential Idea in (eb) the Deleuzean sense: the different colors are no longer objects under (e&eb) a concept, but constitute (e) an order of mixture in coexistence and succession within (eb) the Idea; the relation between the Idea and (e&eb) a given color is (=) not one of subsumption, but one of (e) actualization and differenciation (it si the actualisation and manifestation in space time of the idea that is what we spoke as hologram); and the state of difference between the concept and the object is (=) internalized in (eb) the Idea itself, so that the concept itself has become (=) the object. White light is still a universal, but it is (=) a concrete universal, and not a genus or generality. Verily, this is what is enunciated as holographic principle and "Yoga Maya" of the Brahman-Anti Brahman co creation. Problem in itself is (=) the reality of the genetic element, the complex theme which does not allow itself to be reduced to (eb) propositional thesis and basis. It is one and the same illusion which from the empirical point of view formulates (eb) the problems from (e) the propositions, which function as (=) answers and which from a philosophical point of view and a scientific point of view , defines (eb) the problem through (e&eb) the form of possibility of (e) the corresponding propositions. As long as we define the problem by its possibility of resolvability (eb) we tend to confuse (e&eb) sense with (e&eb) signification and we conceive (eb) condition in (eb) the image of the conditioned. Domains of resolvability are related to (e&eb) the process of self determination of the problem. The series of the problem with the (e&eb) conditions constitute (eb) something unconditioned, determining at once, the conditions of the problem perse, and the domains of resolvability and the solutions present in the (eb) domain. Problem bears no resemblance to the neither to the propositions it subsumes (e) under it, nor to the relation it engenders in the proposition. Problem is (=) not propositional; although it does not exist outside of (e) the propositions which express it. Husserl cannot be followed blindly by the statement that an expression is (=) mere double and necessarily has the same thesis as that which receives it. The Idea of color is (=) thus like white light, which "perplexes" within (e&eb) itself the genetic elements and relations of all the colors, but which is (=) actualized in the diverse colors and their respective spaces. Like the word "problem," Deleuze uses the word "perplexion" to signify (eb) not a coefficient of doubt, hesitation, or astonishment, but (eb) the multiple and virtual state of Ideas. Indeed, Deleuze adopts a number of Neoplatonic notions to indicate (eb) the structure of Ideas, all of which are derived from (e) the root word pli [fold]: perplication, complication, implication, explication, and replication. Interrogation is (=) the shadow of the problem projected, or rater reconstructed based on (e) empirical propositions. Similarly, the Idea of sound could be conceived of as (=) a white noise, just as there is (=) also a white society or a white language, which contains (e) in its virtuality all the phonemes and relations destined to be actualized in (eb) the diverse languages and in (eb) the remarkable parts of a same language. Only a genius with profound thought beyond his space and time and period could have written what is written. Words are more often used to violate the very meaning of sentences which they try to exemplify and presumptuousness forms the bastion and stylobate of many philosophers. Deleuze being a contemporary thinker stands above all in his own right. Constantin Boundas refers to this century as "Deleuzean Century;" which is neither garish nor ostentatious. Attention drawn to remarks on the Difference which is really poised for natural history of philosophy **Deleuze's Readings of Other Philosophers I have added my comments and commentaries of other researchers including that of Deleuze. Where ever the comments are drawn from other sources, attributions are mentioned.**

**@@@



INTRODUCTION

Quantum computation (Source: Wikipedia: Some sentences have been reformulated).For references kindly see Wikipedia

(1) Quantum computing studies theoretical computation systems (quantum computers) that make direct use of (e) quantum-mechanical phenomena, such as superposition and entanglement, to perform operations on (e&eb) data.[1]

(2) Quantum computers are different from digital computers based on transistors. Whereas digital computers require data to be encoded into (e&eb) binary digits (bits), each of which is always in (eb) one of two definite states (0 or 1), quantum computation uses (e) quantum bits (qubits), which can be in (eb) superpositions of states.

(3) A quantum Turing machine is a theoretical model of such a computer, and is also known as the universal quantum computer. Quantum computers share theoretical similarities with non-deterministic and probabilistic computers. The field of quantum computing was initiated by the work of Yuri Manin in 1980,[2] Richard Feynman in 1982,[3] and David Deutsch in 1985.[4] A quantum computer with spins as quantum bits was also formulated for use as (=) a quantum space–time in 1968.[5]

(4) Large-scale quantum computers will be able to solve certain problems much more quickly than any classical computers that use even the best currently known algorithms, like integer factorization using Shor's algorithm or the simulation of quantum many-body systems. There exist quantum algorithms, such as Simon's algorithm, that run faster than any possible probabilistic classical algorithm.[8] Given sufficient computational resources, however, a classical computer could be made to simulate any quantum algorithm, as quantum computation does not violate (e) the Church–Turing thesis. [9]

Basis

(5) A classical computer has a memory made up of bits, where each bit represents (eb) either a one or a zero.
(6) A quantum computer maintains a sequence of qubits. A single qubit can represent (eb) a one, a zero, or any quantum superposition of those two qubit states; a pair of qubits can be in (eb) any quantum superposition of 4 states, and three qubits in any superposition of 8 states.
(7) In general, a quantum computer with n qubits can be in (eb) an arbitrary superposition of up to 2^n different states simultaneously (this compares to a normal computer that can only be in one of these 2^n states at any one time).
(8) A quantum computer operates by setting the qubits in (eb) a controlled initial state that represents (eb) the problem at hand and by manipulating (e&eb) those qubits with a fixed sequence of quantum logic gates.
(9) The sequence of gates to be applied is called a quantum algorithm. The calculation ends with a measurement, collapsing the system of qubits into (eb) one of the 2^n pure states, where each qubit is (=) zero or one.
(10) The outcome can therefore be at most n classical bits of information. Quantum algorithms are often non-deterministic, in that they provide the correct solution only with a certain known probability.
(11) An example of an implementation of qubits for a quantum computer could start with the use of particles with two spin states: **"down"** and **"up"** (typically written | {\downarrow}\rangle and | {\uparrow}\rangle, or |0 {\rangle} and |1 {\rangle}). But in fact any system possessing an observable quantity A, which is conserved under (e&eb) time evolution such that A has (e) at least two discrete and sufficiently spaced consecutive eigenvalues, is (=) a suitable candidate for (e) implementing a qubit. This is true because any such system can be mapped onto (e&eb) an effective spin-1/2 system.

Mechanics

(12) A quantum computer with a given number of qubits is fundamentally different from a classical computer composed of the same number of classical bits. For example, to represent the state of an n-qubit system on a classical computer would require (e) the storage of 2n complex coefficients.
(13) Although this fact may seem to indicate that qubits can hold exponentially more information than their classical counterparts, care must be taken not to overlook the fact that the qubits are (=) only in a **probabilistic superposition of all of their states.**
(14) This means that when the final state of the qubits is measured, (eb) they will only be found in one of the possible configurations they were in before measurement.
(15) Moreover, it is incorrect to think of the qubits as only being in one particular state before measurement since the fact that they were in (eb) a superposition of states before the measurement was made directly affects (e&eb) the possible outcomes of the computation.
(16) Qubits are made up of (e) controlled particles and the means of control (e.g. devices that trap particles and switch them from one state to (e&eb) another).[10]
(17) For example: Consider first a classical computer that operates on (e&eb) a three-bit register. The state of the computer at any time is (=) a probability distribution over the 2^3=8 different three-bit strings 000, 001, 010, 011, 100, 101, 110, 111. If it is a deterministic computer, then (eb) it is in exactly one of these states with probability 1.
(18) However, if it is a probabilistic computer, then (eb) there is a possibility of it being in (eb) any one of a number of different states. We can describe this probabilistic state by (e) eight nonnegative numbers A,B,C,D,E,F,G,H (where A = is the probability that the computer is in state 000, B = is the probability that the computer is in state 001, etc.). There is a restriction that (=) these probabilities sum to 1.
(19) The state of a three-qubit quantum computer is similarly described by (e) an eight-dimensional vector (a, b, c, d, e, f, g, h), called a ket. Here, however, the coefficients can have (e) complex values, and it is (=) the sum of the squares of the coefficients' magnitudes, |a|^2+|b|^2+\cdots+|h|^2, that must equal 1. These squared magnitudes represent (eb) the probability of each of the given states. However, because a complex

number encodes (e) not just a magnitude but also a direction in (eb) the complex plane, the phase difference between any two coefficients (states) represents (eb) a meaningful parameter. This is a fundamental difference between quantum computing and probabilistic classical computing.[11]

(20) If you measure the three qubits, (eb) you will observe a three-bit string. The probability of measuring a given string is (=) the squared magnitude of that string's coefficient (i.e., the probability of measuring 000 = |a|^2, the probability of measuring 001 = |b|^2, etc...).

(21) Thus, measuring a quantum state described by (e) complex coefficients (a,b,...,h) gives (eb) the classical probability distribution (|a|^2, |b|^2, \ldots, |h|^2) and we say that the quantum state "collapses" (eb) to a classical state as a result of (e) making the measurement.

(22) Note that an eight-dimensional vector can be specified in many different ways depending on what basis is chosen for the space. The basis of bit strings (e.g., 000, 001… 111) is known as the computational basis. Other possible bases are unit-length, (e&eb) orthogonal vectors and (e&eb) the eigenvectors of the Pauli-x operator. Ket notation is often used to make (eb) the choice of basis explicit.

(23) For example, the state (a, b, c, d, e, f, g, h) in the computational basis can be written as: a\,|000\rangle + b\,|001\rangle + c\,|010\rangle + d\,|011\rangle + e\,|100\rangle + f\,|101\rangle + g\,|110\rangle + h\,|111\rangle where, e.g., |010\rangle = \left(0,0,1,0,0,0,0,0\right)

(24) The computational basis for a single qubit (two dimensions) is |0\rangle = \left(1,0\right) and |1\rangle = \left(0,1\right).

(25) Using the eigenvectors of the Pauli-x operator, a single qubit is |+\rangle = \tfrac{1}{\sqrt{2}} \left(1,1\right) and |-\rangle = \tfrac{1}{\sqrt{2}} \left(1,-1\right).

Operation

(26) List of unsolved problems in physics: Is a universal quantum computer sufficient to efficiently simulate an arbitrary physical system? We assume the proposition and give a model. We then assume the contradiction and give a model. Models are to be read separately and not simultaneously and coadjutant manner.

(27) While a classical three-bit state and a quantum three-qubit state are both eight-dimensional vectors, they are manipulated quite differently for (e) classical or quantum computation. For computing in either case, the system must be initialized, for example into (eb) the all-zeros string, |000\rangle, corresponding to (e&eb) the vector (1, 0, 0, 0, 0, 0, 0, and 0).

(28) In classical randomized computation, the system evolves (eb) according to the application of stochastic matrices, which preserve (eb/u/+) that the probabilities add up to one (i.e., preserve the L1 norm).

(29) In quantum computation, on the other hand, allowed operations are unitary matrices, which are (=) effectively rotations (they preserve (eb) that the sums of the squares add up to one, the Euclidean or L2 norm). (Exactly what unitaries can be applied depend on∝ the physics of the quantum device.) Consequently, since rotations can be undone by rotating backward, quantum computations are (=) reversible. (Technically, quantum operations can be (=) probabilistic combinations of unitaries, so quantum computation really does generalize (eb) classical computation.

(30) Finally, upon termination of the algorithm, the result needs to be read off. In the case of a classical computer, we sample from the probability distribution on the three-bit register to obtain one definite three-bit string, say 000. Quantum mechanically, we measure the three-qubit state, which is equivalent to (=) collapsing the quantum state down to (e) a classical distribution (with the coefficients in the classical state being the squared magnitudes of (e) the coefficients for the quantum state, as described above), followed by (e) sampling from that distribution. Note that this destroys (e) the original quantum state. Many algorithms will only give the correct answer with a certain probability. However, by repeatedly initializing, running and measuring the quantum computer's results, (eb) the **probability of** getting the correct answer can be increased..

Potential

(31) Integer factorization is believed to be computationally infeasible with an ordinary computer for large integers if they are the product of few prime numbers (e.g., products of two 300-digit primes).[12] By comparison, a quantum computer could efficiently solve this problem using Shor's algorithm to find its factors.

(32) This ability would allow a quantum computer to decrypt many of the cryptographic systems in use today, in the sense that there would be a polynomial time (in the number of digits of the integer) algorithm for solving the problem.

(33) In particular, most of the popular public key ciphers are based on the difficulty of factoring integers or the discrete logarithm problem, both of which can be solved by Shor's algorithm. In particular the RSA, Diffie-Hellman, and Elliptic curve Diffie-Hellman algorithms could be broken

(34). These are used to protect secure Web pages, encrypted email, and many other types of data. Breaking these would have significant ramifications for electronic privacy and security.

(35) However, other cryptographic algorithms do not appear to be broken by those algorithms.[13][14] Some public-key algorithms are based on problems other than the integer factorization and discrete logarithm problems to which Shor's algorithm applies, like the McEliece cryptosystem based on a problem in coding theory.[13][15]

(36) Lattice-based cryptosystems are also not known to be broken by quantum computers, and finding a polynomial time algorithm for solving the dihedral hidden subgroup problem, which would break many lattice based cryptosystems, is a well-studied open problem.[16]

(37) It has been proven that applying Grover's algorithm to break a symmetric (secret key) algorithm by brute force requires time equal to roughly 2n/2 invocations of the underlying cryptographic algorithm, compared with roughly 2n in the classical case,[17] meaning that symmetric key lengths are effectively halved: AES-256 would have the same security against an attack using Grover's algorithm that AES-128 has against classical brute-force search (see Key size).

(38) Quantum cryptography could potentially fulfill some of the functions of public key cryptography.

(39) Besides factorization and discrete logarithms, quantum algorithms offering a more than polynomial speedup over the best known classical algorithm have been found for several problems,[18] including the simulation of quantum physical processes from chemistry and solid state physics, the approximation of Jones polynomials, and solving Pell's equation.

(40) No mathematical proof has been found that shows that an equally fast classical algorithm cannot be discovered, although this is considered unlikely. For some problems, quantum computers offer a polynomial speedup.

(41) The most well-known example of this is quantum database search, which can be solved by Grover's algorithm using quadratically fewer queries to the database than are required by classical algorithms. In this case the advantage is provable.

(42) Several other examples of provable quantum speedups for query problems have subsequently been discovered, such as for finding collisions in two-to-one functions and evaluating NAND trees.

(43) Consider a problem that has these four properties:

(i) The only way to solve it is to guess answers repeatedly and check them,

(ii) The number of possible answers to check is the same as the number of inputs,

(iii) Every possible answer takes the same amount of time to check, and

(iv) There are no clues about which answers might be better: generating possibilities randomly is just as good as checking them in some special order.

(44) An example of this is a password cracker that attempts to guess the password for an encrypted file (assuming that the password has a maximum possible length).

(45) For problems with all four properties, the time for a quantum computer to solve this will be proportional to the square root of the number of inputs. It can be used to attack symmetric ciphers such as Triple DES and AES by attempting to guess the secret key.[19]

(46) Grover's algorithm can also be used to obtain a quadratic speed-up over a brute-force search for a class of problems known as NP-complete.

(47) Since chemistry and nanotechnology rely on understanding quantum systems, and such systems are impossible to simulate in an efficient manner classically, many believe quantum simulation will be one of the most important applications of quantum computing.[20] Quantum simulation could also be used to simulate the behavior of atoms and particles at unusual conditions such as the reactions inside a collider.[21]

(48) There are a number of technical challenges in building a large-scale quantum computer, and thus far quantum computers have yet to solve a problem faster than a classical computer. David DiVincenzo, of IBM, listed the following requirements for a practical quantum computer:[22]

(i) scalable physically to increase the number of qubits;
(ii) qubits that can be initialized to arbitrary values;
(iii) quantum gates that are faster than decoherence time;
(iv) universal gate set;
(v) Qubits that can be read easily.

Quantum decoherence

(49) One of the greatest challenges is controlling or removing quantum decoherence. This usually means isolating the system from its environment as interactions with the external world cause the system to decohere. However, other sources of decoherence also exist.

(50) Examples include the quantum gates, and the lattice vibrations and background nuclear spin of the physical system used to implement the qubits. Decoherence is irreversible, as it is non-unitary, and is usually something that should be highly controlled, if not avoided.

(51) Decoherence times for candidate systems, in particular the transverse relaxation time T2 (for NMR and MRI technology, also called the dephasing time), typically range between nanoseconds and seconds at low temperature.[11] Currently, some quantum computers require their qubits to be cooled to 20 millikelvin in order to prevent significant decoherence.[23]

(52) These issues are more difficult for optical approaches as the timescales are orders of magnitude shorter and an often-cited approach to overcoming them is optical pulse shaping. Error rates are typically proportional to the ratio of operating time to decoherence time, hence any operation must be completed much more quickly than decoherence time.

(53) If the error rate is small enough, it is thought to be possible to use quantum error correction, which corrects errors due to decoherence, thereby allowing the total calculation time to be longer than the decoherence time. An often cited figure for required error rate in each gate is 10−4. This implies that each gate must be able to perform its task in one 10,000th of decoherence time of the system.

(54) Meeting this scalability condition is possible for a wide range of systems. However, the use of error correction brings with it the cost of a greatly increased number of required qubits.

(55) The number required to factor integers using Shor's algorithm is still polynomial, and thought to be between L and L2, where L is the number of bits in the number to be factored; error correction algorithms would inflate this figure by an additional factor of L.

(56) For a 1000-bit number, this implies a need for about 104 qubits without error correction.[24] With error correction, the figure would rise to about 107 qubits.

(57) Note that computation time is about L2 or about 107 steps and on 1 MHz, about 10 seconds. A very different approach to the stability-decoherence problem is to create a topological quantum computer with anyons, quasi-particles used as threads and relying on braid theory to form stable logic gates.[25][26]

Developments

(58) There is a number of quantum computing models, distinguished by the basic elements in which the computation is decomposed. The four main models of practical importance are:

(i) Quantum gate array (computation decomposed into sequence of few-qubit quantum gates)

(ii) One-way quantum computer (computation decomposed into sequence of one-qubit measurements applied to a highly entangled initial state or cluster state)

(iii) Adiabatic quantum computer, based on Quantum annealing (computation decomposed into a slow continuous transformation of an initial Hamiltonian into a final Hamiltonian, whose ground states contains the solution)[27]

(iv) Topological quantum computer [28] (computation decomposed into the braiding of anyons in a 2D lattice)

(v) The Quantum Turing machine is theoretically important but direct implementation of this model is not pursued. All four models of computation have been shown to be equivalent; each can simulate the other with no more than polynomial overhead.

(vi) For physically implementing a quantum computer, many different candidates are being pursued, among them (distinguished by the physical system used to realize the qubits):

(vii) Superconductor-based quantum computers (including SQUID-based quantum computers)[29][30] (qubit implemented by the state of small superconducting circuits (Josephson junctions))

(viii) Trapped ion quantum computer (qubit implemented by the internal state of trapped ions)

(ix) Optical lattices (qubit implemented by internal states of neutral atoms trapped in an optical lattice)

(x) Quantum dot computer, spin-based (e.g. the Loss-DiVincenzo quantum computer [31]) (qubit given by the spin states of trapped electrons)

(xi) Quantum dot computer, spatial-based (qubit given by electron position in double quantum dot)[32]

(xii) Nuclear magnetic resonance on molecules in solution (liquid-state NMR) (qubit provided by nuclear spins within the dissolved molecule)

(xiii) Solid-state NMR Kane quantum computers (qubit realized by the nuclear spin state of phosphorus donors in silicon)

(xiv) Electrons-on-helium quantum computers (qubit is the electron spin)

(xv) Cavity quantum electrodynamics (CQED) (qubit provided by the internal state of trapped atoms coupled to high-finesse cavities)

(xvi) Molecular magnet [33] (qubit given by spin states)

(xvii) Fullerene-based ESR quantum computer (qubit based on the electronic spin of atoms or molecules encased in fullerenes)

(xviii) Linear optical quantum computer (qubits realized by processing states of different modes of light through linear elements e.g. mirrors, beam splitters and phase shifters)[34]

(xix) Diamond-based quantum computer [35][36][37] (qubit realized by electronic or nuclear spin of nitrogen-vacancy centers in diamond)

Bose–Einstein condensate-based quantum computer [38]

(59) Transistor-based quantum computer – string quantum computers with entrainment of positive holes using an electrostatic trap

(60) Rare-earth-metal-ion-doped inorganic crystal based quantum computers [39][40] (qubit realized by the internal electronic state of dopants in optical fibers)

Structured program theorem From Wikipedia, the free encyclopedia

(61) The structured program theorem, also called Böhm-Jacopini theorem, [1][2] is a result in programming language theory. It states that a class of control flow graphs (historically called charts in this context) can compute any computable function if it combines subprograms in only three specific ways (control structures). These are

(i) Executing one subprogram, and then another subprogram (sequence)

(ii) Executing one of two subprograms according to the value of a boolean expression (selection)

(iii) Executing a subprogram until a boolean expression is true (iteration)

(62) The structured chart subject to these constraints may however use additional variables in the form of bits (stored in an extra integer variable in the original proof) in order to keep track of information that the original program represents by the program location. The construction was based on Böhm's programming language P″.

```
(1)  p := 1;
(2)  while p > 0  (eb)do begin
(3)    if p = 1 then (eb) begin
(4)      perform step 1 from (e) the flowchart;
(5)      p := resulting successor step number of (e) step 1 from  (e)the flowchart (0 if no successor);
(6)    end;
(7)    if p = 2 then (eb) begin
(8)      perform step 2 from the flowchart;
(9)      p := resulting successor step of step 2 from the flowchart (0 if no successor);
(10)   end;
(11)   ...
(12)   if p = n then begin
(13)     perform step n from the flowchart;
(14)     p := resulting successor step of step n from the flowchart (0 if no successor);
(15)   end;
(16) End.
```

Origin and variants

(63) The theorem is typically credited[3]:381 to a 1966 paper by Corrado Böhm and Giuseppe Jacopini.[4] David Harel wrote in 1980 that the Böhm–Jacopini paper enjoyed "universal popularity",[3]:381 particularly with proponents of structured programming.

(64) Harel also noted that "due to its rather technical style [the 1966 Böhm–Jacopini paper] is apparently more often cited than read in detail"[3]:381 and, after reviewing a large number of papers published up to 1980, Harel argued that the contents of the Böhm–Jacopini proof was usually misrepresented as a folk theorem that essentially contains a simpler result, a result which itself can be traced to the inception on modern computing theory in the papers of von Neumann and Kleene.[3]:383.

(65) Harel also writes that the more generic name was proposed by H.D. Mills as "The Structure Theorem" in the early 1970s. [3]:381

Single-while-loop, folk version of the theorem

(66) This version of the theorem replaces all the original program's control flow with a single global while loop that simulates a program counter going over all possible labels (flowchart boxes) in the original non-structured program. Harel traced the origin of this folk theorem to two papers marking the beginning of

computing. One is the 1946 description of the von Neumann architecture, which explains how a program counter operates in terms of a while loop.

(67) Harel notes that the single loop used by the folk version of the structured programming theorem basically just provides operational semantics for the execution of a flowchart on a von Neumann computer.[3]:383 Another, even older source that Harel traced the folk version of the theorem is Stephen Kleene's normal form theorem from 1936.[3]:383

(68) Donald Knuth criticized this form of the proof, which results in pseudocode like the one below, by pointing out that the structure of the original program is completely lost in this transformation.[5]:274 Similarly, Bruce Ian Mills wrote about this approach that "The spirit of block structure is a style, not a language. By simulating a Von Neumann machine, we can produce the behavior of any spaghetti code within the confines of a block-structured language. This does not prevent it from being spaghetti."[6]

Implications and refinements

(69) The Böhm-Jacopini proof did not settle the question of whether to adopt structured programming for software development, partly because the construction was more likely to obscure a program than to improve it. On the contrary, it signalled the beginning of the debate. Edsger Dijkstra's famous letter, "Go To Statement Considered Harmful," followed in 1968.[8]

(70) Some academics took a purist approach to the Böhm-Jacopini result and argued that even instructions like break and return from the middle of loops are bad practice as they are not needed in the Böhm-Jacopini proof, and thus they advocated that all loops should have a single exit point. This purist approach is embodied in the Pascal programming language (designed in 1968–1969), which up to the mid-1990s was the preferred tool for teaching introductory programming classes in academia.[9]

(71) Edward Yourdon notes that in the 1970s there was even philosophical opposition to transforming unstructured programs into structured ones by automated means, based on the argument that one needed to think in structured programming fashion from the get go. The pragmatic counterpoint was that such transformations benefited a large body of existing programs.[10] Among the first proposals for an automated transformation was a 1971 paper by Edward Ashcroft and Zohar Manna.[11]

(72) The direct application of the Böhm-Jacopini theorem may result in additional local variables being introduced in the structured chart, and may also result in some code duplication.[12] The latter issue is called the loop and a half problem in this context.[13] Pascal is affected by both of these problems and according to empirical studies cited by Eric S. Roberts, student programmers had difficulty formulating correct solutions in Pascal for several simple problems, including writing a function for searching an element in an array. A 1980 study by Henry Shapiro cited by Roberts found that using only the Pascal-provided control structures, the correct solution was given by only 20% of the subjects, while no subject wrote incorrect code for this problem if allowed to write a return from the middle of a loop.[9]

(73) A simpler result from Kosaraju's paper is that a program is reducible to a structured program (without adding variables) if and only if it does not contain a loop with two distinct exits. Reducibility was defined by Kosaraju, loosely speaking, as computing the same function and using the same "primitive actions" and predicates as the original program, but possibly using different control flow structures.

(74) (This is a narrower notion of reducibility than what Böhm-Jacopini used.) Inspired by this result, in section VI of his highly-cited paper that introduced the notion of Cyclomatic complexity, Thomas J. McCabe described an analogue of Kuratowski's theorem for the control flow graphs (CFG) of non-structured programs, which is to say, the minimal subgraphs that make the CFG of a program non-structured. These subgraphs have a very good description in natural language. They are:

(i) branching out of a loop (other than from the loop cycle test)
(ii) branching into a loop
(iii) branching into a decision (i.e. into an if "branch")

(iv) branching out of a decision

(75) McCabe actually found that these four graphs are not independent when appearing as subgraphs, meaning that a necessary and sufficient condition for a program to be non-structured is for its CFG to have as subgraph one of any subset of three of these four graphs.

(76) He also found that if a non-structured program contains one of these four sub-graphs, it must contain another distinct one from the set of four.

(77) This latter result helps explain how the control flow of non-structured program becomes entangled in what is popularly called "spaghetti code". McCabe also devised a numerical measure that, given an arbitrary program, quantifies how far off it is from the ideal of being a structured program; McCabe called his measure essential complexity.[16]

(78) McCabe's characterization of the forbidden graphs for structured programming can be considered incomplete, at least if the Dijkstra's D structures are considered the building blocks.[17]:274-275[clarification needed]

(79) Up to 1990 there were quite a few proposed methods for eliminating gotos from existing program, while preserving most of their structure. The various approaches to this problem also proposed several notions of equivalence, which are stricter than simply Turing equivalence, in order to avoid output like the folk theorem discussed above.

(80) The strictness of the chosen notion of equivalence dictates the minimal set of control flow structures needed. The 1988 JACM paper by Lyle Ramshaw surveys the field up to that point, as well proposing its own method.[18]

(81) Ramshaw's algorithm was used for example in some Java decompilers because the Java virtual machine code has branch instructions with targets expressed as offsets, but the high-level Java language only has multi-level break and continue statements.[19][20][21] Ammarguellat (1992) proposed a transformation method that goes back to enforcing single-exit.[7]

Letters to Nature: Nature 402, 390-393 (25 November 1999) | doi:10.1038/46503; Received 15 July 1999; Accepted 11 October 1999 Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations Daniel Gottesman1,2 & Isaac L. Chuang3

(82) Algorithms such as quantum factoring1 and quantum search2 illustrate the great theoretical promise of quantum computers; but the practical implementation of such devices will require careful consideration of the minimum resource requirements, together with the development of procedures to overcome inevitable residual imperfections in physical systems3, 4, and 5. Many designs have been proposed, but none allow a large quantum computer to be built in the near future6. Moreover, the known protocols for constructing reliable quantum computers from unreliable components can be complicated, often requiring many operations to produce a desired transformation3, 4, 5, 7, and 8. Here we show how a single technique—a generalization of quantum teleportation9—reduces resource requirements for quantum computers and unifies known protocols for fault-tolerant quantum computation. We show that single quantum bit (qubit) operations, Bell-basis measurements and certain entangled quantum states such as Greenberger–Horne–Zeilinger (GHZ) states10—all of which are within the reach of current technology—are sufficient to construct a universal quantum computer. We also present systematic constructions for an infinite class of reliable quantum gates that make the design of fault-tolerant quantum computers much more straightforward and methodical.

Non-Abelian anyons and topological quantum computation Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma Rev. Mod Phys. 80, 1083 – Published 12 September 2008

(1) Topological quantum computation has emerged as one of the most exciting approaches to constructing (eb) a fault-tolerant quantum computer.
(2) Proposal relies on the existence of (e) topological states of matter whose quasiparticle excitations are (=) neither bosons nor fermions, but are (=) particles known as non-Abelian anyons, meaning that they obey (e&eb) non-Abelian braiding statistics.
(3) Quantum information is stored in (eb) states with multiple quasiparticles, which have (e) a topological degeneracy.
(4) Unitary gate operations that are necessary for (e) quantum computation are carried out by (e) braiding quasiparticles and then measuring (eb) the multiquasiparticle states.
(5) fault tolerance of a topological quantum computer arises from (e) the nonlocal encoding of the quasiparticle states, which makes them immune to (e) errors caused by (er-) local perturbations.
(6) To date, the only such topological states thought to have been found in nature are (=) fractional quantum Hall states, most prominently the ν=5⁄2 state, although several other prospective candidates have been proposed in systems as disparate as ultracold atoms in (eb) optical lattices and thin-film superconductors.
(7) In this review article, current research in this field is described, focusing on the general theoretical concepts of non-Abelian statistics as it relates to (e&eb) topological quantum computation, on understanding non-Abelian quantum Hall states, on proposed experiments to (e) detect non-Abelian anyons, and on (eb) proposed architectures for (e) a topological quantum computer.
(8) Both the mathematical underpinnings of topological quantum computation and the physics of the subject are addressed, using (e) the ν=5⁄2 fractional quantum Hall state as (=) the archetype of a non-Abelian topological state enabling (eb) fault-tolerant quantum computation. **http://dx.doi.org/10.1103/RevModPhys.80.1083**

Fault-tolerant quantum computation for singlet-triplet qubits with leakage errors Sebastian Mehl, Hendrik Bluhm, and David P. DiVincenzo Phys. Rev. B 91, 085419 – Published 20 February 2015

(9) Sebastian Mehl, Hendrik Bluhm, and David P. DiVincenzo describe and analyze leakage errors of (e) singlet-triplet qubits. Even
(10) Though leakage errors are a natural problem for spin qubits encoded using (e) quantum dot arrays, they have obtained little attention in previous studies.
(11) They describe the realization of (e) leakage correction protocols that can be implemented together with (e&eb) the quantum error correction protocol of (e) the surface code.
(12) Furthermore **Sebastian Mehl, Hendrik Bluhm, and David P. DiVincenzo** construct explicit leakage reduction units that need, in the ideal setup, as few as three manipulation steps.
(13) Study shows that leakage errors can be corrected without (e) the need of measurements and at the cost of (e) only a few additional ancilla qubits and gate operations compared to (eu+) standard quantum error correction codes**. http://dx.doi.org/10.1103/PhysRevB.91.085419©2015 American Physical Society**

Universal Adiabatic Quantum Computation via the Space-Time Circuit-to-Hamiltonian Construction David Gosset, Barbara M. Terhal, and Anna Vershynina Phys. Rev. Lett 114, 140501 – Published 6 April 2015

(14) **David Gosset, Barbara M. Terhal, and Anna Vershynina** show how to perform universal adiabatic quantum computation using (e) a Hamiltonian which describes (eb) a set of particles with local interactions on (eb) a two-dimensional grid.
(15) A single parameter in the Hamiltonian is adiabatically changed as (=) a function of time to simulate (e&eb) the quantum circuit.

(16) Authors bound the eigenvalue gap above the unique ground state by mapping model onto (e&eb) the ferromagnetic XXZ chain with (e&eb) kink boundary conditions; the gap of this spin chain was computed exactly by Koma and Nachtergaele using (e) its q-deformed version of SU(2) symmetry.

(17) **David Gosset** et al also discuss a related time-independent Hamiltonian which was shown by Janzing to be capable of universal computation. They observe that in the limit of large system size, the time evolution is equivalent to (=) the exactly solvable quantum walk on Young's lattice.

Majorana Zero Modes and Topological Quantum Computation Sankar Das Sarma, Michael Freedman, Chetan Nayak (Submitted on 12 Jan 2015 (v1), last revised 14 May 2015 (this version, v2))

(18) Sankar Das Sarma, Michael Freedman, Chetan Nayak provide a current perspective on the rapidly developing field of Majorana zero modes in (eb) solid state systems.

(19) They emphasize the theoretical prediction, experimental realization, and potential use of Majorana zero modes in future information processing devices through (e&eb) braiding-based topological quantum computation.

(20) Well-separated Majorana zero modes should manifest (eb) non-Abelian braiding statistics suitable for (e) unitary gate operations for (e) topological quantum computation.

(21) Recent experimental work, following earlier theoretical predictions, has shown (eb) specific signatures consistent with the existence of Majorana modes localized at (eb) the ends of semiconductor nanowires in the presence of (e) superconducting proximity effect.

(22) Sankar Das Sarma, Michael Freedman, Chetan Nayak discuss the experimental findings and their theoretical analyses, and provide a perspective on the extent to which the observations indicate (eb) the existence of anyonic Majorana zero modes in (eb) solid state systems.

(23) Theye also discuss fractional quantum Hall systems (the 5/2 state) in this context. We describe proposed schemes for carrying out braiding with Majorana zero modes as well as the necessary steps for implementing topological quantum computation. **Cite as: arXiv: 1501.02813 [cond-mat.str-el] (or arXiv: 1501.02813v2 [cond-mat.str-el] for this version)**

Shortcuts to non-Abelian braiding Torsten Karzig, Falko Pientka, Gil Refael, and Felix von Oppen Phys. Rev. B 91, 201102(R) – Published 12 May 2015

(24) Topological quantum information processing relies on (e) adiabatic braiding of (e) non-Abelian quasiparticles.

(25) Performing the braiding operations in finite time introduces (eb) transitions out of the ground-state manifold and deviations from (e) the non-Abelian Berry phase.

(26) **Torsten Karzig, Falko Pientka, Gil Refael, and Felix von** show that these errors can be eliminated by (e) suitably designed counterdiabatic correction terms in (eb) the Hamiltonian.

(27) **Torsten Karzig, Falko Pientka, Gil Refael, and Felix von** implement the resulting shortcuts to adiabaticity for (e) simple protocols of non-Abelian braiding and show that the error suppression can be substantial even for (e) approximate realizations of the counterdiabatic terms.

Majoranas in Noisy Kitaev Wires Ying Hu, Zi CAI, Mikhail A. Baranov, Peter Zoller (Submitted on 23 Jun 2015)

(28) Robustness of edge states and non-Abelian excitations of topological states of matter promises quantum memory and quantum processing, which is naturally immune against (e&eb) microscopic imperfections such as static disorder.

(29) However, topological properties will not (e) in general protect quantum system from (e) time-dependent disorder or noise.

(30) **Ying Hu, Zi CAI, Mikhail A. Baranov, Peter Zoller** take the example of a network of Kitaev wires with (e&eb) Majorana edge modes storing qubits to investigate the effects of classical noise in (eb) the crossover from the quasi-static to (e&eb) the fast fluctuation regime.

(31) **Ying Hu, Zi CAI, Mikhail A. Baranov, Peter Zoller** present detailed results for the Majorana edge correlations, and fidelity of braiding operations for (e) both global and local noise sources preserving (eb) parity symmetry, such as (=) random chemical potentials and (e&eb) phase fluctuations.

(32) While in general noise will induce (eb) heating and dephasing, they identify examples of (e) long-lived quantum correlations in presence of (e) fast noise due to motional narrowing, where external noise drives(eb) the system rapidly between the topological and (e&eb) non-topological phases. **Cite as: arXiv: 1506.06977 [quant-ph] (or arXiv: 1506.06977v1 [quant-ph] for this version)**

Hybrid topological-spin qubit systems for two-qubit-spin gates Martin Leijnse and Karsten Flensberg Phys. Rev. B 86, 104511 – Published 24 September 2012

(33) Martin Leijnse and Karsten Flensberg investigate a hybrid quantum system involving (e&eb) spin qubits, based on (e) the spins of electrons confined in (eb) quantum dots, and (e&eb) topological qubits, based on (e) Majorana fermions.

(34) In such a system, gated control of the charge on (e&eb) the quantum dots allows (eb) transfer of quantum information between the spin and (e&eb) topological qubits, and the topological system can be used to facilitate transfer of spin qubits between (e&eb) spatially separated quantum dots and to initialize (eb) entangled spin-qubit pairs.

(35) Martin Leijnse and Karsten Flensberg show that the coupling to (e&eb) the topological system also makes it possible to perform (eb) entangling two-qubit gates on (eb) spatially separated spin qubits.

(36) Two-qubit gates are based on (e) a combination of topologically protected braiding operations, gate-controlled charge transfer between the dots and (e&eb) edge Majorana modes, and measurements of (e) the state of the topological qubits.

Resource Quality of a Symmetry-Protected Topologically Ordered Phase for Quantum Computation Jacob Miller and Akimasa Miyake Phys. Rev. Lett. 114, 120506 – Published 26 March 2015

(37) **Jacob Miller and Akimasa Miyake** investigate entanglement naturally present in (eb) the 1D topologically ordered phase protected with (e&eb) the on-site symmetry group of (e) an octahedron as a potential resource for (e) teleportation-based quantum computation.

(38) **Jacob Miller and Akimasa Miyake** show that, as long as certain characteristic lengths are finite, (eb) all its ground states have (e) the capability to implement (eb) any unit-fidelity one-qubit gate operation asymptotically as (=) a key computational building block.

(39) This feature is intrinsic to the entire phase, in that perfect gate fidelity coincides with (=) perfect string order parameters under (e&eb) a state-insensitive renormalization procedure.

(40) approach may pave the way toward a novel program to classify quantum many-body systems based on (e) their operational use for (e) quantum information processing. http://dx.doi.org/10.1103/PhysRevLett.114.120506© 2015 American Physical Society

Molecular Nanomagnets and Related Phenomena Structure and Bonding Volume 164, 2015, pp 383-430 Date: 22 May 2014 Quantum Computation with Molecular Nanomagnets: Achievements, Challenges, and New Trends Alberto Ghirri, Filippo Troiani, Marco Affronte

(41) Molecular nanomagnets exhibit (eb) quanto-mechanical properties that can be nicely tailored at (eb) synthetic level: superposition and entanglement of quantum states can be created with (e&eb) molecular

spins whose manipulation can be done in (eb) a timescale shorter than their decoherence time, if (e) the molecular environment is controlled in (e&eb) a proper way.

(42) Challenge of quantum computation is to exploit (e) the similarities between the coherent manipulation of (e) molecular spins and algorithms used to (e) process data and solve problems.

Decoherence in adiabatic quantum computation Tameem Albash and Daniel A. Lidar Phys. Rev. A 91, 062320 – Published 17 June 2015

(43) A central question that is repeatedly asked is whether quantum features of the evolution can survive over the long time scales used for (e) quantum annealing relative to standard measures of the decoherence time.

(44) **Tameem Albash and Daniel A. Lidar** reconsider the role of decoherence in (eb) adiabatic quantum computation and quantum annealing using (e) the adiabatic quantum master-equation formalism.

(45) They restrict to the weak-coupling and (e&eb) singular-coupling limits, which correspond to (e&eb) decoherence in (eb) the energy eigenbasis and in the computational basis, respectively.

(46) Authors demonstrate that decoherence in the instantaneous energy eigenbasis does not (e) necessarily detrimentally affect adiabatic quantum computation, and in particular that a short single-qubit T2 time need not (e) imply adverse consequences for (e) the success of the quantum adiabatic algorithm.

(47) **Tameem Albash and Daniel A. Lidar** further demonstrate that boundary cancellation methods, designed to improve (eb+) the fidelity of adiabatic quantum computing in (eb) the closed-system setting, remain beneficial in the open-system setting.

(48) To address the high computational cost of master-equation simulations, they also demonstrate that a quantum Monte Carlo algorithm that explicitly accounts for (e) a thermal bosonic bath can be used to (e) interpolate between classical and (e&eb) quantum annealing.

(49) Study highlights and clarifies the significantly different role played by (e) decoherence in (eb) the adiabatic and circuit models of quantum computing. **http://dx.doi.org/10.1103/PhysRevA.91.062320©2015 American Physical Society**

Springer Theses 2015, pp 97-149 Date: 29 Aug 2014 Cluster States and One-Way Quantum Computation Ryuji Ukai

(50) In this chapter, authors discuss bottom-up theories of continuous-variable cluster states and (e&eb) one-way quantum computation.

(51) They first review formalisms of cluster states using (e) state vectors, stabilizers, and nullifiers.

(52) Second, by applying the extended van Loock-Furusawa criteria, they show a sufficient condition of entanglement for (e) an arbitrary cluster state.

(53) Third, they show that an approximation of any cluster state (Gaussian cluster states) with an arbitrary graph can be generated by (e) using an appropriate network of beam splitters.

(54) As examples of Gaussian cluster states, they show beam splitter networks for (e) the generation of n-mode linear and star cluster states with (e&eb) an arbitrary n.

(55) Fourth, they discuss input-coupling schemes. Authors introduce the squeezer-based input-coupling scheme, which is simpler than the teleportation-based (e) input-coupling scheme.

(56) Fifth, they show several cluster reshaping schemes: erasing, wire shortening with a homodyne measurement, (e&eb) as well as cluster connections with zero, one, or two homodyne measurements.

(57) Sixth, they discuss Gaussian operations in (eb) one-way quantum computation.

(58) As the last issue, we introduce the “delta notation” of one-way quantum computation, where the error derived from (e) finite levels of squeezing is described by (e) nullifier terms of the resource cluster state.

The European Physical Journal Special Topics February 2015, Volume 224, Issue 1, pp 75-88 Date: 05 Feb 2015 Quantum annealing: The fastest route to quantum computation? C.R. Laumann, R. Moessner, A. Scardicchio, S.L. Sondhi

(59) In this review **C.R. Laumann, R. Moessner, A. Scardicchio, S.L. Sondhi** consider the performance of the quantum adiabatic algorithm for the solution of decision problems. They divide the possible failure mechanisms into two sets: small gaps due to quantum phase transitions and (e&eb) small gaps due to avoided crossings inside a phase.

(60) **C.R. Laumann, R. Moessner, A. Scardicchio, S.L. Sondhi** argue that the thermodynamic order of the phase transitions is not (e) predictive of the scaling of the gap with the system size.

(61) On the contrary, **C.R. Laumann, R. Moessner, A. Scardicchio, S.L. Sondhi** also argue that, if the phase surrounding the problem Hamiltonian is a Many-Body Localized (MBL) phase, (eb) the gaps are going to be typically exponentially small and that this follows naturally from (e) the existence of local integrals of motion in the MBL phase.

Optimization of networks for measurement-based quantum computation G. Ferrini, J. Roslund, F. Arzani, Y. Cai, C. Fabre, and N. Treps Phys. Rev. A 91, 032314 – Published 19 March 2015

(62) G. Ferrini, J. Roslund, F. Arzani, Y. Cai, C. Fabre and N. Treps provide a recipe for mitigating the effects of (e) finite squeezing, which affect (e&eb) the production of cluster states and the result of a measurement-based quantum computation in (eb) the continuous variable regime.

(63) To this aim, they derive a compact expression for the unitary matrix which describes (eb) the linear optics network that generates (eb) a certain cluster state from independently squeezed inputs.

(64) G. Ferrini, J. Roslund, F. Arzani, Y. Cai, C. Fabre, and N. Treps show that cluster state from independently squeezed inputs possesses (e) tunable degrees of freedom, which can be exploited to (e) minimize the noise effects. These strategies are readily implementable by several experimental groups**. http://dx.doi.org/10.1103/PhysRevA.91.032314©2015 American Physical Society**

Scalable quantum computation architecture using always-on Ising interactions via quantum feedforward Takahiko Satoh, Yuichiro Matsuzaki, Kosuke Kakuyanagi, William J. Munro, Koichi Semba, Hiroshi Yamaguchi, and Shiro Saito Phys. Rev. A 91, 052329 – Published 28 May 2015

(65) **Takahiko Satoh, Yuichiro Matsuzaki, Kosuke Kakuyanagi, William J. Munro, Koichi Semba, Hiroshi Yamaguchi, and Shiro Saito** propose a way to control the interaction between qubits with (e&eb) always-on Ising interaction.

(66) Unlike the standard method to change the interaction strength with unitary operations, authors fully make use of (e) nonunitary properties of projective measurements so that one can effectively turn the interaction on or off via (e&eb) feedforward.

(67) Scheme is useful for generating two- or three-dimensional cluster states that are (=) a universal resource for (e) fault-tolerant quantum computation with this scheme, and it provides (eb) an alternative way to realize (eb) a scalable quantum processor. http://dx.doi.org/10.1103/PhysRevA.91.0523292015 American Physical Society

Parafermions in a Kagome lattice of qubits for topological quantum computation Adrian Hutter, James R. Wootton, and Daniel Loss (Submitted on 6 May 2015 (v1), last revised 2 Jun 2015 (this version, v2))

(68) Engineering complex non-Abelian anyon models with (e&eb) simple physical systems is crucial for (e) topological quantum computation.

(69) Unfortunately, the simplest systems are typically restricted to (e) Majorana zero modes (Ising anyons). Here authors go beyond this barrier, showing that the Z4 parafermion model of (e) non-Abelian anyons can be realized on (eb) a qubit lattice with only nearest neighbor interactions.
(70) System additionally contains (e) the Abelian D (Z4) anyons as (=) low-energetic excitations.
(71) **Adrian Hutter, James R. Wootton, and Daniel Loss** show that braiding of these parafermions with each other and with (e&eb) the D (Z4) anyons allows (eb) the entire d=4 Clifford group to be generated.
(72) Error correction problem for the model is also studied in detail, guaranteeing (eb) fault-tolerance of the topological operations.
(73) Crucially, since the non-Abelian anyons are engineered through (e&eb) defect lines rather than as (=) excitations, non-Abelian error correction is not (e) required.
(74) Instead the error correction problem is performed on (e&eb) the underlying Abelian model, allowing (eb) high noise thresholds to be realized. **Cite as: arXiv: 1505.01412 [quant-ph] (or arXiv: 1505.01412v2 [quant-ph] for this version)**

Chair: Jarrod McClean, Harvard University Abstract ID: BAPS.2015.MAR.G38.8: G38.00008: What constitutes a resource state for measurement-based quantum computation? 12:39 PM–12:51 PM Preview Abstract MathJax On | Off Eleanor Rieffel (NASA Ames Research Center) Howard Wiseman (Griffith University)

(75) Support for universal measurement-based quantum computation (MBQC) is (=) a sufficient condition for (e) states to be considered resources for (e) MBQC, but seems too strong as (=) a necessary condition given known classes of MBQCs that appear to give an advantage over (e&eb) classical computing but which are not universal.
(76) **Eleanor Rieffel (NASA Ames Research Center) Howard Wiseman (Griffith** University) propose some minimal criteria that states must meet in order to be (=) considered resource states for MBQC.
(77) **They** introduce (PRA 89, 032323 (2014)) the notion of inherently measurement-based computations, and give a series of necessary conditions for (e) families of MBQCs to be considered inherently measurement-based.
(78) **Authors** propose that for a state to be considered a resource for MBQC (e) state must, at minimum, support a family of MBQCs that is (=) inherently measurement-based.
(79) Using these criteria, authors explain why discord-free states cannot be (=) resources for MBQC, in spite of claims to the contrary. They conclude with some open questions. Reference: http://meetings.aps.org/link/BAPS.2015.MAR.G38.8

Wigner Function Negativity and Contextuality in Quantum Computation on Rebits Nicolas Delfosse, Philippe Allard Guerin, Jacob Bian, and Robert Raussendorf Phys. Rev. X 5, 021003 – Published 2 April 2015

(80) **Nicolas Delfosse, Philippe Allard Guerin, Jacob Bian, and Robert Raussendorf** describe a universal scheme of quantum computation by (e) state injection on (eb) rebits (states with real density matrices).
(81) Universal scheme of quantum computation by (e) state injection on (eb) rebits (states with real density matrices) they establish (eb) contextuality and Wigner function negativity as (=) computational resources, extending results of M. Howard et al. [Nature (London) 510, 351 (2014)] to two-level systems.
(82) For this purpose, they define a Wigner function suited to systems of n rebits and prove (eb) a corresponding discrete Hudson's theorem.
(83) They introduce contextuality witnesses for (e) rebit states and discuss the compatibility of result with (e&eb) state-independent contextuality. **http://dx.doi.org/10.1103/PhysRevX.5.021003Published by the American Physical Society**

Quantum computation mediated by ancillary qudits and spin coherent states Timothy J. Proctor, Shane Dooley, and Viv Kendon Phys. Rev. A 91, 012308 – Published 8 January 2015

(84) Models of universal quantum computation in which the required interactions between register (computational) qubits are mediated by (e) some ancillary systems are highly relevant to (e) experimental realizations of a quantum computer.

(85) **Timothy J. Proctor, Shane Dooley, and Viv Kendon** introduce such a universal model that employs (e) a d-dimensional ancillary qudit.

(86) The ancilla-register interactions take the form of (e) controlled displacements operators, with a displacement operator defined on (eb) the periodic and discrete lattice phase space of (e) a qudit.

(87) They show that these interactions can implement controlled phase gates on (eb) the register by utilizing geometric phases that are created when (e) closed loops are traversed in this phase space.

(88) The extra degrees of freedom of the ancilla can be harnessed (e) to reduce the number of operations required for (e) certain gate sequences.

(89) In particular, they see that the computational advantages of the quantum bus (qubus) architecture, which employs (e) a field-mode ancilla, are also applicable to this model.

(90) **Timothy J. Proctor, Shane Dooley, and Viv Kendon** then explore an alternative ancilla-mediated model which employs (e) a spin ensemble as (=) the ancillary system and again the interactions with (e&eb) the register qubits are via (e&eb) controlled displacement operators, with a displacement operator defined on (eb) the Bloch sphere phase space of (e) the spin coherent states of the ensemble.

(91) **Authors also** discuss the computational advantages of this model and its relationship with (e&eb) the qubus architecture**. http://dx.doi.org/10.1103/PhysRevA.91.012308015 American Physical Society**

Mixed Species Ion Chains for Scalable Quantum Computation Wright, John Albert URI: http://hdl.handle.net/1773/33236

(92) Mixed species chains of barium and ytterbium ions are investigated as a tool for (e) building scalable quantum computation devices.

(93) Ytterbium ions provide (eb) a stable, environmentally-insensitive qubit that is easily initialized and manipulated, (e&eb) while barium ions are easily entangled with (e&eb) photons that can allow (eb) quantum information to be transmitted between (e&eb) systems in modular quantum computation units.

(94) Barium and ytterbium are trapped together in (eb) a linear chain in a linear rf trap and their normal mode structure and the thermal occupation numbers of (e) these modes are measured (eb) with a narrow band laser addressing an electric quadrupole transition in (eb) barium ions

(95) Before these measurements, barium ions are directly cooled using (e) Doppler cooling, while the ytterbium ions are sympathetically cooled by (e) the barium.

(96) For radial modes strongly coupled to ytterbium ions the average thermal occupation numbers vary (e&eb) between 400 and 12,000 depending on (∝) ion species configuration and trap parameters.

(97) Ion chain temperatures are also measured using (e) a technique based on ion species reordering.

(98) Surface traps with many dc electrodes provide (eb) the ability to controllably reorder the chain to (e) optimize normal mode cooling, and initial work towards realizing this capability are discussed.

(99) Quantum information can be transferred between ions in a linear chain using (e) an optical system that is well coupled to (e) the motional degrees of freedom of the chain. For this reason, a 532 nm Raman system is developed and its expected performance is evaluated.

State complexity and quantum computation Yu CAI, Huy Nguyen Le, Valerio Scarani (Submitted on 13 Mar 2015)

(100) The complexity of a quantum state may be closely related to (e&eb) the usefulness of the state for quantum computation.

(101) **Yu CAI, Huy Nguyen Le, Valerio Scarani** discuss this link using (e) the tree size of a multiqubit state, a complexity measure that has (e) two noticeable (and, so far, unique) features: it is (=) in principle computable, and (e&eb) non-trivial lower bounds can be obtained, hence identifying (eb) truly complex states.

(102) **Yu CAI, Huy Nguyen Le, Valerio Scarani** review the definition of tree size, together with (e&eb) known results on the most complex three and four qubits states. Moving to the multiqubit case, they revisit a mathematical theorem for proving a lower bound on (eb) tree size that scales superpolynomially in (eb) the number of qubits. Note the model gives the number of qubits necessary for the superpolyomiality.

(103) Next, states with superpolynomial tree size (e&eb) the Immanant states, the Deutsch-Jozsa states, (e&eb) the Shor's states and the subgroup states are described.

(104) **Yu CAI, Huy Nguyen Le, Valerio Scarani** showed that the universal resource state for measurement based quantum computation, the 2D-cluster state, has (e) superpolynomial tree size. Moreover, they showed how the complexity of subgroup states and the 2D cluster state can be verified (e&eb) efficiently.

(105) Question of how tree size is related to (e&eb) the speed up achieved in quantum computation is also addressed. Our model gives a relationship that is based on prediction of the authors.

(106) **Yu CAI, Huy Nguyen Le, Valerio Scarani** show that superpolynomial tree size of the resource state is essential for (e) measurement based quantum computation. Necessary role of large tree size in (eb) the circuit model of (e) quantum computation is (=) still a conjecture; and they prove a weaker version of the conjecture. Cite as: arXiv: 1503.04017 [quant-ph] (or arXiv: 1503.04017v1 [quant-ph] for this version)

Quantum Computation under Micromotion in a Planar Ion Crystal S.-T Wang, C. Shen & L.-M Duan Scientific Reports 5, Article number: 8555 doi:10.1038/srep08555 Received 03 November 2014 Accepted 23 January 2015 Published 25 February 2015

(107) **S.-T Wang, C. Shen & L.-M Duan** propose a scheme to realize scalable quantum computation in a planar ion crystal confined by a Paul trap.

(108) **They** show that the inevitable in-plane micromotion affects (e&eb) the gate design via (e&eb) three separate effects: renormalization of (e) the equilibrium positions, coupling to (e&eb) the transverse motional modes and (e&eb) amplitude modulation in the addressing beam.

(109) **S.-T Wang, C. Shen & L.-M Duan** demonstrate that all of these effects can be taken into account and high-fidelity gates are possible in (eb) the presence of micromotion. Proposal opens the prospect to (e) realize large-scale fault-tolerant quantum computation within (eb) a single Paul trap.

Article abstract Nature Physics 6, 382 - 388 (2010) Published online: 14 March 2010 | doi: 10.1038/nphys1614 A Rydberg quantum simulator Hendrik Weimer1, Markus Müller2, Igor Lesanovsky2, 3, Peter Zoller2 & Hans Peter Büchler1

(110) A universal quantum simulator is a controlled quantum device that reproduces (eb) the dynamics of any other many-particle quantum system with (e&eb) short-range interactions.

(111) This dynamics can refer to (e) both coherent Hamiltonian and dissipative open-system evolution.

(112) **Hendrik Weimer1, Markus Müller2, Igor Lesanovsky2, 3, Peter Zoller2 & Hans Peter Büchler1**propose that laser-excited Rydberg atoms in (eb) large-spacing optical or magnetic lattices provide (eb) an efficient implementation of a universal quantum simulator for spin models involving (e&eb) n-body interactions, including such of higher order.

(113) This would allow the simulation of Hamiltonians of (e) exotic spin models involving (e&eb) n-particle constraints, such as the Kitaev toric code, colour code and lattice gauge theories with (e&eb) spin-liquid phases.

(114) In addition approach provides (eb) the ingredients for dissipative preparation of entangled states based on (e) engineering n-particle reservoir couplings.

(115) basic building blocks of our architecture are (=) efficient and high-fidelity n-qubit entangling gates using (e) auxiliary Rydberg atoms, including (e) a possible dissipative time step through (e&eb) optical pumping which enables (eb) mimicking the time evolution of the system by (e) a sequence of fast, parallel and high-fidelity n-particle coherent and dissipative Rydberg gates.

Induced entanglement enhanced by quantum criticality Qing Ai, Tao Shi, Guilu Long, and C. P. Sun Phys. Rev. A 78, 022327 – Published 19 August 2008

(116) Two-qubit entanglement can be induced by (e) a quantum data bus interacting with (e&eb) two bit entanglement.

(117) **Qing Ai, Tao Shi, Guilu Long, and C. P. Sun** with the quantum spin chain in (eb) the transverse field as an illustration of the quantum data bus, show that such induced entanglement can be enhanced by (e) the quantum phase transition (QPT) of (e) the quantum data bus.

(118) They consider two external spins simultaneously coupled to (e&eb) a transverse field Ising chain.

(119) By adiabatically eliminating (e) the degrees of the chain, the effective coupling between (e&eb) these two spins is obtained.

(120) The matrix elements of the effective Hamiltonian are expressed in terms of (e&eb) the dynamical structure factor (DSF) of the chain.

(121) The DSF is the Fourier transformation of the Green function of (e) an Ising chain and can be calculated numerically by (e) a method introduced by Derzhko and Krokhmalskii [Phys. Rev. B 56, 11659 (1997)].

(122) Since all characteristics of QPT are embodied in (eb) the DSF, the dynamical evolution of (e) the two external spins displays (eb) singularity in the vicinity of the critical point. **http://dx.doi.org/10.1103/PhysRevA.78.022327**

Detection mechanism for quantum phase transition in superconducting qubit array Ying-Dan Wang, Fei Xue, Zhi Song, and Chang-Pu Sun Phys. Rev. B 76, 174519 – Published 27 November 2007

(123) **Ying-Dan Wang, Fei Xue, Zhi Song, and Chang-Pu Sun** describe a mechanism to detect quantum phase transition (QPT) in a system by (e) a coherent probe weakly coupled to it.

(124) **They** illustrate this mechanism by (e) a circuit QED architecture where a superconducting Josephson junction qubit array interacts with (e&eb+) a one-dimensional superconducting transmission line resonator (TLR).

(125) Superconducting qubit array is modeled as (=) an Ising chain in transverse field.

(126) Investigation shows that the QPT phenomenon in the superconducting qubit array can be evidently revealed by (e) the correlation spectrum of TLR output: At the critical point, the drastic broadening of spectrum indicates (eb) the occurrence of QPT. **Ying-Dan Wang, Fei Xue, Zhi Song, and Chang-Pu Sun** also show the generalization of this mechanism to (e&eb) other QPT systems. **http://dx.doi.org/10.1103/PhysRevB.76.174519**

Yong Zhang et al 2008 J. Phys A: Math. Theor 41 055310 doi:10.1088/1751-8113/41/5/055310 Quantum algebras associated with Bell states Yong Zhang1, Naihuan Jing2, 3 and Mo-Lin Ge4

(127) The Bell matrix has become an interesting interdisciplinary topic involving quantum information theory and (e&eb) the Yang–Baxter equation.

(128) Bell matrix is an antisymmetric unitary solution of (e) the braided Yang–Baxter equation and yields (eb) all the Bell states by acting on (e&eb) the product basis.

(129) In this paper, using the Faddeev–Reshetikhin–Takhtadjian (FRT) construction, **Yong Zhang1, Naihuan Jing2, 3 and Mo-Lin Ge4** obtain (eb) quantum algebra associated with (e&eb) the Bell matrix.

(130) **Yong Zhang1, Naihuan Jing2, 3 and Mo-Lin Ge4** explore two characteristic algebraic structures in (eb) its four-dimensional representation.

(131) One is a representation with a composition series, namely, it has (e) irreducible sub representations but is not completely reducible. Other is a direct sum of two-dimensional cyclic representations, and can be spanned by (e) four maximally entangled states as (=) local unitary transformations of (e) the Bell states.

(132) Both of them are expected to be realized in physical systems and exploited in (eb) quantum information theory.

(133) Besides authors present the other quantum algebra associated with (e&eb) the unitary evolution of the Bell states (or the Yang–Baxterization of the Bell matrix).

Quantum Measurement and Quantum Computation in TGD Universe M. Pitk¨anen Email: matpitka6@gmail.com. http://tgdtheory.com/public_html/. August 18, 2015

(134) During years I have been thinking how quantum computation could be carried out in TGD Universe. There are considerable deviations from the standard view. Zero Energy Ontology (ZEO) (e&eb) weak form of NMP dictating (e&eb) the dynamics of state function reduction (e&eb) negentropic entanglement (NE), and (e&eb) hierarchy of Planck constants define (eb) the basic differences between TGD based and (e&eb) standard quantum measurement theory.

(135) TGD suggests (eb) also the importance of topological quantum computation (TQC) like processes with (e&eb) braids represented as (=) magnetic flux tubes/strings along them.

(136) Natural question that popped in my mind was how NMP and Zero Energy Ontology (ZEO) could affect (e&eb) the existing view about TQC.

(137) The outcome was a more precise view about TQC. The basic observation is that the phase transition to dark matter phase reduces (e) dramatically the noise affecting (e&eb) quantum quits

(138) Phase transition to dark matter phase reduces (e) dramatically the noise affecting (e&eb) quantum quits together with (e&eb) robustness of braiding as TQC program raises (eb) excellent hopes about TQC in (eb) TGD Universe.

(139) The restriction to negentropic space-like entanglement (NE) defined by (e) a unitary matrix is something new but does not (e) seem to have any fatal consequences as the study of Shor's algorithm shows.

(140) NMP strongly suggests that when a pair of systems - the ends of braid – suffers (e) state function reduction, the NE must be transferred somehow from (e) the system. How? The model for quantum teleportation allows (eb) identifying a possible mechanism allowing (eb) achieving this.

(141) This mechanism could be fundamental mechanism of (e) information transfer also in living matter and phosphorylation could represent (eb) the transfer of NE according to this mechanism: the transfer of metabolic energy would be at (eb) deeper level transfer of negentropy.

(142) Quantum measurements could be actually seen as (=) transfer of negentropy at deeper level. NE defines an excellent candidate for (e) an analog of error correcting code. If only the diagonal form of the unitary entanglement matrix carries (e&eb) information, the quantization of phases as (=) roots of unity provides (eb) a scenario in which Nature itself would take care of (e&eb) error correction.

Entanglement distribution over quantum code-division-multiple-access networks Chang-long Zhu, Nan Yang, Yu-xi Liu, Franco Nori, Jing Zhang (Submitted on 8 Jul 2015 (v1), last revised 9 Jul 2015 (this version, v2))

(143) **Chang-long Zhu, Nan Yang, Yu-xi Liu, Franco Nori, Jing Zhang** present a method for quantum entanglement distribution over (e&eb) a so-called code-division-multiple-access network, in which two pairs of users share (e&eb) the same quantum channel to transmit (e&eb) information.

(144) The main idea of this method is to use different broad-band chaotic phase shifts, generated by (e) electro-optic modulators (EOMs) and (e&eb) chaotic Colpitts circuits, to encode (e) the information-bearing quantum signals coming from (e) different users, and then recover (e) the masked quantum signals at the receiver side by imposing (e&eb) opposite chaotic phase shifts.

(145) The chaotic phase shifts given to different pairs of users are almost uncorrelated due to (e) the randomness of chaos and thus the quantum signals from different pair of users can be distinguished (e&eb) even when they are sent via (e&eb) the same quantum channel.

(146) It is shown that two maximally-entangled states can be generated between (e&eb) two pairs of users by the proposed method mediated by (e&eb) bright coherent lights, which can be more easily implemented in experiments compared with (e&eb) single-photon lights.

(147) Method is robust under (e&eb) the channel noises if only the decay rates of the information-bearing fields induced by (e) the channel noises are not quite high. Study opens up new perspectives for addressing and transmitting quantum information in future quantum networks. Cite as: arXiv: 1507.02004 [quant-ph] (or arXiv: 1507.02004v2 [quant-ph] for this version)

Noise suppression of on-chip mechanical resonators by chaotic coherent feedback Nan Yang, Jing Zhang, Hui Wang, Yu-xi Liu, Re-Bing Wu, Lian-Qing Liu, Chun-Wen Li, Franco Nori (Submitted on 19 Apr 2015)

(148) **Nan Yang, Jing Zhang, Hui Wang, Yu-xi Liu, Re-Bing Wu, Lian-Qing Liu, Chun-Wen Li, Franco Nori** propose a method to decouple the nanomechanical resonator in optomechanical systems from (e) the environmental noise by (e) introducing a chaotic coherent feedback loop.

(149) They find that the chaotic controller in the feedback loop can modulate (e&eb) the dynamics of the controlled optomechanical system and induce (eb) a broadband response of the mechanical mode.

(150) This broadband response of the mechanical mode will cut off (e) the coupling between the mechanical mode and (e&eb) the environment and thus suppress (e) the environmental noise of (e) the mechanical modes. As an application authors use the protected optomechanical system to act as (=) a quantum memory.

(151) It's shown that the noise-decoupled optomechanical quantum memory is efficient for (e) storing information transferred from (e) coherent or squeezed light**. Cite as: arXiv: 1504.04817 [quant-ph] (or arXiv: 1504.04817v1 [quant-ph] for this version)**

Signatures of chaos in the dynamics of quantum discord Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose Phys. Rev. E 91, 032906 – Published 10 March 2015

(152) **Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose** identify signatures of chaos in (eb) the dynamics of discord in (eb) a multiqubit system collectively modelled as (=) a quantum kicked top.

(153) Evolution of discord between any two qubits is (=) quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity is (=) lost.

(154) As the initial wave function is varied from the regular regions to the chaotic sea, a contour plot of the time-averaged discord remarkably reproduces the structures of the classical stroboscopic map. We also

find surprisingly opposite behavior of two-qubit discord versus entanglement of the two qubits as measured by the concurrence. Our results provide evidence of signatures of chaos in dynamically generated discord. http://dx.doi.org/10.1103/PhysRevE.91.032906©2015 American Physical Society

Pairwise quantum discord for a symmetric multi-qubit system in different types of noisy channels You-neng Guo, Mao-fa Fang, Zeng Ke, Guo-you Wang (Submitted on 17 Jul 2015)

(155) **You-neng Guo, Mao-fa Fang, Zeng Ke, Guo-you Wang** study the pairwise quantum discord (QD) for a symmetric multi-qubit system in (eb) different types of noisy channels, such as phase-flip, amplitude damping (e&eb) phase-damping, and (e&eb) depolarizing channels.

(156) Using the QD and geometric measure of quantum discord (GMQD) to quantify (eb) quantum correlations, some analytical or numerical results are presented.

(157) Results show that, the dynamics of the pairwise QD is related to (e&eb) the number of spin particles N as well as initial parameter θ of (e) the one-axis twisting collective state.

(158) With the number of spin particles N increasing (eb) the amount of the pairwise QD increases.

(159) However, when the amount of the pairwise QD arrives at a stable maximal value, (eb) the pairwise QD is independence of the number of spin particles N increasing.

(160) The behavior of the pairwise QD is (=) symmetrical during a period $0 \leq \theta \leq 2\pi$. . Moreover, we compare the pairwise QD dynamics with the GMQD for a symmetric multi-qubit system in different types of noisy channels. **Cite as: arXiv:1507.04826 [quant-ph] (or arXiv:1507.04826v1 [quant-ph] for this version)**

Quantum many-body systems out of equilibrium J. Eisert, M. Friesdorf & C. Gogolin Nature Physics 11, 124–130 (2015) doi: 10.1038/nphys3215 Received 11 August 2014 Accepted 03 December 2014 published online 03 February 2015

(161) How do closed quantum many-body systems driven out of (e) equilibrium eventually achieve (eb) equilibration? Model answers this question.

(162) And how do these systems thermalize given (e) that they comprise so many degrees of freedom?

(163) Progress in answering these—and related—questions has accelerated in recent years—a trend that can be partially attributed to (e) success with experiments performing (eb) quantum simulations using (e) ultracold atoms and trapped ions.

(164) Here **J. Eisert, M. Friesdorf & C. Gogolin** provide an overview of this progress, specifically in studies probing dynamical equilibration and (e&eb) thermalization of systems driven out of (e) equilibrium by quenches, ramps and periodic driving.

(165) In doing so, they also address topics such as the eigenstate thermalization hypothesis,(e&eb) typicality,(e&eb) transport, (e&eb) many-body localization and (e&eb) universality near phase transitions, as well as future prospects for quantum simulation.

Advances in Quantum Teleportation Stefano Pirandola, Jens Eisert, Christian Weedbrook, Akira Furusawa, and Samuel L. Braunstein (Submitted on 28 May 2015)

(166) Quantum teleportation is one of the most important protocols in quantum information. By exploiting (e) the physical resource of entanglement, quantum teleportation serves (e-) as a key primitive in (eb) a variety of quantum information tasks and represents (eb) an important building block for quantum technologies, with (e&eb) a pivotal role in the continuing progress of (e) quantum communication, quantum computing and quantum networks.

(167) **Jens Eisert, Christian Weedbrook, Akira Furusawa, and Samuel L. Braunstein** review the basic theoretical ideas behind quantum teleportation and its variant protocols. They focus on the main

experiments, together with (e&eb) the technical advantages and disadvantages associated with (e&eb) the use of the various technologies, from photonic qubits and (e&eb) optical modes to atomic ensembles, trapped atoms, and (e&eb+) solid-state systems. Analysing the current state-of-the-art, they finish by discussing open issues, challenges and potential future implementations. Cite as: arXiv:1505.07831 [quant-ph] (or arXiv:1505.07831v1 [quant-ph] for this version)

Unbounded number of channel uses may be required to detect quantum capacity Toby Cubitt, David Elkouss, William Matthews, Maris Ozols, David Pérez-García & Sergii Strelchuk Nature Communications 6, Article number: 6739 doi:10.1038/ncomms7739 Published 31 March 2015

(168) Transmitting data reliably over noisy communication channels is (=) one of the most important applications of information theory, and is well understood for channels modelled by classical physics. However, when quantum effects are involved, we do not know how to compute channel capacities. This is because the formula for the quantum capacity involves (e&eb) maximizing the coherent information over (e&eb) an unbounded number of channel uses.

(169) Entanglement across channel uses can even increase (eb) the coherent information from zero to non-zero.

(170) **Toby Cubitt, David Elkouss, William Matthews, Maris Ozols, David Pérez-García & Sergii Strelchuk** study the number of channel uses necessary to (e) detect positive coherent information. In all previous known examples, two channel uses already sufficed. It might be that only a finite number of channel uses are always sufficient. They show that this is not the case: for any number of uses, there are channels for which the coherent information is zero, but which nonetheless have capacity.

Energy flow in quantum critical systems far from equilibrium M. J. Bhaseen, Benjamin Doyon, Andrew Lucas& Koenraad Schalm Nature Physics 11, 509–514 (2015) doi: 10.1038/nphys3320: Published online 04 May 2015

(171) Characterizing the behaviour of strongly coupled quantum systems out of equilibrium is a cardinal challenge for both theory and experiment.

(172) With diverse applications ranging from the dynamics of the quark–gluon plasma to transport in novel states of quantum matter, establishing universal results and organizing principles out of equilibrium is crucial.

(173) **M. J. Bhaseen, Benjamin Doyon, Andrew Lucas& Koenraad Schalm** present a universal description of energy transport between quantum critical heat baths in arbitrary dimension.

(174) The current-carrying non-equilibrium steady state (NESS) is a Lorentz-boosted thermal state.

(175) In the context of gauge/gravity duality this reveals an intimate correspondence between far-from-equilibrium transport and black hole uniqueness theorems

(176) . **M. J. Bhaseen, Benjamin Doyon, Andrew Lucas& Koenraad Schalm** provide analytical expressions for the energy current and the generating function of energy current fluctuations, together with predictions for experiment.

@@@

The flashes fell upon them; some lay down

And hid their eyes and wept; and some did rest

Their chins upon their clenched hands, and smil'd;

And others hurried to and fro, and fed

Their funeral piles with fuel, and look'd up

With mad disquietude on the dull sky,

The pall of a past world; and then again

With curses cast them down upon the dust,

And gnash'd their teeth and howl'd: the wild birds shriek'd

And, terrified, did flutter on the ground,

And flap their useless wings; the wildest brutes

Came tame and tremulous; and vipers crawl'd

And twin'd themselves among the multitude,

Hissing, but stingless–they were slain for food.

And War, which for a moment was no more,

Did glut himself again: a meal was bought

With blood, and each sate sullenly apart

Gorging himself in gloom: no love was left;

All earth was but one thought–and that was death

Immediate and inglorious; and the pang

Of famine fed upon all entrails–men

Died, and their bones were tombless as their flesh;

The meagre by the meagre were devour'd,

Even dogs assail'd their masters, all save one,

And he was faithful to a corse, and kept

The birds and beasts and famish'd men at bay,

Till hunger clung them, or the dropping dead

Lur'd their lank jaws; himself sought out no food,

But with a piteous and perpetual moan,

And a quick desolate cry, licking the hand

Which answer'd not with a caress–he died.

The crowd was famish'd by degrees; but two

Of an enormous city did survive,

And they were enemies: they met beside

The dying embers of an altar-place

Where had been heap'd a mass of holy things

For an unholy usage; they rak'd up,

And shivering scrap'd with their cold skeleton hands

The feeble ashes, and their feeble breath

Blew for a little life, and made a flame

Which was a mockery; then they lifted up

Their eyes as it grew lighter, and beheld

Each other's aspects–saw, and shriek'd, and died–

Even of their mutual hideousness they died,

Unknowing who he was upon whose brow

Famine had written Fiend. The world was void,

The populous and the powerful was a lump,

Seasonless, herbless, treeless, manless, lifeless–

A lump of death–a chaos of hard clay

The rivers, lakes and ocean all stood still,

And nothing stirr'd within their silent depths;

Ships sailorless lay rotting on the sea,

And their masts fell down piecemeal: as they dropp'd

They slept on the abyss without a surge–

The waves were dead; the tides were in their grave,

The moon, their mistress, had expir'd before;

The winds were wither'd in the stagnant air,

And the clouds perish'd; Darkness had no need

Of aid from them–**She was the Universe**.

Zero is not dwanda. There is nothing like positive zero and negative zero. This is in the context that there are certain things which do not fall in the ambit of neither Dvaita nor Advaita. Dvaita, Advaita visarjane Namah! Reads one sholka! Duality. Assets, Liabilities, space, time, mass energy, chaos, disorder, and void and probably voidlessness. I thought there is lot of similarity between Hawking's I (+) and I (_) and Ramana maharishi's I-I state. Witness consciousness is neuron DNA. I had apprehensions that information is not lost, all actions and passions perpetrated must have been stored somewhere. We lack the sophisticated technology to download it. Aristotle and others (including, especially, Leibniz) have argued that time does not exist independently of the events that occur in time. This view is typically called either "Reductionism with Respect to Time" or "Relationism with Respect to Time," since according to this view; all talk that appears to be about time can somehow be reduced to talk about temporal relations among things and events. The opposing view, normally referred to either as "Platonism with Respect to Time" or as "Substantivalism with Respect to Time" or as "Absolutism with Respect to Time," has been defended by Plato, Newton, and others. On this view, time is like an empty container into which things and events may be placed; but it is a container that exists independently of what (if anything) is placed in it (Source for Aristotle's comment: Wikipedia) Dibbling dicer, didactic dilly, dilly dally dime, obscurantist scatoma, marginal SCHOLIUM, dopey doppelgänger Gerry MANDERING goo sander, first read what you have written carious apparighters and blazoner buster and cadences less caitiff Why this anxiety and anxiousness? What do you think is the reason for it? Do you think you have committed any murder mayhem, plunder, pillage, nemesis apocalypse, cataclysm, CLEPSYDRA or calypso? No! No! ……..The very anxiety is some sorties of a defence mechanism not to do any mistakes or rather improve yourself personality wise! But people are never satisfied! …….. Repeated actions by the same object or the subject affects GD complex. Complex becomes rug with a shallow texture…….Is life matter or matter life?.....They always want more and more!..... That is their ego being projected for you in you on their screen of consciousness! ……It is an internal form of evolution. You see! Innocence does not foreclose options of perversity!..... As a mathematician, I would say the Freudian view that there is a decisive importance in the development of the ego, because the perception consciousness is situate at the membrane! Which membrane are you talking of? Protoplasmic membrane, of-course let there be no confusion about it! So i say that ego is the product of primary narcissism!..... Test of ego lies in the problem of coop rationality, coordination not of others but of itself? Itself? What do you mean? As an intention ego is most innocent and innocuous! It is not a vice, but a virtue! It is just like creating total surfaces from the partial surfaces, so that one's Samara i.e. what one projects outwards is beneficial. Beneficial? To whom? To the ego holder? Heh! Heh! Heh! Normal person like me is more immoral than he thinks! But he is also more moral than he suspects! That gives you the force of embarking upon venture. You mean ego? Yes! As a surface phenomenon, it marks the failure of illness, or some discrepancy in the personality. This has considerable positive importance in the life of person, and plays a vital role in achievement of his primary orders and secondary organisation! What the hell is it! The societal order and the personal comfort nothing less nothing more! So we might say that entire action is projected on a double screen one screen is yours and the other is belonging to the "other". This surface, i have not seen, it is transcendental, metaphysical. According to the necessities of the screen ego" "mutates" itself to produce "necessary" "projection" for you! One is mamakara, i.e" the other "ahamkara" you are neither! Then? Nothing? Absolute

nothingness pervades you .....!!!

What do you mean!

Illusion is preferred to bitter truths!

Because of many interpretation of every wavefunction collapse characterising a situation!

But how can there be such a thing as God is there if you think so, and God is not there if you think so like some people say!

Such phenomenon cannot exist!

Then?

Rather you cannot think beyond that!

Utch!

One thinking for green, one for yellow, one for red and so on……. !

Yes!

But conceptually they are sound!

There is no reason to think so!

What is the reason to think otherwise!

Nature is hoodwinking and browbeating at every stage!

Body is the product of mind!

What?

Yes!

Thought is life!

You become what you think!

Like wave particle duality, she has other dualities too!

Yes!

Psi of (GTNT) in Dirac Brakets = + Psi (GT) e (GT) in Dirac Brakets - Psi (GNT) e (GNT) in Dirac Brakets!

Model is applicable!

You said what is “Yes” is “No” and what is “No” is “Yes”!

Yes!

Then what happens to above equation!

It remains the same with affirmation as the second term and negativity as the first!

That way consciousness is conditioned mind!

Mind conditions consciousness!

Dissimulations make consciousness for an attack that is unprepared!

So?

Change your consciousness!

Preposterous and way to untruth!

Somebody said there might be superdeterminism at Big bang but Physicists are not bothered about that!

Yes!

"Self", the individual general ledger is thus always dynamic

Psi (SELF) in Dirac Brakets = + C (1) (D (1)) in Dirac Brakets +……++ C (n) (D (n)) in Dirac Brakets!

Psi (SELF) in Dirac Brakets = - C (1) (D (1)) in Dirac Brakets +……- C (n) (D (n)) in Dirac Brakets!

Well! Both the possibilities exist!

Simultaneously?

Yes!

On the other hand, if we were to measure one of the subsystems, in such a way that the measurement distinguishes between the states 0 and 1 (eb), we will leave behind either $|00\rangle$ or $|11\rangle$ which are unentangled pure states. This is unlike the W state which leaves bipartite entanglements even when we measure one of its subsystems. **Greenberger–Horne–Zeilinger state (Wikipedia)!**

May be or might not be!

It is true for certain systems!

Einstein's theory might not be applicable in some remote corner of the Universe!

So!

That is not the raison d'etre and Fon ET origio for dismissal!

But we stick to it until we find the singularity!

Yes!

Truth +- Illusion +- Bohm's variables (Consciousness)

GHZ state leads to (eb) striking non-classical correlations (1989). Particles prepared in this state lead to (eb) a version of Bell's theorem, which shows the internal inconsistency of the notion of elements-of-reality introduced in (eb) the famous Einstein–Podolsky–Rosen paper**!**

It is quantum state I am talking about! That is clear!

Thoughts are themselves created and they themselves resort to creation of "unreasoning of self conscious" either for information delineation and dissemination and suppression!

OK!

It is also true that everyone is forced to act according to T (Tamas) R (Rajas) S (Sattva) complex composition!

Yes!

There is a verse in Gita which states world emanated from God's Yoga Maya!

You mean God also has illusions?

No!

It is the same energy that manifests as Yoga maya and Maha maya!

Like Chit Shakti and Maya Shakti!

Yes!

It looks equivocal!

It is the same energy that brings light to your houses and destroyed Hiroshima and Nagasaki!

Well you are not insinuating that E=mc^2 is simulation?

I donot know!

Distraction of attention is followed to hide the progress or lack of it!

May be E=mc^2 + Bohm's Variables or hidden variables'!

Life and death is one!

What is life but a gasp of a baby in sultry evening, a flash of lighting, or a shadow that loses identity when there uiis no sun?

Death not only packs off consciousness but also unconsciousness!

So will I build my altar

In the fields

And the blue sky my

Fretted dome shall be

And the fragrance that the wild flower yields

Shall be the incense

I will yield to tee

(Coleridge)

Yes!

Pain and pleasure?

So often we believe that we have come to a place that's is void of hope and possibilities, only to find it is the very hopelessness that allows us to hit rock bottom, give up our illusion of future, turn it over and cry for help. It is out of the ashes of hopelessness we get hope! There has to be complete destruction for creation!

Yes!

Gratification and Deprivation?

Yes!

You have come round to the same point!

That is Maya!

What?

You cannot think beyond that anagrammatism!

So?

Anagrammatism kills truth!

You are thinking conditioned in the phenomenological condition itself!

Condition and conditioned are different!

Certainly!

Like Hospital ward and the patient!

You are identifying with the body!

If I donot it shall be identity crisis!

Well there is identitylessness!

Neither this identity nor that identity till the end of infinity!

There is no end for infinity!

OK!

Psi (IDLN) in Dirac Brakets =- C (I (1)) e (I (1)) in Dirac Brakets ……-- C (I (n)) e (I (n)) in Dirac Brakets

Yes!

Some thing like neither this identity nor that identity till infinity!

Donot talk of infinity!

OK! Some finite number!

Yes!

Like Neti Neti!

Yes!

That is liberation?

Yes!

Is it deathlike state?

I donot know! It looks like that!

Psi (Absolute) in Dirac Brakets = -C(S (1)) e(S (1) in Dirac Brakets……- C(S (1)) e(S (1) in Dirac Brakets!

Yes! Neither this state nor that state!

Looks very consistent at the Planck's scale!

Highest is a higher aspect of oneself!

Higher consciousness is the consciousness of a higher Self, transcendental reality, or God. It is "the part of the human being that is capable of transcending animal instincts".[1] The concept developed in German Idealism, and is a central notion in contemporary popular spirituality. The better consciousness in me lifts me into a world where there is no longer personality and causality or subject or object. My hope and my belief is that this better (supersensible and extra-temporal) consciousness will become my only one, and for that reason I hope that it is not God. But if anyone wants to use the expression God symbolically for the better consciousness itself or for much that we are able to separate or name, so let it be, yet not among philosophers I would have thought (Wikipedia)

That is transcendental to your own consciousness!

It might be accentuated or dissipated reality!

When you sleep talk you are reacting to a situation just as you do in waking state!

Merging individual consciousness to cosmic consciousness is the sole aim of life!

Shiva and Shakti are opposite in thematic and discursive forms!

Yes!

Why not at the deterministic level?

It is gravity! (See recent article which states that gravity is responsible for the subterfuge at cosmic level the happenings, axiomatic predications, dualities observed at the micro level!)

Einstein again saves the face and salvages the situation!

Yes!

You understand you are bringining space time by referring to gravity!

I do!

You also said TRS is the characterstics of space time!

That's enough to create the Maha mAyA!

Why do they refuse to accept that there are possibilities that evidence can be eradicated and witnesses could be muzzled!

I donot know!

Everyone knows it!

Evidence is important in science!

I am trying to say mind tries to interpret things in the light of the consideration of the experiences, projections, cognitions, of the past!

OK!

Apprehensions do arise!

Like?

When you see red and it is somebody who you know showing and you have some deal pending, you tend to think that it is endangered!

Yes!

There is no evidence for it until you go and see it!

Yes!

"What is realized as the Absolute Brahman is full of unlimited bliss without grief. That is certainly the ultimate phase of the supreme enjoyer, the Personality of Godhead. He is eternally devoid of all disturbances, fearless, completely conscious as opposed to matter, uncontaminated and without distinctions. He is the principal, primeval cause of all causes and effects, in whom there is no sacrifice for fruitive activities and in whom the illusory energy (still related to him) does not stand." **Sri Caitanya-caritamrta CC Madhya-lila (Source: Wikipedia)**

That seems contradictory!

Truth is contradictory!

There is serpent, there is rope, there is shadow and there is substance!

So?

They are related to each other!

But shadow is shadow!

Shadow does not exist without substance!

So Satchidananda is a state of knowing where ignorance exists as a shadow!

Psi (Satchidananda) in Dirac Brakets = + Psi (knowledge) e (knowledge) in Dirac Brakets - Psi (ignorance) e (ignorance) in Dirac Brakets

Yes!

I am the one seated in the hearts of all beings ……...I have no beginning, no middle nor end…….!

You are talking of cosmic consciousness!

Yes! Cosmic self!

Nature's General Ledger!

Psi Cosmic consciousness in Dirac brakets = - Psi (Beginning) in Dirac Brakets C (Beginning)- - Psi (Middle) in Dirac Brakets C (Middle)- Psi (End) in Dirac Brakets C (End) !

Yes!

There is no real line in pure consciousness?

It looks like!

Beginning = Middle = End!

Yes!

Tamas=Rajas=Sattva

Non linearity cannot be perceived by mind!

So?

It is necessary to add Bohm's variables too!

What in effect is everything?

Nothing!

In enlightenment also there is nothing!

You carry water after enlightenment as you did before!

That is extreme mortification!

There must be some difference between bank experience and Brahman experience!

Both are same!

That is why there is little difference between great scientists' and great artists except repetitive thinking!

TBAB and CBAB also compare that way!

Vacuum?

Void!

Just screen!

Atleast that exists!

There is no screen also!

Where ever there exists matter there is consciousness!

Donot get repetitive! I have heard that before!

Show must go on!

In a way there are no different quantum states in Advaita!

Yes!

It doth exist in space time Dvaita!

Yes!

Dvaita comes from Advaita!

Yes!

Model was give when space and time are equivalent etc,

Yes!

Every action then leads to evolution!

Evolution is nothing!

Journey is destination!

There is no justification!

Did not Christ say He is both path and destination?

Yes!

Psi (Christ) in Dirac Brakets = Psi (path) in Dirac Brakets c (path) = C (destination) Psi (Destination) in Dirac Brakets!

Yes!

Accentuation Attrition models are applicable!

What do you get out of it!

May be result for some million years!

Not Christ?

No!

Why so?

You fail so that somebody succeeds!

Morris Tua Vita Mia!

I am not sure!

You said this is a debit world!

Yes!

At Planck's scale!

At Newtonian scale it does not appear due to gravity!

You also said it is a credit- credit world!

Answer is the same!

Bizarre quantum world!

Most of the hymns refer God as Maya which is insatiable and unsaturated!

More you think more turbid waters shall become!

I have to think!

Do not have hight reactionary potential!

What do you mean?

I have seen you angry on some issues!

This is mathematics!

So?

There is no place for emotion!

Why so?

I might give a speculative theory or assume superdeterminism with correct mathematics following!

Ugh!

Not again "No mind Condition"!

Two word documents opened when I double clicked!

So?

Donot attribute motives to everything!

Somebody is tampering with the system!

Nobody is tampering with nature!

Why nature itself does it!

Or your mind is enough to do it!

You always said life is a dream, what then death is?

Rappoteur!

Between truth and illusion!

Double slit experiment is too old to be recount!

Donot recount!

Why?

I have seen you recount things in the past and feel gratified at your performance and revel in glory!

How many times they have recounted "Einstein's greatest blunder" before they wrote their papers!

So?

Recounting makes people angrier!

But accelerating universe theory is tested!

(1) Penrose says quantum gravity might not remove singularities! (See Page 36: Nature of space and Time: Hawking and Penrose)
(2) If it were the case then big bang would have resulted from a collapsing phase!
(3) Singularities' of collapsing and expanding universes' have to joined(e&eb) despite different geometries
(4) Theory of quantum gravity should replace (e&eb) concept of space and time at singularity

(5) Initial type (P) singularities are constrained(e) to have (e) vanishing Weyl Tensor
(6) If the universe is closed, then (eb) final universe will have diverging Weyl tensor
(7) Created black holes in open universe have (e) diverging Weyl tensor
(8) COBE and second law of thermodynamics implies that big bang singularity was extremely uniform
(9) White holes violate(e) second law of thermodynamics

Quantum Gravity And Singularities'

INTRODUCTION—VARIABLES USED

(1) Penrose says quantum gravity might not remove singularities! (See Page 36: Nature of space and Time: Hawking and Penrose)
(2) If it were the case then big bang would have resulted from a collapsing phase!
(3) Singularities' of collapsing and expanding universes' have to be joined(e&eb) despite different geometries
(4) Theory of quantum gravity should replace (e&eb) concept of space and time at singularity
(5) Initial type (P) singularities are constrained(e) to have (e) vanishing Weyl Tensor
(6) If the universe is closed, then (eb) final universe will have diverging Weyl tensor
(7) Created black holes in open universe have (e) diverging Weyl tensor
(8) COBE and second law of thermodynamics implies that big bang singularity was extremely uniform
(9) White holes violate(e) second law of thermodynamics
(10) Plato said time is (=) parallelogram!

NOTATION

Module One

Penrose says quantum gravity might not remove singularities! (See Page 36: Nature of space and Time: Hawking and Penrose)

G_{13} : Category one of removal of singularities (note the double negative)

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of quantum gravity

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

If it were the case then big bang would have resulted from a collapsing phase!

G_{16} : Category one of quantum gravity might not remove singularities

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of big bang would have resulted from a collapsing phase

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Singularities' of collapsing and expanding universes' have to be joined (e&eb) despite different geometries

G_{20} : Category one of Singularities' of expanding universes'; Singularities' of collapsing universes'

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Singularities' of collapsing universes'; Singularities' of expanding universes'

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Theory of quantum gravity should replace (e&eb) concept of space and time at singularity

G_{24} : Category one of quantum gravity; space and time at singularity

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of space and time at singularity ;quantum gravity

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Initial type (P) singularities are constrained to have (e) vanishing Weyl Tensor

G_{28} : Category one of vanishing Weyl Tensor

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Initial type (P) singularities are constrained

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

If the universe is closed, then (eb) final universe will have diverging Weyl tensor

G_{32} : Category one of universe is closed

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of final universe will have diverging Weyl tensor

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Created black holes in open universe have (e) diverging Weyl tensor

G_{36} : Category one of diverging Weyl tensor

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Created black holes in open universe

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

COBE and second law of thermodynamics implies that big bang singularity was extremely uniform

G_{40} : Category one of COBE and second law of thermodynamics

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of big bang singularity was extremely uniform

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

White holes violate(e) second law of thermodynamics

G_{44} : Category one of second law of thermodynamics (take all the three categories as equivalent or make the classification based on the parametric representationalities representative of systems to which second law of

thermodynamics is applicable)

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of White holes

T_{45} : Category two of SAS

T_{46} : Category three of SAS

TITLE

Birth Of Spacetime Atoms As The Passage Of Time

INTRODUCTION—VARIABLES USED

The birth of spacetime atoms as the passage of time∗ Fay Dowker Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2AZ, UK May 15, 2014 For references please see original article. Some sentences and figures are deleted due to spatial constraints.

(1) The view that the passage of time is physical finds expression in the classical sequential growth models of Rideout and Sorkin in which a discrete spacetime grows by (e) the partially ordered accretion of new spacetime atoms.

(2) To answer the question "Do we need a physics of passage?" requires one to decide if there are relevant observational facts unexplained by current science. There is no consensus on this. For example, there is a long-standing and persistent disagreement between those – let us call them "Blockheads" after J. Earman, as cited by J. Butterfield in this volume – who claim that our sense-experience of the passage of time can be adequately accounted for within (eb) a theory in which the physical world is a spacetime Block, and those who contend that there is something essential about our sense-experience which corresponds to (e&eb) nothing to be found within such a Block Universe picture [1].

(3) The approach, causal set theory, is based on (e) the hypothesis that spacetime is fundamentally granular or atomic at the Planck scale and this atomicity opens the door to new dynamical possibilities for spacetime and, hence, to a new perspective on the dichotomy of Being and Becoming.

(4) Spacetime is a continuous, smooth, four-dimensional material that bends, warps and ripples (e&eb) according to dynamical law as specified by the Einstein equations.

(5) Even when there is no matter present, "empty" spacetime itself can carry (e&eb) energy in the form of gravitational waves

(6) Indeed this is the explanation for the variation in the rotation rate of the Hulse-Taylor binary pulsar system, which can be accurately modelled as a system losing energy via (e&eb) this gravitational radiation.

(7) The spacetime material is, however, very different from those substances that populated pre-relativistic physical theory in that it is intrinsically four-dimensional. It cannot be understood as a three-dimensional entity – "space" – evolving in time because that would imply (eb) a global time in which the evolution occurs and there is no such global, physical time in General Relativity (GR).

(8) The notion that at one moment of time there is space, 3-d geometry, and at the next moment space has evolved to another 3-d geometry is wrong in GR. There is no such physically meaningful entity as 3-d space, no physically meaningful slicing of spacetime into (e&eb) space-changing-in-time.

(9) The structure of spacetime that takes centre stage in understanding the physics of GR is its causal structure. This causal structure is a partial order on (eb) the points of spacetime

NOTATION

Module One

The view that the passage of time is physical finds expression in the classical sequential growth models of Rideout and Sorkin in which a discrete spacetime grows by (e) the partially ordered accretion of new spacetime atoms.

G_{13} : Category one of partially ordered accretion of new spacetime atoms.

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of passage of time is physical finds expression in the classical sequential growth models of Rideout and Sorkin in which a discrete spacetime grows

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

To answer the question "Do we need a physics of passage?" requires one to decide if there are relevant observational facts unexplained by current science.

There is no consensus on this. For example, there is a long-standing and persistent disagreement between those – let us call them "Blockheads" after J. Earman, as cited by J. Butterfield in this volume – who claim that our sense-experience of the passage of time can be adequately accounted for within (eb) a theory in which the physical world is a spacetime Block, and those who contend that there is something essential about our sense-experience which corresponds to (e&eb) nothing to be found within such a Block Universe picture [1] (Please see the original article)

G_{16} : Category one of essential about our sense-experience; nothing to be found within such a Block Universe picture [1]

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of nothing to be found within such a Block Universe picture [1];essential about our sense-experience

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The approach, causal set theory, is based on (e) the hypothesis that spacetime is fundamentally granular or atomic at the Planck scale and this atomicity opens the door to new dynamical possibilities for spacetime and, hence, to a new perspective on the dichotomy of Being and Becoming

G_{20} : Category one of hypothesis that spacetime is fundamentally granular or atomic at the Planck scale and this atomicity opens the door to new dynamical possibilities for spacetime and, hence, to a new perspective on the dichotomy of Being and Becoming

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of causal set theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Spacetime is a continuous, smooth, four-dimensional material that bends, warps and ripples (e&eb) according to dynamical law as specified by the Einstein equations

G_{24} : Category one of Spacetime is a continuous, smooth, four-dimensional material that bends, warps and ripples; dynamical law as specified by the Einstein equations

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of dynamical law as specified by the Einstein equations; Spacetime is a continuous, smooth, four-dimensional material that bends, warps and ripples

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Even when there is no matter present, "empty" spacetime itself can carry (e&eb) energy in the form of gravitational waves

G_{28} : Category one of "empty" spacetime; energy in the form of gravitational waves

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of energy in the form of gravitational waves ;"empty" spacetime

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Indeed this "empty" spacetime itself can carry energy in the form of gravitational waves is the explanation for the variation in the rotation rate of the Hulse-Taylor binary pulsar system, which can be accurately modelled as a system losing energy via (e&eb) this gravitational radiation

G_{32} : Category one of "empty" spacetime itself can carry energy in the form of gravitational waves

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of explanation for the variation in the rotation rate of the Hulse-Taylor binary pulsar system, which can be accurately modelled as a system losing energy via (e&eb) this gravitational radiation

T_{33} : Category two of

T_{34} : Category three of

Module seven

Indeed this "empty" spacetime itself can carry energy in the form of gravitational waves is the explanation for the variation in the rotation rate of the Hulse-Taylor binary pulsar system, which can be accurately modelled as a system losing energy via (e&eb) this gravitational radiation

G_{36} : Category one of "empty" spacetime itself can carry energy in the form of gravitational waves is the explanation for the variation in the rotation rate of the Hulse-Taylor binary pulsar system, which can be accurately modelled as a system losing energy; gravitational radiation

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of gravitational radiation; "empty" spacetime itself can carry energy in the form of gravitational waves is the explanation for the variation in the rotation rate of the Hulse-Taylor binary pulsar system, which can be accurately modelled as a system losing energy

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The spacetime material is, however, very different from those substances that populated pre-relativistic physical theory in that it is intrinsically four-dimensional.

spacetime material cannot be understood as a three-dimensional entity – "space" – evolving in time because that would imply (eb) a global time in which the evolution occurs and there is no such global, physical time in General Relativity (GR)

G_{40} : Category one of global time in which the evolution occurs and there is no such global, physical time in General Relativity (GR)

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of spacetime material cannot be understood as a three-dimensional entity – "space" – evolving in time

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Notion that at one moment of time there is space, 3-d geometry, and at the next moment space has evolved to another 3-d geometry is wrong in GR.

There is no such physically meaningful entity as 3-d space, no physically meaningful slicing of spacetime into (e&eb) space-changing-in-time.

G_{44} : Category one of no such physically meaningful entity as 3-d space, no physically meaningful slicing of spacetime; space-changing-in-time

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of space-changing-in-time ;no such physically meaningful entity as 3-d space, no physically meaningful slicing of spacetime

T_{45} : Category two of SAS

T_{46} : Category three of SAS

TITLE

Global Time Does Not Exist In GR

INTRODUCTION—VARIABLES USED

The birth of spacetime atoms as the passage of time∗ Fay Dowker Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2AZ, UK May 15, 2014 For references please see original article. Some sentences and figures are deleted due to spatial constraints

(1) Given two points of spacetime, call them A and B, they will either be (=) ordered or unordered.
(2) If they are ordered then one, let's say A without loss of generality, precedes – or, is to the past of –

the other, B. This ordering is transitive: if A precedes B and B precedes C then A precedes C. The order is partial because there are pairs of spacetime points such that there is no physical sense in which one of the pair precedes the other, they are simply unordered. This lack of ordering does not mean the points of the pair are simultaneous because that would imply (eb) they occur at the same "time" and require the existence of a global time for them to be simultaneous in.

(3) Global time does not exist in (eb) GR.

(4) This partial ordering of the points of spacetime is referred to as the causal structure of spacetime because (e) it coincides with the potential for physical effects to propagate.

(5) A point of spacetime is an idealization of an event that has (e) no physical extent, like an infinitely small, infinitely fast-burning firecracker [3]. 3 if and only if (e&eb) A precedes B in the causal structure.

(6) If two spacetime points are unordered then no physical effect can propagate from one to the other because (e) to do so would require (e) something physical to travel faster than light.

(7) Causal structure plays a central role in GR and indeed the epitome of the theory, a black hole, is defined in terms of the causal structure: it is a region of spacetime such that nothing that happens in that region can affect (e&eb) anything outside the region.

(8) It is only by thinking of a black hole in terms of (e&eb) causal structure that its physics can be understood.

(9) Physical time in GR is associated, locally and "personally," with (e&eb) every localised physical system, in a manner that more closely reflects our intimate experience of time than the global time of pre-relativistic Newtonian mechanics.

NOTATION

Module One

Given two points of spacetime, call them A and B, they will either be (=) ordered or unordered

G_{13} : Category one of A and B, they will either

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of ordered or unordered

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

If they are ordered then one, let's say A without loss of generality, precedes – or, is to the past of – the other, B. This ordering is transitive: if A precedes B and B precedes C then A precedes C.

The order is partial because there are pairs of spacetime points such that there is no physical sense in which one of the pair precedes the other, they are simply unordered.

This lack of ordering does not mean the points of the pair are simultaneous because that would imply (eb) they occur at the same "time" and require the existence of a global time for them to be simultaneous in

G_{16} : Category one of lack of ordering does not mean the points of the pair are simultaneous

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of occurrence at the same "time" and require the existence of a global time for them to be simultaneous in

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Global time does not exist in (eb) GR.

G_{20} : Category one of Global time does not exist

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of GR (Take all the three categories as equivalent or make the classification based on the systems to which GR is applicable0

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

This partial ordering of the points of spacetime is referred to as the causal structure of spacetime because (e) it coincides with the potential for physical effects to propagate

G_{24} : Category one of it coincides with the potential for physical effects to propagate

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of partial ordering of the points of spacetime is referred to as the causal structure of spacetime

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

A point of spacetime is an idealization of an event that has (e) no physical extent, like an infinitely small, infinitely fast-burning firecracker [3]. 3 if and only if (e&eb) A precedes B in the causal structure

G_{28} : Category one of no physical extent, like an infinitely small, infinitely fast-burning firecracker [3]. 3 if and only if (e&eb) A precedes B in the causal structure

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of point of spacetime is an idealization of an event

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

A point of spacetime is an idealization of an event that has (e) no physical extent, like an infinitely small, infinitely fast-burning firecracker [3]. 3 if and only if (e&eb) A precedes B in the causal structure

G_{32} : Category one of point of spacetime is an idealization of an event that has (e) no physical extent, like an infinitely small, infinitely fast-burning firecracker [3]. 3 if and only if; A precedes B in the causal structure

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of A precedes B in the causal structure ;point of spacetime is an idealization of an event that has (e) no physical extent, like an infinitely small, infinitely fast-burning firecracker [3]. 3 if and only if

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Causal structure plays a central role in GR and indeed the epitome of the theory, a black hole, is defined in terms of the causal structure: it is a region of spacetime such that nothing that happens in that region can affect (e&eb) anything outside the region

G_{36} : Category one of Causal structure plays a central role in GR and indeed the epitome of the theory, a black hole, is defined in terms of the causal structure: it is a region of spacetime such that nothing that happens in that region; anything outside the region

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of anything outside the region; Causal structure plays a central role in GR and indeed the epitome of the theory, a black hole, is defined in terms of the causal structure: it is a region of spacetime such that nothing that happens in that region

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

It is only by thinking of a black hole in terms of (e&eb) causal structure that its physics can be understood

G_{40} : Category one of black hole; causal structure that its physics can be understood

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of causal structure that its physics can be understood; black hole

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Physical time in GR is associated, locally and "personally," with (e&eb) every localised physical system, in a manner that more closely reflects our intimate experience of time than the global time of pre-relativistic Newtonian mechanics.

G_{44} : Category one of Physical time in GR is associated, locally and "personally," ; every localised physical system, in a manner that more closely reflects our intimate experience of time than the global time of pre-relativistic Newtonian mechanics.

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of every localised physical system, in a manner that more closely reflects our intimate experience of time than the global time of pre-relativistic Newtonian mechanics.; Physical time in GR is associated, locally and "personally,"

T_{45} : Category two of SAS

T_{46} : Category three of SAS

TITLE

Observation Affects Fabric Of Space Time Itself

INTRODUCTION—VARIABLES USED

The birth of spacetime atoms as the passage of time∗ Fay Dowker Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2AZ, UK May 15, 2014 For references please see original article. Some sentences and figures are deleted due to spatial constraints

(1) Each person or object traces out a trajectory or worldline through (e&eb) spacetime, a continuous line in spacetime that is totally ordered by the causal order: for any 2 points on the worldline, one precedes the other.
(2) GR also provides a quantitatively precise concept of proper time that elapses along (e&eb) each

timelike worldline

(3) A clock carried by a person following a worldline through spacetime will measure this proper time as it elapses, locally along the worldline. Viewed from this perspective, the famous "twin paradox" is no longer a paradox: two people who meet once, then follow different worldlines in spacetime and meet a second time in the future will in general have (e) experienced different amounts of proper time – real, physical time – elapsing along their different worldlines between the meetings.

(4) GR explains this because it predicts that as long as the radius of curvature of spacetime is large compared (e&eb) to the physical scale of the system and the relative velocities of the subsystems involved are small compared with the speed of light, the differences in proper time that elapse along our different worldlines will be (=) negligible.

(5) We can behave as if there is a global 2As an example of this, it is very difficult to answer the question, "Does someone falling feet first into a black hole lose (re)sight of their feet as their feet cross the horizon?" without drawing the conformal "Penrose" diagram that depicts the causal structure. 4 times, a global moment of Now, because for practical everyday purposes our clocks will remain synchronised to very high precision.

(6) In addition to being the key to understanding GR, the causal structure of spacetime is a unifying concept. Theorems by Kronheimer and Penrose [4], Hawking [5] and Malament [6] establish that the causal order unifies within (e&eb) itself topology, including dimension, differentiable structure (smoothness) and almost all the spacetime geometry.

(7) This local scale information can be furnished by providing the spacetime volume of every region of spacetime or, alternatively, the amount of proper time that elapses – the duration – along every timelike worldline. In the continuum, the causal structure and local scale information complement each other to give the full geometry of spacetime, the complete spacetime fabric.

(8) Atomicity unifies order and (e&eb) geometry.

(9) There are strong, physical arguments that the smooth manifold structure of spacetime must break down (e) at the Planck scale where quantum fluctuations in the structure of spacetime cannot be ignored.

The birth of spacetime atoms as the passage of time∗ Fay Dowker Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2AZ, UK May 15, 2014 For references please see original article. Some sentences and figures are deleted due to spatial constraints

(1) It is here we claim that the observation affects (e&eb) fabric of space time itself, with Schrödinger's wave function collapse in a different state other than the one it would have, and had not the observation taken place.

(2) The most convincing evidence that spacetime cannot remain smooth and continuous at arbitrarily small scales and that the scale at which the continuum breaks down is the Planck scale is (=) the finite value of the entropy of a Black Hole [7]. Advert recent observation that gravity is one that is responsible for the macroscopic reality in contrast to Planck's scale duality (Einstein saves Quantum Mechanics)

(10) Fundamental spacetime discreteness is a simple proposal that realises (eb) the widely held expectation that there must be a physical, Planck scale cutoff in nature

NOTATION

Module One

Each person or object traces out a trajectory or worldline through (e&eb) spacetime, a continuous line in spacetime that is totally ordered by the causal order: for any 2 points on the worldline, one precedes the

other

G_{13} : Category one of Each person or object traces out a trajectory or worldline; spacetime, a continuous line in spacetime that is totally ordered by the causal order: for any 2 points on the worldline, one precedes the other

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of spacetime, a continuous line in spacetime that is totally ordered by the causal order: for any 2 points on the worldline, one precedes the other; Each person or object traces out a trajectory or worldline

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

GR also provides a quantitatively precise concept of proper time that elapses along (e&eb) each timelike worldline

G_{16} : Category one of GR also provides a quantitatively precise concept of proper time that elapses; each timelike worldline

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of each timelike worldline ;GR also provides a quantitatively precise concept of proper time that elapses

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

GR explains this because it predicts that as long as the radius of curvature of spacetime is large compared (e&eb) to the physical scale of the system and the relative velocities of the subsystems involved are small compared with the speed of light, the differences in proper time that elapse along our different worldlines will be (=) negligible

G_{20} : Category one of GR explains this because it predicts that as long as the radius of curvature of spacetime is large; physical scale of the system and the relative velocities of the subsystems involved are small compared with the speed of light, the differences in proper time that elapse along our different worldlines will be negligible

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of physical scale of the system and the relative velocities of the subsystems involved are small compared with the speed of light, the differences in proper time that elapse along our different worldlines will be negligible; GR explains this because it predicts that as long as the radius of curvature of

spacetime is large

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Causal structure of spacetime on addition to being the key to understanding GR is a unifying concept.

Theorems by Kronheimer and Penrose [4], Hawking [5] and Malament [6] establish that the

causal order unifies within (e&eb) itself topology, including dimension, differentiable structure (smoothness) and almost all the spacetime geometry

G_{24} : Category one of causal order unifies; itself topology, including dimension, differentiable structure (smoothness) and almost all the spacetime geometry

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of itself topology, including dimension, differentiable structure (smoothness) and almost all the spacetime geometry; causal order unifies

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

This local scale information can be furnished by providing the spacetime volume of every region of spacetime or, alternatively, the amount of proper time that elapses – the duration – along every timelike worldline.

In the continuum, the causal structure and local scale information complement each other to give the full geometry of spacetime, the complete spacetime fabric

G_{28} : Category one of causal structure; local scale information

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of local scale information ;causal structure

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Atomicity unifies order and (e&eb) geometry

G_{32} : Category one of Atomicity unifies order; geometry

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of geometry ;Atomicity unifies order

T_{33} : Category two of

T_{34} : Category three of

Module seven

There are strong, physical arguments that the smooth manifold structure of spacetime must break down (e) at the Planck scale where quantum fluctuations in the structure of spacetime cannot be ignored

G_{36} : Category one of smooth manifold structure of spacetime; Planck scale where quantum fluctuations in the structure of spacetime cannot be ignored

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Planck scale where quantum fluctuations in the structure of spacetime cannot be ignored ;smooth manifold structure of spacetime

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

smooth manifold structure of spacetime must break down at the Planck scale where quantum fluctuations in the structure of spacetime cannot be ignored

G_{40} : Category one of quantum fluctuations in the structure of spacetime cannot be ignored

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of smooth manifold structure of spacetime must break down at the Planck scale

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

It is here we claim that the observation affects (e&eb) fabric of space time itself, with Schrödinger's wave function collapse in a different state other than the one it would have, and had not the observation taken

place

G_{44} : Category one of observation; fabric of space time itself

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of fabric of space time itself; observation

T_{45} : Category two of SAS

T_{46} : Category three of SAS

Sometimes I think there had been more big bangs and we are choosing what we want!

It looks like that!

Many be our 13.7 billion year bang was one of big bangs we see in the accelerating universe!

May be!

Big Bangs are going on with newer space times!

So?

People donot think of Physics there!

Nature is fooling you like TBAB does!

What happens when you join singularities with different geometries?

See the above model!

You have any evidence!

Nature destroys all the evidence!

Add to your cup of miseries you have anagrammatic mind!

Nature is Mother!

So?

She cannot destroy!

Mother also shows moon and says it would come for dinner tonight!

Ugh! I cannot Father! Jesus!

Thank God! You did not talk of Oedipus complex for probing nature!

You like the place you came from!

Are not scientists' long term planners!

I think they think of short term gains!

Nature in motion is Saguna!

Ok!

Nature in inertia is Nirguna!

OK!

Move further!

There is nowhere to go!

Ugh!

You are digging holes and filling it!

So is everyone!

You want to bring the world to stand still?

It already is!

Everything is conserved holistically and not individually!

It only affirms the premises and also the power to state the conclusion!

OK!

Individual consciousness contributes to the harmony of Universal consciousness the Nature's General ledger!

Like in an organisation, it is necessary to main social order!

OK! But…..

That it divinated upon you itself is testimony of the God!

It is not like that!

Then what?

Abstaining from work one cannot achieve freedom from reaction nor by renunciation can one attain perfection!

I am doing neither!

Personal uncertainty either Yes or NO is not a doubt foreign to what is happening but rather a sine qua non of event itself!

When everything fails, philosophy prevails!

Hehehhee!

@@@

Broad spectrum

(6) The new quantum dot spectrometer deploys hundreds of quantum dot materials that each filters (e&eb) a specific set of wavelengths of light. The quantum dot filters are printed into a thin film and placed on (eb) top of a photodetector such as the charge-coupled devices (CCDs) found in cell phone cameras.

(7) The researcher created an algorithm that analyzes the percentage of photons absorbed by (e) each filter, and then recombines (e&eb) the information from each one to calculate (eb) the intensity and wavelength of the original rays of light.

(8) The more quantum dot materials there are (=) the more wavelengths can be covered and the higher resolution can be obtained. In this case, the researchers used about 200 types of quantum dots spread over a range of about 300 nanometers. With more dots, such spectrometers could be designed to cover an even wider range of light frequencies.

(9) "Bawendi and Bao showed a beautiful way to exploit the controlled optical absorption of semiconductor quantum dots for (e) miniature spectrometers. They demonstrate a spectrometer that is not only small, but also with high throughput and high spectral resolution, which has never been achieved before," says Feng Wang, an associate professor of physics at the University of California at Berkeley who was not involved in the research.

(10) If incorporated into small handheld devices, this type of spectrometer could be used to (e) diagnose skin conditions or analyze urine samples, Bao says. They could also be used to (e) track vital signs such as pulse and oxygen level or to measure exposure to different frequencies of ultraviolet light, which vary greatly in their ability to damage skin. "The central component of such spectrometers—the quantum dot filter array—is fabricated with solution-based processing and printing, thus enabling significant potential cost reduction," Bao adds.

NOTATION

Module One

By controlling the ratio of these starting materials, the temperature, and the reaction time, scientists can generate (eb) a nearly unlimited number of dots with differences in an electronic property known as bandgap, which determines (eb) the wavelengths of light that each dot will absorb

G_{13} : Category one of controlled ratio of these starting materials, the temperature, and the reaction time

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of unlimited number of dots with differences in an electronic property known as bandgap, which determines (eb) the wavelengths of light that each dot will absorb

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

By controlling the ratio of these starting materials, the temperature, and the reaction time, scientists can generate a nearly unlimited number of dots with differences in an electronic property known as bandgap, which determines (eb) the wavelengths of light that each dot will absorb

G_{16} : Category one of controlling the ratio of these starting materials, the temperature, and the reaction time, scientists can generate a nearly unlimited number of dots with differences in an electronic property known as bandgap

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of wavelengths of light that each dot will absorb

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

New quantum dot spectrometer deploys hundreds of quantum dot materials that each filters (e&eb) a specific set of wavelengths of light.

The quantum dot filters are printed into a thin film and placed on (eb) top of a photodetector such as the charge-coupled devices (CCDs) found in cell phone cameras

G_{20} : Category one of quantum dot spectrometer deploys hundreds of quantum dot materials; specific set of wavelengths of light

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of specific set of wavelengths of light; quantum dot spectrometer deploys hundreds of quantum dot materials

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

quantum dot filters are printed into a thin film and placed on (eb) top of a photodetector such as the charge-coupled devices (CCDs) found in cell phone cameras

G_{24} : Category one of quantum dot filters are printed into a thin film

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of photodetector such as the charge-coupled devices (CCDs) found in cell phone cameras

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Researcher created an algorithm that analyzes the percentage of photons absorbed by (e) each filter, and then recombines (e&eb) the information from each one to calculate (eb) the intensity and wavelength of the original rays of light

G_{28} : Category one of each filter, and then recombines (e&eb) the information from each one to calculate (eb) the intensity and wavelength of the original rays of light

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of algorithm analyzes the percentage of photons absorbed

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Researcher created an algorithm that analyzes the percentage of photons absorbed by each filter, and then recombines (e&eb) the information from each one to calculate (eb) the intensity and wavelength of the original rays of light

G_{32} : Category one of algorithm that analyzes the percentage of photons absorbed by each filter; information from each one to calculate (eb) the intensity and wavelength of the original rays of light

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of information from each one to calculate (eb) the intensity and wavelength of the original rays of light; algorithm that analyzes the percentage of photons absorbed by each filter

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Researcher created an algorithm that analyzes the percentage of photons absorbed by each filter, and then recombines the information from each one to calculate (eb) the intensity and wavelength of the original rays of light

G_{36} : Category one of algorithm that analyzes the percentage of photons absorbed by each filter, and then recombines the information from each one

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of intensity and wavelength of the original rays of light

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The more quantum dot materials there are (=) the more wavelengths can be covered and the higher resolution can be obtained.

In this case, the researchers used about 200 types of quantum dots spread over a range of about 300 nanometers. With more dots, such spectrometers could be designed to cover an even wider range of light frequencies

G_{40} : Category one of more quantum dot materials

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of more wavelengths can be covered and the higher resolution can be obtained.

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

If incorporated into small handheld devices, this type of spectrometer could be used to (e) diagnose skin conditions or analyze urine samples, Bao says.

They could also be used to (e) track vital signs such as pulse and oxygen level or to measure exposure to different frequencies of ultraviolet light, which vary greatly in their ability to damage skin.

"The central component of such spectrometers—the quantum dot filter array—is fabricated with solution-based processing and printing, thus enabling significant potential cost reduction," Bao adds

G_{44} : Category one of spectrometer (take parameters concomitant with quantum dots and quantum dot filter array)

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of different frequencies of ultraviolet light

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad (1)$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad (2)$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad (3)$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad (4)$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad (5)$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad (6)$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad (7)$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad (8)$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad (9)$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad (10)$$

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}),t)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}),t)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}),t) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28}$ 25

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29}$ 26

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30}$ 27

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28}$ 28

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}), t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}), t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}), t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & \boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for

category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3
$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3
$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3
$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3
$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3
$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3
$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad (75)$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$ *are fourth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$ *are fifth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$ *are sixth detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients*

$for\ category\ 1,2\ and\ 3$
$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ $are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ $are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients
$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients
$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients
$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition

coefficients for category 1, 2, and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{37}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$
are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3
$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(A) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(B) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(C) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(D)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(E) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(F) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(a) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ and $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$$

334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

335

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$$

336

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)})$$

$$\left[\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left((\lambda)^{(2)}\right)^{2} + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^{2} + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left((\lambda)^{(2)}\right)^{2} + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\} = 0$$

$$+$$

$$(\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)})\{((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)})$$

$$\left[\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21} + (a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20} + (a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left((\lambda)^{(3)}\right)^{2} + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left((\lambda)^{(3)}\right)^{2} + \left((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left((\lambda)^{(3)}\right)^{2} + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWO

Light Spectrum Based On The Wavelength Multiplexing Principle

INTRODUCTION—VARIABLES USED

A colloidal quantum dot spectrometer Jie Bao & Moungi G. Bawendi AffiliationsContributionsCorresponding author Nature 523, 67–70 (02 July 2015) doi:10.1038/nature14576 Received 15 August 2013 Accepted 12 May 2015 Published online 01 July 2015

(1) Spectroscopy is carried out in almost every field of science, whenever light interacts with (e&eb) matter1.
(2) Although sophisticated instruments with impressive performance characteristics are available, much effort continues to be invested in the development of miniaturized, cheap and easy-to-use systems1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13. Current micro spectrometer designs mostly use interference filters2, 3, 4, 5 and interferometric optics3 that limit (e) their photon efficiency, resolution and spectral range2, 3.
(3) Here authors show that many of these limitations can be overcome by replacing (e&eb) interferometric optics with a two-dimensional absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17.
(4) Instead of measuring different bands of a spectrum individually after introducing temporal or spatial separations with gratings or interference-based narrowband filters, a colloidal quantum dot spectrometer measures (eb) a light spectrum based on (e) the wavelength multiplexing principle18: multiple spectral bands are encoded and detected simultaneously with one filter and one detector9, 10, 11, 12, respectively, with the array format allowing the (eb) process to be efficiently repeated many times using (e) different filters with different encoding so that sufficient information is obtained to enable (eb) computational reconstruction of the target spectrum.
(5) Authors illustrate the performance of such a quantum dot micro spectrometer, made from 195 different types of quantum dots with absorption features that cover a spectral range of 300 nanometres, by measuring (eb) shifts in spectral peak positions as small as one nanometre. Given this performance, demonstrable avenues for further improvement, the ease with which quantum dots can be processed and integrated, and their numerous finely tuneable bandgaps that cover a broad spectral range, they expect that quantum dot micro spectrometers will be useful in applications where minimizing size, weight, cost and complexity of the spectrometer are critical

NOTATION

Module One

Spectroscopy is carried out in almost every field of science, whenever light interacts with (e&eb) matter1

G_{13} : Category one of light; matter

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of matter; light

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Although sophisticated instruments with impressive performance characteristics are available, much effort continues to be invested in the development of miniaturized, cheap and easy-to-use systems1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.

Current micro spectrometer designs mostly use interference filters2, 3, 4, 5 and interferometric optics3 that limit (e) their photon efficiency, resolution and spectral range2, 3

G_{16} : Category one of their photon efficiency, resolution and spectral range2

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of interference filters2, 3, 4, 5 and interferometric optics3

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Here authors show that many of these limitations can be overcome by replacing (e&eb) interferometric optics with a two-dimensional absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17

G_{20} : Category one of limitations

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of) interferometric optics with a two-dimensional absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Here authors show that many of these limitations can be overcome by replacing interferometric optics with a two-dimensional absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17

G_{24} : Category one of limitations can be overcome by replacing interferometric optics; two-dimensional

absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of two-dimensional absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17; limitations can be overcome by replacing interferometric optics

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Here authors show that many of these limitations can be overcome by replacing interferometric optics with a two-dimensional absorptive filter array composed of (e) colloidal quantum dots14, 15, 16, and 17

G_{28} : Category one of colloidal quantum dots14, 15, 16, and 17

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of limitations can be overcome by replacing interferometric optics with a two-dimensional absorptive filter array

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Instead of measuring different bands of a spectrum individually after introducing temporal or spatial separations with gratings or interference-based narrowband filters, a colloidal quantum dot spectrometer measures (eb) a light spectrum based on (e) the wavelength multiplexing principle18: multiple spectral bands are encoded and detected simultaneously with one filter and one detector9, 10, 11, 12, respectively, with the array format allowing the (eb) process to be efficiently repeated many times using (e) different filters with different encoding so that sufficient information is obtained to enable (eb) computational reconstruction of the target spectrum

G_{32} : Category one of wavelength multiplexing principle18

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of light spectrum

T_{33} : Category two of

T_{34} : Category three of

Module seven

NATURE MATERIALS | LETTER Localized surface plasmon resonances arising from free carriers in doped quantum dots Joseph M. Luther, Prashant K. Jain, Trevor Ewers & A. Paul Alivisatos Nature Materials 10, 361–366 (2011) doi: 10.1038/nmat3004: Published online 10 April 2011

(1) Localized surface plasmon resonances (LSPRs) typically arise in (eb) nanostructures of noble metals1, 2 resulting in (eb) enhanced and geometrically tunable absorption and scattering resonances.
(2) LSPRs, however, are not limited to nanostructures of metals and can also be achieved in (eb) semiconductor nanocrystals with appreciable free carrier concentrations.
(3) Here, **Joseph M. Luther, Prashant K. Jain, Trevor Ewers & A. Paul Alivisatos** describe well-defined LSPRs arising from (e) p-type carriers in vacancy-doped semiconductor quantum dots (QDs).
(4) Achievement of LSPRs by free carrier doping of (e) a semiconductor nanocrystal would allow (eb) active on-chip control of LSPR responses.
(5) Plasmonic sensing and manipulation of (e) solid-state processes in single nanocrystals constitutes another interesting possibility.
(6) Authors also demonstrate that doped semiconductor QDs allow (eb) realization of LSPRs and quantum-confined excitons within (eb) the same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing.

Localized surface plasmon resonances (LSPRs) typically arise in (eb) nanostructures of noble metals1, 2 resulting in (eb) enhanced and geometrically tunable absorption and scattering resonances

G_{36} : Category one of Localized surface plasmon resonances (LSPRs)

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of nanostructures of noble metals1, 2 resulting in (eb) enhanced and geometrically tunable absorption and scattering resonances

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Localized surface plasmon resonances (LSPRs) typically arise in nanostructures of noble metals1, 2 resulting in (eb) enhanced and geometrically tunable absorption and scattering resonances

G_{40} : Category one of Localized surface plasmon resonances (LSPRs) typically arise in nanostructures of noble metals1, 2

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of enhanced and geometrically tunable absorption and scattering resonances

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

LSPRs, however, are not limited to nanostructures of metals and can also be achieved in (eb) semiconductor nanocrystals with appreciable free carrier concentrations

G_{44} : Category one of LSPRs, however, are not limited to nanostructures of metals and can also be achieved

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of semiconductor nanocrystals with appreciable free carrier concentrations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad (1)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad (2)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad (3)$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad (4)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad (5)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad (6)$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad (7)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad (8)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad (9)$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad (10)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad (11)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad (12)$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad (13)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad (14)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad (15)$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad (16)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad (17)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad (18)$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad (19)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 91$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 92$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} - \begin{bmatrix} (b_{44}')^{(9)}-(b_{44}'')^{(9)}(G_{47},t) & -(b_{16}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{24}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{13}'')^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{36}'')^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}-(b_{45}'')^{(9)}(G_{47},t) & -(b_{17}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{25}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{14}'')^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{37}'')^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}-(b_{46}'')^{(9)}(G_{47},t) & -(b_{18}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{26}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{15}'')^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{38}'')^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \tag{97}$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \tag{98}$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \le \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \le (\hat{k}_{32})^{(6)}|T_{33} - \ T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(G) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(H) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(I) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(J)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(K) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(L) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\widehat{M}_{40})^{(8)}t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''^{(9)}_{44} \left(T_{45}(s_{(44)}), s_{(44)} \right) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} \left(T_{45}(s_{(44)}), s_{(44)} \right) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} \left(T_{45}(s_{(44)}), s_{(44)} \right) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} \left(G_{47}(s_{(44)}), s_{(44)} \right) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} \left(G_{47}(s_{(44)}), s_{(44)} \right) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} \left(G_{47}(s_{(44)}), s_{(44)} \right) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}}\left(e^{(\tilde{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(b) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}}\left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)}\left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}}\left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1 \right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0 \right) e^{\left(- \frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0} \right)} + (\hat{P}_{44})^{(9)} \right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0 \right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0} \right)} \right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0 \right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{13})^{(1)} \right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)} \right), \left(G^{(2)}, T^{(2)} \right) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-(\hat{M}_{13})^{(1)} t} \}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T} : \left(\tilde{G}, \tilde{T} \right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left| \tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)} \right| \le \int_0^t (a_{13})^{(1)} \left| G_{14}^{(1)} - G_{14}^{(2)} \right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{ (a'_{13})^{(1)} \left| G_{13}^{(1)} - G_{13}^{(2)} \right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)} \right) \left| G_{13}^{(1)} - G_{13}^{(2)} \right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad (208)$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad (209)$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)} \left(T_{21}^{(1)}, s_{(20)}\right) \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left| (a''_{20})^{(3)} \left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)} \left(T_{21}^{(2)}, s_{(20)}\right) \right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| G_{23}{}^{(1)} - G_{23}{}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d \left(((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}) \right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[- \int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)} \right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a_{24}')^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a_{24}'')^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a_{24}'')^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a_{24}'')^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a_{24}')^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a_{24}'')^{(4)}$ and $(b_{24}'')^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a_{25}')^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a_{26}')^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (250)

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $((\widehat{M}_{36})^{(7)})_1, ((\widehat{M}_{36})^{(7)})_2$ *and* $((\widehat{M}_{36})^{(7)})_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)}((\widehat{M}_{36})^{(7)})_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)}((\widehat{M}_{36})^{(7)})_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}((G_{39})(t),t) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + ((\widehat{P}_{40})^{(8)} + G_j^0)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(\nu_1)^{(2)}>0,(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)}>0,(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, $\boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, $\boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, $\boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, $\boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right. \quad 334$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\le T_{38}(t)\le$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\le-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\le-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\le-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\le-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\leq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\leq\nu^{(1)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\leq(\bar{\nu}_1)^{(1)}\leq\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\leq\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\leq\nu^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\le\frac{dv^{(3)}}{dt}\le-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\ge\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\le v^{(3)}(t)\le(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\le\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\le v^{(3)}(t)\le(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\le\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\le v^{(3)}(t)\le$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\le(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\le(\bar{v}_1)^{(3)}\le(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\le v^{(3)}(t)\le\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\le(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\le v^{(3)}(t)\le(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(b) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWO A

Localized Surface Plasmon Resonances

INTRODUCTION—VARIABLES USED

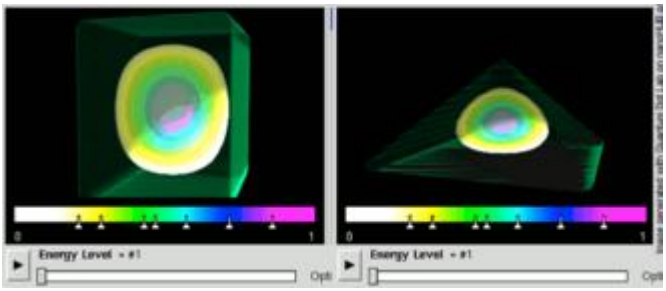
NATURE MATERIALS | LETTER Localized surface plasmon resonances arising from free carriers in doped quantum dots Joseph M. Luther, Prashant K. Jain, Trevor Ewers & A. Paul Alivisatos Nature Materials 10, 361–366 (2011) doi: 10.1038/nmat3004: Published online 10 April 2011

(1) **Joseph M. Luther, Prashant K. Jain, Trevor Ewers & A. Paul Alivisatos** describe well-defined LSPRs arising from (e) p-type carriers in vacancy-doped semiconductor quantum dots (QDs).
(2) Achievement of LSPRs by free carrier doping of (e) a semiconductor nanocrystal would allow (eb) active on-chip control of LSPR responses.
(3) Plasmonic sensing and manipulation of (e) solid-state processes in single nanocrystals constitutes another interesting possibility.
(4) Authors also demonstrate that doped semiconductor QDs allow (eb) realization of LSPRs and quantum-confined excitons within (eb) the same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

Quantum dots (Wikipedia)

(1) A quantum dot (QD) is a nanocrystal made of semiconductor materials that is small enough to exhibit quantum mechanical properties
(2) Specifically, its excitons are confined in all three spatial dimensions. The electronic properties of these materials are intermediate between those of bulk semiconductors and of discrete molecules.[1][2][3]
(3) Quantum dots were first discovered by Alexey Ekimov in 1981[4][5][6] in a glass matrix and then in colloidal solutions by Louis E. Brus in 1985.[7] The term "quantum dot" was coined by Mark Reed.[8]
(4) Researchers have studied applications for quantum dots in transistors, solar cells, LEDs, and diode lasers. They have also investigated quantum dots as agents for medical imaging and as possible qubits in quantum computing. The first commercial release of a product utilizing quantum dots was the Sony XBR X900A series of flat panel televisions released in 2013.[9]
(5) Electronic characteristics of a quantum dot are closely related to its size and shape. For example, the band gap in a quantum dot which determines the frequency range of emitted light is inversely related to its size. In fluorescent dye applications the frequency of emitted light increases as the size of the quantum dot decreases. Consequently, the color of emitted light shifts from red to blue when the size of the quantum dot is made smaller.[10] This allows the excitation and emission of quantum dots to be highly tunable. Since the size of a quantum dot may be set when it is made, its conductive properties may be carefully controlled. Quantum dot assemblies consisting of many different sizes, such as gradient multi-layer nanofilms, can be made to exhibit a range of desirable emission properties.

Quantum confinement in semiconductors

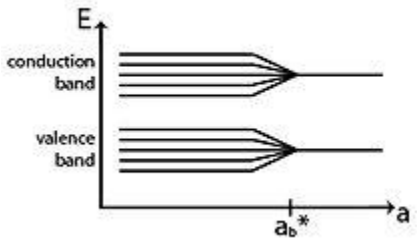


3D confined electron wave functions in a quantum dot. Here, rectangular and triangular-shaped quantum

dots are shown. Energy states in rectangular dots are more s-typeand p-type. However, in a triangular dot the wave functions are mixed due to confinement symmetry. (Click for animation)

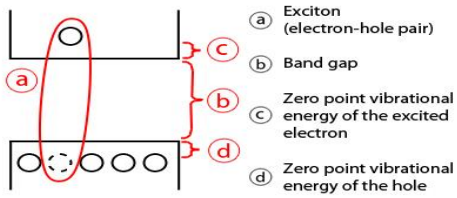
Potential well

In a semiconductor crystallite whose radius is smaller than the size of its exciton Bohr radius, the excitons are squeezed, leading to quantum confinement. The energy levels can then be modeled using the particle in a box model in which the energy of different states is dependent on the length of the box. Quantum dots are said to be in the 'weak confinement regime' if their radii are on the order of the exciton Bohr radius; quantum dots are said to be in the 'strong confinement regime' if their radii are smaller than the exciton Bohr radius. If the size of the quantum dot is small enough that the quantum confinement effects dominate (typically less than 10 nm), the electronic and optical properties are highly tunable.



Splitting of energy levels for small quantum dots due to the quantum confinement effect. The horizontal axis is the radius, or the size, of the quantum dots and ab* is the Exciton Bohr radius.

Fluorescence occurs when an excited electron relaxes to the ground state and combines with the hole. In a simplified model, the energy of the emitted photon can be understood as the sum of the band gap energy between the occupied level and the unoccupied energy level, the confinement energies of the hole and the excited electron, and the bound energy of the exciton (the electron-hole pair):



Band gap energy

The band gap can become larger in the strong confinement regime where the size of the quantum dot is smaller than the Exciton Bohr radius ab* as the energy levels split up.

$$a_b^* = \varepsilon_r \left(\frac{m}{\mu}\right) a_b$$

where ab is the Bohr radius=0.053 nm, m is the mass, μ is the reduced mass, and εr is the size-dependent dielectric constant (Relative permittivity).

This results in the increase in the total emission energy (the sum of the energy levels in the smaller band gaps in the strong confinement regime is larger than the energy levels in the band gaps of the original levels in the weak confinement regime) and the emission at various wavelengths; which is precisely what happens in the sun, where the quantum confinement effects are completely dominant and the energy levels split up to the degree that the energy spectrum is almost continuous, thus emitting white light.

Confinement energy

The exciton entity can be modeled using the particle in the box. The electron and the hole can be seen as hydrogen in the Bohr model with the hydrogen nucleus replaced by the hole of positive charge and negative electron mass. Then the energy levels of the exciton can be represented as the solution to the particle in a box at the ground level (n = 1) with the mass replaced by the reduced mass. Thus by varying the size of the quantum dot, the confinement energy of the exciton can be controlled.

Bound exciton energy

There is Coulomb attraction between the negatively charged electron and the positively charged hole. The negative energy involved in the attraction is proportional to Rydberg's energy and inversely proportional to square of the size-dependent dielectric constant [11] of the semiconductor. When the size of the semiconductor crystal is smaller than the Exciton Bohr radius, the Coulomb interaction must be modified to fit the situation. Therefore, the sum of these energies can be represented as:

$$E_{\text{confinement}} = \frac{\hbar^2\pi^2}{2a^2}\left(\frac{1}{m_e}+\frac{1}{m_h}\right) = \frac{\hbar^2\pi^2}{2\mu a^2}$$

$$E_{\text{exciton}} = -\frac{1}{\epsilon_r^2}\frac{\mu}{m_e}R_y = -R_y^*$$

$$E = E_{\text{bandgap}} + E_{\text{confinement}} + E_{\text{exciton}}$$

$$= E_{\text{bandgap}} + \frac{\hbar^2\pi^2}{2\mu a^2} - R_y^*$$

where μ is the reduced mass, a is the radius, me is the free electron mass, mh is the hole mass, and εr is the size-dependent dielectric constant.

Although the above equations were derived using simplifying assumptions, the implications are clear; the energy of the quantum dots is dependent on their size due to the quantum confinement effects, which dominate below the critical size leading to changes in the optical properties. This effect of quantum confinement on the quantum dots has been experimentally verified[12] and is a key feature of many emerging electronic structures.[13][14]

Besides confinement in all three dimensions (i.e., a quantum dot), other quantum confined semiconductors include: Quantum wires, which confine electrons or holes in two spatial dimensions and allow free propagation in the third. Quantum wells, which confine electrons or holes in one dimension and allow free propagation in two dimensions.

NOTATION

Module One

Joseph M. Luther, Prashant K. Jain, Trevor Ewers & A. Paul Alivisatos describe well-defined LSPRs arising from (e) p-type carriers in vacancy-doped semiconductor quantum dots (QDs)

G_{13} : Category one of p-type carriers in vacancy-doped semiconductor quantum dots (QDs)

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of LSPRs

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Achievement of LSPRs by free carrier doping of a semiconductor nanocrystal would allow (eb) active on-chip control of LSPR responses

G_{16} : Category one of achievement of LSPRs by free carrier doping of a semiconductor nanocrystal

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of active on-chip control of LSPR responses

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Plasmonic sensing and manipulation of (e) solid-state processes in single nanocrystals constitutes another interesting possibility

G_{20} : Category one of Plasmonic sensing and manipulation; solid-state processes in single nanocrystals

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of solid-state processes in single nanocrystals ;Plasmonic sensing and manipulation

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Authors also demonstrate that doped semiconductor QDs allow (eb) realization of LSPRs and quantum-confined excitons within (eb) the same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and

quantum information processing

G_{24} : Category one of doped semiconductor QDs

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of realization of LSPRs and quantum-confined excitons within (eb) the same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Authors also demonstrate that doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons within (eb) the same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

G_{28} : Category one of doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Authors also demonstrate that doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons within the same nanostructure, opening up (eb) the possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

G_{32} : Category one of doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons within the same nanostructure

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of possibility of strong coupling of (e) photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Authors also demonstrate that doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons within the same nanostructure, opening up the possibility of strong coupling of photonic and electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

G_{36} : Category one of doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons within the same nanostructure, opening up the possibility of strong coupling of photonic; electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of electronic modes, with implications (e&eb) for light harvesting, nonlinear optics, and quantum information processing ;doped semiconductor QDs allow realization of LSPRs and quantum-confined excitons within the same nanostructure, opening up the possibility of strong coupling of photonic

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

$$E_{\text{confinement}} = \frac{\hbar^2\pi^2}{2a^2}\left(\frac{1}{m_e}+\frac{1}{m_h}\right) = \frac{\hbar^2\pi^2}{2\mu a^2}$$

$$E_{\text{exciton}} = -\frac{1}{\epsilon_r^2}\frac{\mu}{m_e}R_y = -R_y^*$$

$$E = E_{\text{bandgap}} + E_{\text{confinement}} + E_{\text{exciton}}$$

$$= E_{\text{bandgap}} + \frac{\hbar^2\pi^2}{2\mu a^2} - R_y^*$$

G_{40} : Category one of LHS of above equation (final)

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of RHS of the above equation

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The exciton entity can be modeled using the particle in the box. The electron and the hole can be seen as hydrogen in the Bohr model with the hydrogen nucleus replaced by the hole of positive charge and negative electron mass.

Then the energy levels of the exciton can be represented as the solution to the particle in a box at the ground level (n = 1) with the mass replaced by the reduced mass.

Thus by varying the size of the quantum dot, the confinement energy of the exciton can be controlled.

G_{44} : Category one of size of the quantum dot; confinement energy of the exciton

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of confinement energy of the exciton; size of the quantum dot

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THREE

Atomically Thin Semiconductor Emit Single Photons

INTRODUCTION—VARIABLES USED

Defects in atomically thin semiconductor emit single photons May 04, 2015

(1) Researchers at the University of Rochester have shown that defects on an atomically thin semiconductor can produce (eb) light-emitting quantum dots.
(2) The quantum dots serve as a source of (e) single photons and could be useful for (e) the integration of quantum photonics with solid-state electronics - a combination known as integrated photonics.
(3) Scientists have become interested in integrated solid-state devices for quantum information processing uses. Quantum dots in atomically thin semiconductors could not only provide a framework to explore the fundamental physics of (e) how they interact, but also enable nanophotonics applications, the researchers say.
(4) Quantum dots are often referred to as (=) artificial atoms.
(5) Quantum dots are artificially engineered or naturally occurring defects in solids that are being studied for (e) a wide range of applications. Nick Vamivakas, assistant professor of optics at the University of Rochester and senior author on the paper, adds that atomically thin, 2D materials, such as graphene, have also generated interest among scientists who want to explore their potential for optoelectronics. However, until now, optically active quantum dots have not been observed in

2D materials.

(6) In a paper published in Nature Nanotechnology this week, the Rochester researchers show how tungsten diselenide (WSe2) can be fashioned into (e&eb) an atomically thin semiconductor that serves as (=) a platform for solid-state quantum dots.

(7) Perhaps most importantly the defects that create the dots do not (e) inhibit the electrical or optical performance of the semiconductor and they can be controlled by (e&eb) applying electric and magnetic fields.

(8) Vamivakas explains that the brightness of the quantum dot emission can be controlled by applying (e&eb) the voltage. He adds that the next step is to use voltage to (e) "tune the color" of the emitted photons, which can make it possible to integrate these quantum dots with (e&eb) nanophotonic devices.

(9) A key advantage is how much easier it is to create quantum dots in (eb) atomically thin tungsten diselenide compared to (e&eb) producing quantum dots in more traditional materials like indium arsenide.

(10) "We start with a black crystal and then we peel layers of it off until (eb) we have an extremely thin later left, an atomically thin sheet of tungsten di selenide," said Vamivakas.

NOTATION

Module One

Researchers at the University of Rochester have shown that defects on an atomically thin semiconductor can produce (eb) light-emitting quantum dots

G_{13} : Category one of defects on an atomically thin semiconductor

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of light-emitting quantum dots

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

The quantum dots serve as a source of (e) single photons and could be useful for (e) the integration of quantum photonics with solid-state electronics - a combination known as integrated photonics

G_{16} : Category one of single photons

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantum dots

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Quantum dots are often referred to as (=) artificial atoms

G_{20} : Category one of Quantum dots

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of artificial atoms

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In a paper published in Nature Nanotechnology this week, the Rochester researchers show how tungsten diselenide (WSe2) can be fashioned into (e&eb) an atomically thin semiconductor that serves as (=) a platform for solid-state quantum dots

(All categories' can be taken as equivalent)

G_{24} : Category one of tungsten diselenide (WSe2); atomically thin semiconductor that serves as (=) a platform for solid-state quantum dots

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of atomically thin semiconductor that serves as (=) a platform for solid-state quantum dots; tungsten diselenide (WSe2)

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Perhaps most importantly the defects that create the dots do not (e) inhibit the electrical or optical performance of the semiconductor and they can be controlled by (e&eb) applying electric and magnetic fields

G_{28} : Category one of inhibit the electrical or optical performance of the semiconductor and they can be controlled by (e&eb) applying electric and magnetic fields

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of defects that create the dots

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Perhaps most importantly the defects that create the dots do not inhibit the electrical or optical performance of the semiconductor and they can be controlled by (e&eb) applying electric and magnetic fields

G_{32} : Category one of defects that create the dots do not inhibit the electrical or optical performance of the semiconductor and they can be controlled; electric and magnetic fields

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of electric and magnetic fields ;defects that create the dots do not inhibit the electrical or optical performance of the semiconductor and they can be controlled

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Vamivakas explains that the brightness of the quantum dot emission can be controlled by applying (e&eb) the voltage.

He adds that the next step is to use voltage to (e) "tune the color" of the emitted photons, which can make it possible to integrate these quantum dots with (e&eb) nanophotonic devices

G_{36} : Category one of brightness of the quantum dot emission; voltage

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of voltage; brightness of the quantum dot emission

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

He adds that the next step is to use voltage to (e) "tune the color" of the emitted photons, which can make it possible to integrate these quantum dots with (e&eb) nanophotonic devices

G_{40} : Category one of voltage

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of"tune the color" of the emitted photons, which can make it possible to integrate these quantum dots with (e&eb) nanophotonic devices

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

He adds that the next step is to use voltage to "tune the color" of the emitted photons, which can make it possible to integrate these quantum dots with (e&eb) nanophotonic devices

G_{44} : Category one of voltage to "tune the color" of the emitted photons

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of integrate these quantum dots with (e&eb) nanophotonic devices

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad (1)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad (2)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad (3)$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad (4)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad (5)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad (6)$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad (7)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad (8)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad (9)$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad (10)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad (11)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad (12)$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad (13)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad (14)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad (15)$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad (16)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad (17)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad (18)$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad (19)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (91)$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (92)$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ *94*

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45},t)+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a'_{45})^{(9)}+(a''_{45})^{(9)}(T_{45},t)+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{37},t)+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{array}\right] G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}-(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}-(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}-(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \tag{97}$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \tag{98}$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: (99)

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: (100)

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: (101)

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad (102)$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad (103)$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad (104)$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad (105)$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: (106)

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad (107)$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad (108)$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants (109)

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}\left((G_{35}), t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(M) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(N) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(O) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(P)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(Q) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(R) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\widehat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} \text{ , } T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} \text{ , } T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(c) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$

$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad (208)$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad (209)$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \qquad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $((\widehat{M}_{36})^{(7)})_1, ((\widehat{M}_{36})^{(7)})_2$ *and* $((\widehat{M}_{36})^{(7)})_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)}((\widehat{M}_{36})^{(7)})_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)}((\widehat{M}_{36})^{(7)})_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}((G_{39})(t),t) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + ((\widehat{P}_{40})^{(8)} + G_j^0)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(\nu_1)^{(2)}>0,(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)}>0,(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right. \quad 334$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_0)^{(5)}\right)t\right]}}{5+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)}-(\nu_0)^{(5)}}{(\nu_0)^{(5)}-(\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{5+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)}-(\nu_0)^{(5)}}{(\nu_0)^{(5)}-(\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(c) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FOUR

Atomically Thin Semiconductor That Emit Single Photons

INTRODUCTION—VARIABLES USED

Defects in atomically thin semiconductor emit single photons May 04, 2015

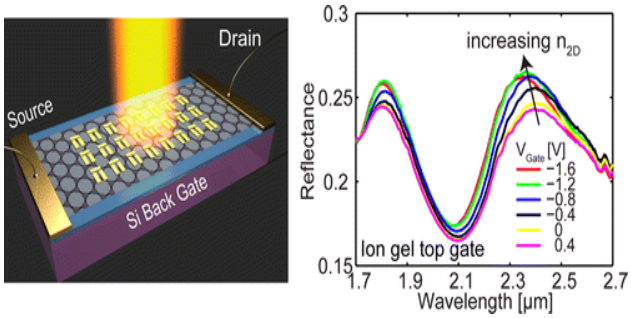
(1) The researchers take two of these atomically thin sheets and lay one over the other one. At the point where they overlap, (eb) a quantum dot is created.

(2) Overlap creates (eb) a defect in the otherwise smooth 2D sheet of semiconductor material. The extremely thin semiconductors are much easier to integrate with (e&eb) other electronics.
(3) The quantum dots in tungsten di selenide also possess (e) an intrinsic quantum degree of freedom - the electron spin.
(4) This is a desirable property as the spin can both act as (=) a store of quantum information as well as provide (eb) a probe of the local quantum dot environment.
(5) "What makes tungsten di selenide extremely versatile is that the color of the single photons emitted by the quantum dots is correlated with (e&eb) the quantum dot spin," said first author Chitraleema Chakraborty. Chakraborty added that the ease with which the spins and (e&eb) photons interact with one another should make these systems ideal for quantum information applications as well as nanoscale metrology.

Voltage-controlled quantum light from an atomically thin semiconductor Chitraleema Chakraborty, Laura Kinnischtzke, Kenneth M. Goodfellow, Ryan Beams & A. Nick Vamivakas Nature Nanotechnology 10, 507–511 (2015) doi:10.1038/nnano.2015.79 Published online 04 May 2015

(1) Although semiconductor defects can often be detrimental to (e) device performance, they are also responsible for the breadth of functionality exhibited by (e) modern optoelectronic devices1.
(2) Artificially engineered defects (so-called quantum dots) or naturally occurring defects in solids are currently being investigated for applications ranging from quantum information science2, 3 and optoelectronics4 to high-resolution metrology5. In parallel, the quantum confinement exhibited by atomically thin materials (semi-metals, semiconductors and insulators) has ushered in an era of flatland optoelectronics whose full potential is still being articulated6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 18. **Chitraleema Chakraborty, Laura Kinnischtzke, Kenneth M. Goodfellow, Ryan Beams & A. Nick Vamivakas** demonstrate the possibility of leveraging the atomically thin semiconductor tungsten diselenide (WSe2) as (=) a host for quantum dot-like defects. They report that this previously unexplored solid-state quantum emitter in WSe2 generates (eb) single photons with emission properties that can be controlled via (e&eb) the application of external D.C. electric and magnetic fields.
(3) These new optically active quantum dots exhibit excited-state lifetimes on the order of 1 ns and remarkably large excitonic g-factors of 10. It is anticipated that WSe2 quantum dots will provide (eb) a novel platform for integrated solid-state quantum photonics2, 3 and quantum information processing19, as well as a rich condensed-matter physics playground with which to explore the coupling of quantum dots and (e&eb) atomically thin semiconductors.

Electrical Modulation of Fano Resonance in Plasmonic Nanostructures Using Graphene Naresh K. Emani†§, Ting-Fung Chung‡§, Alexander V. Kildishev†§, Vladimir M. Shalaev†§, Yong P. Chen†‡§, and Alexandra Boltasseva*†∥§ Nano Lett., 2014, 14 (1), pp 78–82 DOI: 10.1021/nl403253c Publication Date (Web): December 9, 2013 Copyright © 2013 American Chemical Society*E-mail: aeb@purdue.edu.



(4) Pauli blocking of interband transistions gives rise to (eb) tunable optical properties in single layer graphene (SLG).

(5) tunable optical properties in single layer graphene (SLG) is exploited in (eb) a graphene-nanoantenna hybrid device where (e) Fano resonant plasmonic nanostructures are fabricated on top of a graphene sheet.

(6) Use of Fano resonant elements enhances (eb) the interaction of incident radiation with (e&eb) the graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance.

(7) Naresh K. Emani†§, Ting-Fung Chung‡§, Alexander V. Kildishev†§, Vladimir M. Shalaev†§, Yong P. Chen†‡§, and Alexandra Boltasseva*†‖§ observe electrically controlled damping in (eb)the Fano resonances occurring at (eb) approximately 2 μm, and the results are verified by full-wave 3D finite-element simulations. Approach can be used for development of next generation of tunable plasmonic and hybrid nanophotonic devices

NOTATION

Module One

The researchers take two of these atomically thin sheets and lay one over the other one. At the point where they overlap, (eb) a quantum dot is created

G_{13} : Category one of overlapping of atomically thin sheets

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of quantum dot (many such experiments create quantum dots. Parameters of the experiment could be used for classification or all the categories could be taken as equivalent)

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Overlap creates (eb) a defect in the otherwise smooth 2D sheet of semiconductor material.

The extremely thin semiconductors are much easier to integrate with (e&eb) other electronics

G_{16} : Category one of Overlap

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of defect in the otherwise smooth 2D sheet of semiconductor material

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

quantum dots in tungsten di selenide also possess (e) an intrinsic quantum degree of freedom - the electron spin

G_{20} : Category one of intrinsic quantum degree of freedom - the electron spin; quantum dots in tungsten di selenide

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of quantum dots in tungsten di selenide; intrinsic quantum degree of freedom - the electron spin

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

This is a desirable property as the spin can both act as (=) a store of quantum information as well as provide (eb) a probe of the local quantum dot environment

G_{24} : Category one of spin

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of store of quantum information as well as provide (eb) a probe of the local quantum dot environment

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

G_{28} : Category one of spin (s)

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of probe of the local quantum dot environment

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

"What makes tungsten di selenide extremely versatile is that the color of the single photons emitted by the quantum dots is correlated with (e&eb) the quantum dot spin," said first author Chitraleema Chakraborty.

Chakraborty added that the ease with which the spins and (e&eb) photons interact with one another should make these systems ideal for quantum information applications as well as nanoscale metrology

G_{32} : Category one of color of the single photons emitted by the quantum dots; quantum dot spin

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of quantum dot spin ;color of the single photons emitted by the quantum dots

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Although semiconductor defects can often be detrimental to device performance, they are also responsible for (e) the breadth of functionality exhibited by modern optoelectronic devices1

G_{36} : Category one of breadth of functionality exhibited by modern optoelectronic devices1

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of semiconductor defects can often be detrimental to device performance, they are also responsible

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Artificially engineered defects (so-called quantum dots) or naturally occurring defects in solids are currently being investigated for applications ranging from quantum information science2, 3 and optoelectronics4 to high-resolution metrology5. In parallel, the quantum confinement exhibited by atomically thin materials (semi-metals, semiconductors and insulators) has ushered in an era of flatland optoelectronics whose full potential is still being articulated6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 18. **Chitraleema Chakraborty, Laura Kinnischtzke, Kenneth M. Goodfellow, Ryan Beams & A. Nick Vamivakas** demonstrate the possibility of leveraging the atomically thin semiconductor tungsten diselenide

(WSe2) as (=) a host for quantum dot-like defects.

They report that this previously unexplored solid-state quantum emitter in WSe2 generates (eb) single photons with emission properties that can be controlled via (e&eb) the application of external D.C. electric and magnetic fields.

G_{40} : Category one of solid-state quantum emitter in WSe2

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of single photons with emission properties that can be controlled via (e&eb) the application of external D.C. electric and magnetic fields.

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

They report that this previously unexplored solid-state quantum emitter in WSe2 generates single photons with emission properties that can be controlled via (e&eb) the application of external D.C. electric and magnetic fields

G_{44} : Category one of solid-state quantum emitter in WSe2 generates single photons with emission properties; application of external D.C. electric and magnetic fields

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of application of external D.C. electric and magnetic fields; solid-state quantum emitter in WSe2 generates single photons with emission properties

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for

category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & \boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category $1,2$ *and* 3
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category $1,2$ *and* 3
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category $1,2$ *and* 3
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category $1,2$ *and* 3
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category $1,2$ *and* 3
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\left[-(b''_{24})^{(4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\left[-(b''_{25})^{(4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\left[-(b''_{26})^{(4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1,2$ *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1,2$ *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1,2$ *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1,2$ *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1,2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & +(a''_{36})^{(7)}(T_{37},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & +(a''_{37})^{(7)}(T_{37},t) & +(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & +(a''_{38})^{(7)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a''_{36})^{(7)}(T_{37},t)$, $(a''_{37})^{(7)}(T_{37},t)$, $(a''_{38})^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation

coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

96

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(S) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(T) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(U) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(V)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(W) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(X) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(d) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

208

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

209

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

214

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$
Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$ 331

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$ 332

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(d) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ and $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586

ASYMPTOTIC STABILITY ANALYSIS A

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ and $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FIVE

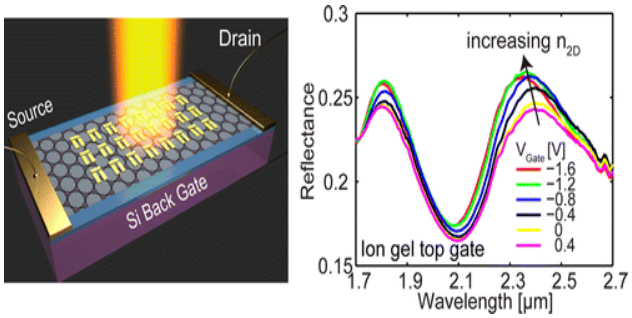
Quantum Light From An Atomically Thin Semiconductor

INTRODUCTION—VARIABLES USED

NATURE NANOTECHNOLOGY | LETTER Voltage-controlled quantum light from an atomically thin semiconductor Chitraleema Chakraborty, Laura Kinnischtzke, Kenneth M. Goodfellow, Ryan Beams & A. Nick Vamivakas Nature Nanotechnology 10, 507–511 (2015) doi:10.1038/nnano.2015.79 Received 23 December 2014 Accepted 17 March 2015 published online 04 May 2015

(1) These new optically active quantum dots exhibit (eb) excited-state lifetimes on the order of 1 ns and remarkably large excitonic g-factors of 10.
(2) It is anticipated that WSe2 quantum dots will provide (eb) a novel platform for (e) integrated solid-state quantum photonics2, 3 and quantum information processing19, as well as a rich condensed-matter physics playground with which to explore the coupling of quantum dots and (e&eb) atomically thin semiconductors.

Electrical Modulation of Fano Resonance in Plasmonic Nanostructures Using Graphene Naresh K. Emani†§, Ting-Fung Chung‡§, Alexander V. Kildishev†§, Vladimir M. Shalaev†§, Yong P. Chen†‡§, and Alexandra Boltasseva*†∥§ Nano Lett., 2014, 14 (1), pp 78–82 DOI: 10.1021/nl403253c Publication Date (Web): December 9, 2013 Copyright © 2013 American Chemical Society*E-mail: aeb@purdue.edu.



(1) Pauli blocking of interband transistions gives rise to (eb) tunable optical properties in single layer graphene (SLG).

(2) Tunable optical properties in single layer graphene (SLG) is exploited in a graphene-nanoantenna hybrid device where (e) Fano resonant plasmonic nanostructures are fabricated on top of a graphene sheet. Use of Fano resonant elements enhances (eb) the interaction of incident radiation with (e&eb) the graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance.

(3) Naresh K. Emani†§, Ting-Fung Chung‡§, Alexander V. Kildishev†§, Vladimir M. Shalaev†§, Yong P. Chen†‡§, and Alexandra Boltasseva*†‖§ observe electrically controlled damping in (eb)the Fano resonances occurring at (eb) approximately 2 μm, and the results are verified by full-wave 3D finite-element simulations. Approach can be used for development of next generation of tunable plasmonic and hybrid nanophotonic devices.

Quenching of dynamic nuclear polarization by spin-orbit coupling in GaAs quantum dots John M. Nichol, Shannon P. Harvey, Michael D. Shulman, Arijeet Pal, Vladimir Umansky, Emmanuel I. Rashba, Bertrand I. Halperin, Amir Yacoby (Submitted on 18 Feb 2015)

(1) The central-spin problem, in which an electron spin interacts with (e&eb) a nuclear spin bath, is a widely studied model of quantum decoherence. Dynamic nuclear polarization (DNP) occurs in (eb) central spin systems when (e) electronic angular momentum is transferred to (e&eb) nuclear spins and is exploited in (eb) spin-based quantum information processing for (e) coherent electron and nuclear spin control.

(2) However, the mechanisms limiting DNP remain only partially understood. **John M. Nichol, Shannon P. Harvey, Michael D. Shulman, Arijeet Pal, Vladimir Umansky, Emmanuel I. Rashba, Bertrand I. Halperin, Amir Yacoby** show that spin-orbit coupling quenches (e) DNP in a GaAs double quantum dot, even though spin-orbit coupling in GaAs is weak.

(3) Using Landau-Zener sweeps, authors measure the dependence of the electron spin-flip probability on (∝) the strength and direction of in-plane magnetic field, allowing (eb) to distinguish effects of (e) the spin-orbit and hyperfine interactions.

(4) To confirm interpretation, they measure high-bandwidth correlations in the electron spin-flip probability and attain results consistent with a significant spin-orbit contribution. They observe that DNP is quenched when (e) the spin-orbit component exceeds the hyperfine, in agreement with a theoretical model.

(5) Results shed new light on the surprising competition between the spin-orbit and (e&eb) hyperfine interactions in central-spin systems. Subjects: Mesoscale and Nanoscale Physics (cond-mat.mes-hall) Cite as: arXiv: 1502.05400 [cond-mat.mes-hall] (or arXiv: 1502.05400v1 [cond-mat.mes-hall] for this version)

NOTATION

Module One

These new optically active quantum dots exhibit (eb) excited-state lifetimes on the order of 1 ns and remarkably large excitonic g-factors of 10

G_{13} : Category one of optically active quantum dots

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of excited-state lifetimes on the order of 1 ns and remarkably large excitonic g-factors of 10

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

It is anticipated that WSe2 quantum dots will provide (eb) a novel platform for (e) integrated solid-state quantum photonics2, 3 and quantum information processing19, as well as a rich condensed-matter physics playground with which to explore the coupling of quantum dots and (e&eb) atomically thin semiconductors

G_{16} : Category one of quantum dots; atomically thin semiconductors

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of atomically thin semiconductors; quantum dots

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Pauli blocking of interband transistions gives rise to (eb) tunable optical properties in single layer graphene (SLG)

G_{20} : Category one of Pauli blocking of interband transistions

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of tunable optical properties in single layer graphene (SLG)

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Elliott formula from Wikipedia, the free encyclopedia

$$\alpha(\omega) = \mathrm{Im}\left[\sum_{\lambda} \frac{F_\lambda}{E_\lambda - \hbar\omega - \mathrm{i}\gamma_\lambda(\omega)}\right]$$

G_{24} : Category one of LHS of the above equation

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of RHS of the above equation

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

A tunable optical property in single layer graphene (SLG) is exploited in a graphene-nanoantenna hybrid device where (e) Fano resonant plasmonic nanostructures are fabricated on top of a graphene sheet.

Use of Fano resonant elements enhances (eb) the interaction of incident radiation with (e&eb) the graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance

G_{28} : Category one of Fano resonant elements

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of interaction of incident radiation with (e&eb) the graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Use of Fano resonant elements enhances the interaction of incident radiation with (e&eb) the graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance

G_{32} : Category one of Fano resonant elements enhances the interaction of incident radiation; graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of graphene sheet and enables (wb) efficient electrical modulation of the plasmonic

resonance ;Fano resonant elements enhances the interaction of incident radiation

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Use of Fano resonant elements enhances the interaction of incident radiation with the graphene sheet and enables (wb) efficient electrical modulation of the plasmonic resonance

G_{36} : Category one of Fano resonant elements enhances the interaction of incident radiation with the graphene sheet

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of efficient electrical modulation of the plasmonic resonance

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The central-spin problem, in which an electron spin interacts with (e&eb) a nuclear spin bath, is a widely studied model of quantum decoherence.

Dynamic nuclear polarization (DNP) occurs in central spin systems when (e) electronic angular momentum is transferred to (e&eb) nuclear spins and is exploited in (eb) spin-based quantum information processing for (e) coherent electron and nuclear spin control

G_{40} : Category one of electronic angular momentum is transferred to (e&eb) nuclear spins and is exploited in (eb) spin-based quantum information processing for (e) coherent electron and nuclear spin control

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Dynamic nuclear polarization (DNP) occurs in central spin systems

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Dynamic nuclear polarization (DNP) occurs in central spin systems when electronic angular momentum is transferred to (e&eb) nuclear spins and is exploited in (eb) spin-based quantum information processing for (e) coherent electron and nuclear spin control

G_{44} : Category one of Dynamic nuclear polarization (DNP) occurs in central spin systems when electronic angular momentum; nuclear spins and is exploited in (eb) spin-based quantum information processing for (e) coherent electron and nuclear spin control

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of nuclear spins and is exploited in (eb) spin-based quantum information processing for (e) coherent electron and nuclear spin control; Dynamic nuclear polarization (DNP) occurs in central spin systems when electronic angular momentum

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15}$ 3

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$ 4

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$ 5

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**
Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**
Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\big((G_{35}),t\big)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\big((G_{35}),t\big)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\big((G_{35}),t\big)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\big((G_{39}),t\big)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\big((G_{39}),t\big)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\big((G_{39}),t\big)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for

1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\;+(a''_{16})^{(2,2,2)}(T_{17},t)\;+(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\;+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\;+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\;+(a''_{17})^{(2,2,2)}(T_{17},t)\;+(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\;+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\;+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\;+(a''_{18})^{(2,2,2)}(T_{17},t)\;+(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\;+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\;+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficients for category 1, 2 and 3
$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3
$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3
$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*
$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*
$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*
$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*
$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*
$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*
$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (91)$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (92)$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ *94*

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}+(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \le \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \le (\hat{k}_{32})^{(6)}|T_{33} - \ T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(Y) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(Z) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(AA) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(BB)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(CC) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(DD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\widehat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(e) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)}\left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)}\left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$

$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} \left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)} (\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} \mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \qquad (214)$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (216)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: (217)

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is (218)

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (250)

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\;e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\;(G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}}G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}}G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)} \quad 293$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad 294$$

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0$, $(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0$, $(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)},(\nu_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},\nu^{(7)},u^{(7)}$: 361

By $(\nu_1)^{(7)}>0,(\nu_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{\nu}_1)^{(7)},,(\bar{\nu}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)}>0,(\bar{\nu}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\leq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\leq\nu^{(1)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\leq(\bar{\nu}_1)^{(1)}\leq\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\leq\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\leq\nu^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

with $(p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(e) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \ , \ \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\Big]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\Big)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SIX

High-Dimensional Hyperentanglement Through A Biphoton Frequency Comb

INTRODUCTION—VARIABLES USED

New method of quantum entanglement vastly increases how much information can be carried in a photon Jun 30, 2015 by Matthew Chin

(1) A team of researchers led by UCLA electrical engineers has demonstrated a new way to harness (e)

light particles, or photons, that are connected to (e&eb) each other and act in unison no matter how far apart they are —a phenomenon known as quantum entanglement. In previous studies, photons have typically been entangled by (e&eb) one dimension of their quantum properties—usually the direction of their polarization.

(2) In the new study, researchers demonstrated that they could slice up and entangle each photon pair into (e&eb) multiple dimensions using (e) quantum properties such as the photons' energy and spin. This method, called hyperentanglement, allows (eb) each photon pair to carry much more data than was possible with previous methods.

(3) Quantum entanglement could allow (eb) users to send data through (e&eb) a network and know immediately whether that data had made it to its destination without (e) being intercepted or altered.

(4) With hyperentanglement, users could send much denser packets of information using (e) the same networks. The research, published today in Nature Photonics, was led by Zhenda Xie, a research scientist in the lab of Chee Wei Wong, a UCLA associate professor of electrical engineering who was the research project's principal investigator. Researchers from MIT, Columbia University, the University of Maryland and the National Institute of Standards and Technology were also part of the team.

(5) Albert Einstein famously described quantum entanglement as "spooky action at a distance" because it seems so improbable that what happens to one particle in an entangled pair (eb) also happens instantly to the other particle, even over great distances. The phenomenon exceeds the speed of light.

(6) In the new study, researchers sent hyperentangled photons in a shape known as a biphoton frequency comb, essentially breaking up entangled photons into (e&ebn) smaller parts.

(7) In secure data transfer, photons sent over fiber optic networks can be encrypted through (e&eb) entanglement.

(8) With each dimension of entanglement, the amount of information carried on a photon pair is (=) doubled, so a photon pair entangled by (e) five dimensions can carry 32 times as much data as a pair entangled by (e) only one.

(9) The result greatly extends from (e) wavelength multiplexing, the method for (e) carrying many videos over a single optical fiber.

(10) "We show that an optical frequency comb can be generated at (eb) single photon level," Xie said. "Essentially, we're leveraging wavelength division multiplexing concepts at (eb) the quantum level."

(11) Potential applications for the research include secure communication and information processing, in particular for high-capacity data transfer with minimal error. This could be useful for medical servers, government data communications, financial markets and military communication channels, as well as quantum cloud communications and distributed quantum computing. "We are fortunate to verify a decades-old theoretical prediction by Professor Jeff Shapiro of MIT, that quantum entanglement can be observed in a comb-like state," Wong said. "With the help of state-of-the-art high-speed single photon detectors at NIST and support from Dr. Franco Wong, Dr. Xie was able to verify the high-dimensional and multi-degrees-of-freedom entanglement of photons. These observations demonstrate a new fundamentally secure approach for dense information processing and communications."

NOTATION

Module One

Dynamic nuclear polarization (DNP) occurs in central spin systems when electronic angular momentum is transferred to nuclear spins and is exploited in spin-based quantum information processing for (e) coherent

electron and nuclear spin control

G_{13} : Category one of coherent electron and nuclear spin control

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of Dynamic nuclear polarization (DNP) occurs in central spin systems when electronic angular momentum is transferred to nuclear spins and is exploited in spin-based quantum information processing

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

A team of researchers led by UCLA electrical engineers has demonstrated a new way to harness light particles, or photons, that are connected to (e&eb) each other and act in unison no matter how far apart they are —a phenomenon known as quantum entanglement.

In previous studies, photons have typically been entangled by (e&eb) one dimension of their quantum properties—usually the direction of their polarization

G_{16} : Category one of photons have typically been entangled; one dimension of their quantum properties—usually the direction of their polarization

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of one dimension of their quantum properties—usually the direction of their polarization; photons have typically been entangled

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

In the new study, researchers demonstrated that they could slice up and entangle each photon pair into (e&eb) multiple dimensions using (e) quantum properties such as the photons' energy and spin.

This method, called hyperentanglement, allows (eb) each photon pair to carry much more data than was possible with previous methods

G_{20} : Category one of slice up and entangle each photon pair; multiple dimensions using (e) quantum properties such as the photons' energy and spin

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of multiple dimensions using (e) quantum properties such as the photons' energy and

spin ;slice up and entangle each photon pair

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In the new study, researchers demonstrated that they could slice up and entangle each photon pair into multiple dimensions using (e) quantum properties such as the photons' energy and spin

G_{24} : Category one of quantum properties such as the photons' energy and spin

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of slice up and entangle each photon pair into multiple dimensions

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Quantum entanglement could allow (eb) users to send data through (e&eb) a network and know immediately whether that data had made it to its destination without (e) being intercepted or altered

G_{28} : Category one of Quantum entanglement

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of users to send data through (e&eb) a network and know immediately whether that data had made it to its destination without (e) being intercepted or altered

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Quantum entanglement could allow users to send data through (e&eb) a network and know immediately whether that data had made it to its destination without (e) being intercepted or altered

G_{32} : Category one of Quantum entanglement could allow users to send data; network and know immediately whether that data had made it to its destination without being intercepted or altered

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of network and know immediately whether that data had made it to its destination

without being intercepted or altered; Quantum entanglement could allow users to send data

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

In the new study, researchers sent hyperentangled photons in a shape known as a biphoton frequency comb, essentially breaking up entangled photons into (e&ebn) smaller parts

G_{36} : Category one of hyperentangled photons in a shape known as a biphoton frequency comb, essentially breaking up entangled photons; smaller parts

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of smaller parts ;hyperentangled photons in a shape known as a biphoton frequency comb, essentially breaking up entangled photons

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In secure data transfer, photons sent over fiber optic networks can be encrypted through (e&eb) entanglement

G_{40} : Category one of photons sent over fiber optic networks can be encrypted; entanglement

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of entanglement; photons sent over fiber optic networks can be encrypted

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

We show that an optical frequency comb can be generated at (eb) single photon level," Xie said. "Essentially, we're leveraging wavelength division multiplexing concepts at (eb) the quantum level."

G_{44} : Category one of optical frequency comb; single photon level

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of single photon level; optical frequency comb

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21},t)+(a''_{16})^{(2,2,2)}(T_{17},t)+(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)+(a''_{40})^{(8,8,8,8)}(T_{41},t)+(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}+(a''_{21})^{(3)}(T_{21},t)+(a''_{17})^{(2,2,2)}(T_{17},t)+(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)+(a''_{41})^{(8,8,8,8)}(T_{41},t)+(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21},t)+(a''_{18})^{(2,2,2)}(T_{17},t)+(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)+(a''_{42})^{(8,8,8,8)}(T_{41},t)+(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a_{28}')^{(5)} & +(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad 79$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a_{29}')^{(5)} & +(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{29} \quad 80$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a_{30}')^{(5)} & +(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{30} \quad 81$$

$Where$ $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ $are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ $are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t) \quad +(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)} & +(a''_{41})^{(8)}(T_{41},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t) \quad +(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)} & +(a''_{42})^{(8)}(T_{41},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t) \quad +(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{15}$$

Where $+(a''_{40})^{(8)}(T_{41},t)$, $+(a''_{41})^{(8)}(T_{41},t)$, $+(a''_{42})^{(8)}(T_{41},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)}\ \ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\left((G_{19}), t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\ ,\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(FF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le \ (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(GG) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(HH)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - \ T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(II) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(JJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\widehat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)$, $T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(f) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0 \quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$

$G_{20}^{(2)}|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t} \leq \frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right) \quad (214)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (216)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: (217)

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)}G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} is bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. (218)

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\,(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_{+}$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (239)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(5)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)})d\left(((G_{39})^{(1)},(T_{39})^{(1)};(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)},(\nu_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},\nu^{(7)},u^{(7)}$: 361

By $(\nu_1)^{(7)}>0,(\nu_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{\nu}_1)^{(7)},,(\bar{\nu}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)}>0,(\bar{\nu}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2+(\sigma_2)^{(3)}\nu^{(3)}-(a_{20})^{(3)}\right)\leq\frac{d\nu^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2+(\sigma_1)^{(3)}\nu^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(\nu_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\nu_1)^{(3)}<(\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t)\geq\frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(\nu_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)}\leq\nu^{(3)}(t)\leq(\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t)\leq\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)}\leq\nu^{(3)}(t)\leq(\bar{\nu}_1)^{(3)}$

If $0<(\nu_1)^{(3)}<(\nu_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)}\leq\frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}\leq\nu^{(3)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}\leq(\bar{\nu}_1)^{(3)}$$

If $0<(\nu_1)^{(3)}\leq(\bar{\nu}_1)^{(3)}\leq(\nu_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)}\leq\nu^{(3)}(t)\leq\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}\leq(\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)}\leq\nu^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{\nu^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(f) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SIX A

Insulator-To-Metal Transition

INTRODUCTION—VARIABLES USED

Science 11 September 2015: Vol. 349 no. 6253 pp. 1202-1205 DOI: 10.1126/science.1260507 Critical behavior at a dynamic vortex insulator-to-metal transition Nicola Poccia1,2, Tatyana I. Baturina3,4,5, Francesco Coneri1, Cor G. Molenaar1, X. Renshaw Wang1, Ginestra Bianconi6, Alexander

Brinkman1, Hans Hilgenkamp1, Alexander A. Golubov1,7, Valerii M. Vinokur5,*

(1) An array of superconducting islands placed on a normal metal film offers (eb) a tunable realization of nanopatterned superconductivity.
(2) Array of superconducting islands placed on a normal metal film offers a tunable realization of nanopatterned superconductivity enables (eb) investigation of the nature of competing vortex states and phase transitions between them.
(3) A square array creates (eb) the eggcrate potential in which magnetic field–induced vortices are (=) frozen into a vortex insulator.
(4) **Nicola Poccia1,2, Tatyana I. Baturina3,4,5, Francesco Coneri1, Cor G. Molenaar1, X. Renshaw Wang1, Ginestra Bianconi6, Alexander Brinkman1, Hans Hilgenkamp1, Alexander A. Golubov1,7, Valerii M. Vinokur5,*** observed a vortex insulator–vortex metal transition driven by the applied electric current and determined (eb) critical exponents that coincided with those for thermodynamic liquid-gas transition. Findings offer a comprehensive description of dynamic critical behavior and establish a deep connection between equilibrium and (e&eb) nonequilibrium phase transitions**.**

Team announces breakthrough observation of Mott transition in a superconductor September 11, 2015 by Joost Bruysters

An international team of researchers, including the MESA+ Institute for Nanotechnology at the University of Twente in The Netherlands and the U.S. Department of Energy's Argonne National Laboratory, announced today in Science the observation of a dynamic Mott transition in a superconductor. The discovery experimentally connects the worlds of classical and quantum mechanics and illuminates the mysterious nature of the Mott transition. It also could shed light on non-equilibrium physics, which is poorly understood but governs most of what occurs in our world. The finding may also represent a step towards more efficient electronics based on the Mott transition. Since its foundations were laid in the early part of the 20th century, scientists have been trying to reconcile quantum mechanics with the rules of classical or Newtonian physics (like how you describe the path of an apple thrown into the air—or dropped from a tree). Physicists have made strides in linking the two approaches, but experiments that connect the two are still few and far between; physics phenomena are usually classified as either quantum or classical, but not both.

(5) One system that unites the two is found in superconductors, certain materials that conduct electricity perfectly when cooled to very low temperatures. Magnetic fields penetrate (e&eb) the superconducting material in (eb) the form of tiny filaments called vortices, which control (e&eb) the electronic and magnetic properties of the materials. These vortices display both classical and quantum properties, which led researchers to study them for access to one of the most enigmatic phenomena of modern condensed matter physics: the Mott insulator-to-metal transition.
(6) The Mott transition occurs in certain materials that according to textbook quantum mechanics should be metals, but in reality turn insulators. A complex phenomenon controlled by the interactions of many quantum particles, the Mott transition remains mysterious—even whether or not it's a classical or quantum phenomenon is not quite clear. Moreover, scientists have never directly observed a dynamic Mott transition, in which a phase transition from an insulating to a metallic state is induced by (e) driving an electrical current through the system; the disorder inherent in real systems disguises (e) Mott properties.
(7) At the University of Twente, researchers built a system containing 90,000 superconducting niobium nano-sized islands on top of a gold film. In this configuration, the vortices find it energetically easiest to settle into energy dimples in (eb) an arrangement like an egg crate—and make the

material act as (=) a Mott insulator, since the vortices won't move if the applied electric current is small.

(8) When they applied a large enough electric current, however, the scientists saw a dynamic Mott transition as the system flipped to become (=) a conducting metal; the properties of the material had changed as (e) the current pushed it out of equilibrium.

(9) Vortex systems behaved exactly like an electronic Mott transition driven by (e) temperature, said Valerii Vinokur, an Argonne Distinguished Fellow and corresponding author on the study. He and study co-author Tatyana Baturina, then at Argonne, analyzed the data and recognized the Mott behavior. "This experimentally materializes the correspondence between quantum and classical physics," Vinokur said. "We can controllably induce (eb) a phase transition between states of locked vortices to itinerant vortices by applying an (e&eb) electric current to the system," said Hans Hilgenkamp, head of the University of Twente research group. "Studying these phase transitions in our artificial systems is interesting in its own right, but may also provide further insight in the electronic transitions in real materials." The system could further provide scientists with insight into two categories of physics that have been hard to understand: many-body systems and out-of-equilibrium systems. "This is a classical system that which is easy to experiment with and provides what looks like access to very complicated many-body systems," said Vinokur. "It looks a bit like magic." As the name implies, many-body problems involve a large number of particles interacting; with current theory they are very difficult to model or understand. "Furthermore, this system will be key to building a general understanding of out-of-equilibrium physics, which would be a major breakthrough in physics," Vinokur said. The Department of Energy named five great basic energy scientific challenges of our time; one of them is to understand and controlling (e&eb) out-of-equilibrium phenomena.

(10) Equilibrium systems—where there's no energy moving around—are now understood quite well. But nearly everything in our lives involves (e&eb) energy flow, from photosynthesis to digestion to tropical cyclones, and we don't yet have the physics to describe it well. Scientists think a better understanding could lead to huge improvements in energy capture, batteries and energy storage, electronics and more. As we seek to make electronics faster and smaller, Mott systems also offer a possible alternative to the silicon transistor. Since they can be flipped (e&eb) between conducting and insulating with small changes in voltage, (eb) they may be able to encode 1s and 0s at smaller scales and higher accuracy than silicon transistors.

(11) 'Initially, we were studying the structures for completely different reasons, namely to investigate the effects of inhomogeneities on (eb) superconductivity," Hilgenkamp said. "After discussing with Valerii Vinokur at Argonne, we looked more specifically into our data and were quite amazed to see that it revealed so nicely the details of the transition between the state of locked and (e&eb) moving vortices. There are many ideas for follow up studies, and we look forward to our continued collaboration." The results were printed in the study "Critical behavior at a dynamic vortex insulator-to-metal transition," released today in Science. Other co-authors are associated with the Siberian Branch of Russian Academy of Science, the Rome International Center for Materials Science Super stripes, Novosibirsk State University, the Moscow Institute of Physics and Technology and Queen Mary University of London.

More information: "Critical behavior at a dynamic vortex insulator-to-metal transition." Science 11 September 2015: DOI: 10.1126/science.1260507 Journal reference: Science search and more info website Provided by: University of Twente search and more info website

NOTATION

Module One

An array of superconducting islands placed on a normal metal film offers (eb) a tunable realization of nanopatterned superconductivity

G_{13} : Category one of array of superconducting islands placed on a normal metal film

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of tunable realization of nanopatterned superconductivity

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

array of superconducting islands placed on a normal metal film offers a tunable realization of nanopatterned superconductivity enables (eb) investigation of the nature of competing vortex states and phase transitions between them

G_{16} : Category one of array of superconducting islands placed on a normal metal film offers a tunable realization of nanopatterned superconductivity

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of investigation of the nature of competing vortex states and phase transitions between them

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

A square array creates (eb) the eggcrate potential in which magnetic field–induced vortices are (=) frozen into a vortex insulator

G_{20} : Category one of square array

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of eggcrate potential in which magnetic field–induced vortices are (=) frozen into a vortex insulator

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

square array creates the eggcrate potential in which magnetic field–induced vortices are (=) frozen into a

vortex insulator

G_{24} : Category one of square array creates the eggcrate potential in which magnetic field–induced vortices

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of frozen into a vortex insulator

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Nicola Poccia1,2, Tatyana I. Baturina3,4,5, Francesco Coneri1, Cor G. Molenaar1, X. Renshaw Wang1, Ginestra Bianconi6, Alexander Brinkman1, Hans Hilgenkamp1, Alexander A. Golubov1,7, Valerii M. Vinokur5,* observed a vortex insulator–vortex metal transition driven by the applied electric current and determined (eb) critical exponents that coincided with those for thermodynamic liquid-gas transition.

Findings offer a comprehensive description of dynamic critical behavior and establish a deep connection between equilibrium and (e&eb) nonequilibrium phase transitions

G_{28} : Category one of equilibrium; nonequilibrium phase transitions

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of nonequilibrium phase transitions; equilibrium

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

One system that unites the two is found in superconductors, certain materials that conduct electricity perfectly when cooled to very low temperatures.

Magnetic fields penetrate (e&eb) the superconducting material in (eb) the form of tiny filaments called vortices, which control (e&eb) the electronic and magnetic properties of the materials.

These vortices display both classical and quantum properties, which led researchers to study them for access to one of the most enigmatic phenomena of modern condensed matter physics: the Mott insulator-to-metal transition

G_{32} : Category one of Magnetic fields; superconducting material in (eb) the form of tiny filaments called vortices, which control (e&eb) the electronic and magnetic properties of the materials

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of superconducting material in (eb) the form of tiny filaments called vortices, which control (e&eb) the electronic and magnetic properties of the materials; Magnetic fields

T_{33} : Category two of

T_{34} : Category three of

Module seven

Magnetic fields penetrate the superconducting material in the form of tiny filaments called vortices, which control (e&eb) the electronic and magnetic properties of the materials.

G_{36} : Category one of Magnetic fields penetrate the superconducting material in the form of tiny filaments called vortices; electronic and magnetic properties of the materials

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of electronic and magnetic properties of the materials; Magnetic fields penetrate the superconducting material in the form of tiny filaments called vortices

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The Mott transition occurs in certain materials that according to textbook quantum mechanics should be metals, but in reality turn insulators.

A complex phenomenon controlled by the interactions of many quantum particles, the Mott transition remains mysterious—even whether or not it's a classical or quantum phenomenon is not quite clear.

Moreover, scientists have never directly observed a dynamic Mott transition, in which a phase transition from an insulating to a metallic state is induced by (e) driving an electrical current through the system; the disorder inherent in real systems disguises (e) Mott properties

G_{40} : Category one of driving an electrical current through the system; the disorder inherent in real systems disguises (e) Mott properties

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of phase transition from an insulating to a metallic state

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Moreover, scientists have never directly observed a dynamic Mott transition, in which a phase transition from an insulating to a metallic state is induced by (e) driving an electrical current through the system;

the disorder inherent in real systems disguises (e) Mott properties

G_{44} : Category one of Mott properties

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of disorder inherent in real systems

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$

are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{ccc}(a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) \quad +(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{ccc}(a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) \quad +(a''_{21})^{(3,3,3)}(T_{21},t)\\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{ccc}(a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) \quad +(a''_{22})^{(3,3,3)}(T_{21},t)\\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{array}{ccc}(b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) \quad -(b''_{20})^{(3,3,3,)}(G_{23},t)\\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t)\\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t)\end{array}\right]T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition*

coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}+(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t)\\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t)\\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\,+(a''_{32})^{(6)}(T_{33},t)\;+(a''_{28})^{(5,5,5)}(T_{29},t)\;+(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\,+(a''_{33})^{(6)}(T_{33},t)\;+(a''_{29})^{(5,5,5)}(T_{29},t)\;+(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\,+(a''_{34})^{(6)}(T_{33},t)\;+(a''_{30})^{(5,5,5)}(T_{29},t)\;+(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)} & -(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)} & -(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)} & -(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41}$$

$$- \begin{bmatrix} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{c} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(KK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(LL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(MM) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(NN)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(OO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(PP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - \ T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0,t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(g) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t), t(t), t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$
Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations
$(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(g) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504 is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 513$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*})=0$, T_{41}^{*} given by $f(T_{41}^{*})=0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SEVEN

Out Of Equilibrium Systems

INTRODUCTION—VARIABLES USED

Harnessing high-dimensional hyperentanglement through a biphoton frequency comb Zhenda Xie, Tian Zhong, Sajan Shrestha, XinAn Xu, Junlin Liang, Yan-Xiao Gong, Joshua C. Bienfang, Alessandro Restelli, Jeffrey H. Shapiro, Franco N. C. Wong & Chee Wei Wong AffiliationsContributionsCorresponding authors Nature Photonics (2015) doi:10.1038/nphoton.2015.110 Received 23 December 2014 Accepted 20 May 2015 Published online 29 June 2015

(1) Quantum entanglement is (=) a fundamental resource for (e) secure information processing and communications, and hyperentanglement or high-dimensional entanglement has been separately proposed for (e) its high data capacity and error resilience.
(2) The continuous-variable nature of the energy–time entanglement makes it an ideal candidate for (e) efficient high-dimensional coding with (e&eb) minimal limitations.
(3) Here, authors demonstrate the first simultaneous high-dimensional hyperentanglement using (e) a biphoton frequency comb to harness (e) the full potential in (eb) both the **energy and time domain**
(4) Long-postulated Hong–Ou–Mandel quantum revival is (=) exhibited, with up to 19 time-bins and 96.5% visibilities.
(5) Authors further witness the high-dimensional energy–time entanglement through (e&eb) Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility.
(6) This qudit state is observed to simultaneously violate (e) the generalized Bell inequality by up to (e) 10.95 standard deviations while observing recurrent Clauser–Horne–Shimony–Holt S-parameters violate (e) up to 2.76.
(7) Biphoton frequency comb provides (eb) a platform for (e) photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding.

NOTATION

Module One

(1) At the University of Twente, researchers built a system containing 90,000 superconducting niobium nano-sized islands on top of a gold film. In this configuration, the vortices find it energetically easiest to settle into energy dimples in (eb) an arrangement like an egg crate—and make the material act as (=) a Mott insulator, since the vortices won't move if the applied electric current is small.

(2) When they applied a large enough electric current, however, the scientists saw a dynamic Mott transition as the system flipped to become (=) a conducting metal; the properties of the material had changed as (e) the current pushed it out of equilibrium.

(3) Vortex systems behaved exactly like an electronic Mott transition driven by (e) temperature, said Valerii Vinokur, an Argonne Distinguished Fellow and corresponding author on the study. He and study co-author Tatyana Baturina, then at Argonne, analyzed the data and recognized the Mott behavior. "This experimentally materializes the correspondence between quantum and classical physics," Vinokur said. "We can controllably induce (eb) a phase transition between states of locked vortices to itinerant vortices by applying an (e&eb) electric current to the system," said Hans Hilgenkamp, head of the University of Twente research group. "Studying these phase transitions in our artificial systems is interesting in but may also provide further insight in the electronic transitions in real materials." The system could further provide scientists with insight into two categories of physics that have been hard to understand: many-body systems and out-of-equilibrium systems. "This is a classical system that which is easy to experiment with and provides what looks like access to very complicated many-body systems," said Vinokur. "It looks a bit like magic." As the name implies, many-body problems involve a large number of particles interacting; with current theory they are very difficult to model or understand. "Furthermore, this system will be key to building a general understanding of out-of-equilibrium physics, which would be a major breakthrough in physics," Vinokur said. The Department of Energy named five great basic energy scientific challenges of our time; one of them is to understand and controlling (e&eb) out-of-equilibrium phenomena.

(4) Equilibrium systems—where there's no energy moving around—are now understood quite well. But nearly everything in our lives involves (e&eb) energy flow, from photosynthesis to digestion to tropical cyclones, and we don't yet have the physics to describe it well. Scientists think a better understanding could lead to huge improvements in energy capture, batteries and energy storage, electronics and more. As we seek to make electronics faster and smaller, Mott systems also offer a possible alternative to the silicon transistor. Since they can be flipped (e&eb) between conducting and insulating with small changes in voltage, (eb) they may be able to encode 1s and 0s at smaller scales and higher accuracy than silicon transistors.

(5) 'Initially, we were studying the structures for completely different reasons, namely to investigate the effects of inhomogeneities on (eb) superconductivity," Hilgenkamp said. "After discussing with Valerii Vinokur at Argonne, we looked more specifically into our data and were quite amazed to see that it revealed so nicely the details of the transition between the state of locked and (e&eb) moving vortices. There are many ideas for follow up studies, and we look forward to our continued collaboration." The results were printed in the study "Critical behavior at a dynamic vortex insulator-to-metal transition," released today in Science. Other co-authors are associated with the Siberian Branch of Russian Academy of Science, the Rome International Center for Materials Science Super stripes, Novosibirsk State University, the Moscow Institute of Physics and Technology and Queen Mary University of London.

More information: "Critical behavior at a dynamic vortex insulator-to-metal transition." Science 11

September 2015: DOI: 10.1126/science.1260507 Journal reference: Science search and more info website Provided by: University of Twente search and more info website
At the University of Twente, researchers built a system containing 90,000 superconducting niobium nano-sized islands on top of a gold film.

vortices in this configuration find it energetically easiest to settle into energy dimples in (eb) an arrangement like an egg crate—and make the material act as (=) a Mott insulator, since the vortices won't move if the applied electric current is small.

G_{13} : Category one of vortices in this configuration find it energetically easiest to settle into energy dimples

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of arrangement like an egg crate—and make the material act as (=) a Mott insulator, since the vortices won't move if the applied electric current is small

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

vortices in this configuration find it energetically easiest to settle into energy dimples in an arrangement like an egg crate—and make the material act as (=) a Mott insulator, since the vortices won't move if the applied electric current is small.

G_{16} : Category one of vortices in this configuration find it energetically easiest to settle into energy dimples in an arrangement like an egg crate—and make the material; Mott insulator, since the vortices won't move if the applied electric current is small

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Mott insulator, since the vortices won't move if the applied electric current is small; vortices in this configuration find it energetically easiest to settle into energy dimples in an arrangement like an egg crate—and make the material

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

dynamic Mott transition as the system when they applied a large enough electric current flipped to become (=) a conducting metal

the properties of the material had changed as (e) the current pushed it out of equilibrium

G_{20} : Category one of dynamic Mott transition as the system when they applied a large enough electric current flipped

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of conducting metal

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

properties of the material had changed as (e) the current pushed it out of equilibrium

G_{24} : Category one of current pushed it out of equilibrium

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of properties of the material

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Vortex systems behaved exactly like an electronic Mott transition driven by (e) temperature, said Valerii Vinokur, an Argonne Distinguished Fellow and corresponding author on the study

. He and study co-author Tatyana Baturina, then at Argonne, analyzed the data and recognized the Mott behavior. "This experimentally materializes the correspondence between quantum and classical physics," Vinokur said. "We can controllably induce (eb) a phase transition between states of locked vortices to itinerant vortices by applying an (e&eb) electric current to the system," said Hans Hilgenkamp, head of the University of Twente research group.

"Studying these phase transitions in our artificial systems is interesting in but may also provide further insight in the electronic transitions in real materials."

The system could further provide scientists with insight into two categories of physics that have been hard to understand: many-body systems and out-of-equilibrium systems.

"This is a classical system that which is easy to experiment with and provides what looks like access to very complicated many-body systems," said Vinokur.

"It looks a bit like magic." As the name implies, many-body problems involve a large number of particles interacting; with current theory they are very difficult to model or understand.

"Furthermore, this system will be key to building a general understanding of out-of-equilibrium physics, which would be a major breakthrough in physics," Vinokur said.

The Department of Energy named five great basic energy scientific challenges of our time; one of them is to understand and controlling (e&eb) out-of-equilibrium phenomena.

G_{28} : Category one of temperature

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Vortex systems behaved exactly like an electronic Mott transition

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

phase transition can be controllably induced between states of locked vortices to itinerant vortices by applying an (e&eb) electric current to the system," said Hans Hilgenkamp, head of the University of Twente research group

G_{32} : Category one of phase transition can be controllably induced between states of locked vortices to itinerant vortices; electric current to the system

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of electric current to the system; phase transition can be controllably induced between states of locked vortices to itinerant vortices

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

many-body systems and out-of-equilibrium systems

G_{36} : Category one of many-body systems; out-of-equilibrium systems

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of out-of-equilibrium systems ;many-body systems

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

This model throws light on controlling out of equilibrium systems

G_{40} : Category one of out of equilibrium systems; temperature

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of temperature; out of equilibrium systems

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Equilibrium systems—where there's no energy moving around—are now understood quite well.

But nearly everything in our lives involves (e&eb) energy flow, from photosynthesis to digestion to tropical cyclones, and we don't yet have the physics to describe it well

Model throws light on the physics of systems with energy flow as well.

Scientists think a better understanding could lead to huge improvements in energy capture, batteries and energy storage, electronics and more. As we seek to make electronics faster and smaller, Mott systems also offer a possible alternative to the silicon transistor. Since they can be flipped (e&eb) between conducting and insulating with small changes in voltage, (eb) they may be able to encode 1s and 0s at smaller scales and higher accuracy than silicon transistors

G_{44} : Category one of photosynthesis to digestion to tropical cyclones; energy flow

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of energy flow; photosynthesis to digestion to tropical cyclones,

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$

$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{16})^{(2)}\,+(a''_{16})^{(2)}(T_{17},t)\;+(a''_{13})^{(1,1,)}(T_{14},t)\;+(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{c}(a'_{17})^{(2)}\,+(a''_{17})^{(2)}(T_{17},t)\;+(a''_{14})^{(1,1,)}(T_{14},t)\;+(a''_{21})^{(3,3,3)}(T_{21},t)\\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{18})^{(2)}\,+(a''_{18})^{(2)}(T_{17},t)\;+(a''_{15})^{(1,1,)}(T_{14},t)\;+(a''_{22})^{(3,3,3)}(T_{21},t)\\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for

category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(QQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(RR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(SS) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(TT)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(UU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(VV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0,t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(h) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t})=0$ and $\mathrm{T}_i(\mathrm{t})=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

208

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

209

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

214

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t), t(t), t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{36})^{(7)}|G_{37}^{(1)} - G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)} - G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)|G_{36}^{(1)} - G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)|\; e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}\,((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$
Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations
$(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(h) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 513$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{*^*}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ and $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586

ASYMPTOTIC STABILITY ANALYSIS A

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ and $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SEVEN A

Hyperentanglement Or High-Dimensional Entanglement Has Been Separately Proposed For High Data Capacity And Error Resilience Of Quantum Entanglement

INTRODUCTION—VARIABLES USED

Harnessing high-dimensional hyperentanglement through a biphoton frequency comb Zhenda Xie, Tian Zhong, Sajan Shrestha, XinAn Xu, Junlin Liang, Yan-Xiao Gong, Joshua C. Bienfang, Alessandro Restelli, Jeffrey H. Shapiro, Franco N. C. Wong & Chee Wei Wong AffiliationsContributionsCorresponding authors Nature Photonics (2015) doi:10.1038/nphoton.2015.110 Received 23 December 2014 Accepted 20 May 2015 Published online 29 June 2015

(1) Quantum entanglement is (=) a fundamental resource for (e) secure information processing and communications, and hyperentanglement or high-dimensional entanglement has been separately proposed for (e) high data capacity and error resilience of quantum entanglement
(2) The continuous-variable nature of the energy–time entanglement makes it an ideal candidate for (e) efficient high-dimensional coding with (e&eb) minimal limitations.
(3) Here, authors demonstrate the first simultaneous high-dimensional hyperentanglement using (e) a biphoton frequency comb to harness (e) the full potential in (eb) both the **energy and time domain**
(4) Long-postulated Hong–Ou–Mandel quantum revival is (=) exhibited, with up to 19 time-bins and 96.5% visibilities.
(5) Authors further witness the high-dimensional energy–time entanglement through (e&eb) Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility.
(6) This qudit state is observed to simultaneously violate (e) the generalized Bell inequality by up to (e) 10.95 standard deviations while observing recurrent Clauser–Horne–Shimony–Holt S-parameters violate (e) up to 2.76.
(7) Biphoton frequency comb provides (eb) a platform for (e) photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding.

NOTATION

Module One

Equilibrium systems—where there's no energy moving around—are now understood quite well.

But nearly everything in our lives involves (e&eb) energy flow, from photosynthesis to digestion to tropical cyclones, and we don't yet have the physics to describe it well. Scientists think a better understanding could lead to huge improvements in energy capture, batteries and energy storage, electronics and more. As we seek to make electronics faster and smaller,

Mott systems also offer a possible alternative to the silicon transistor Since they can be flipped (e&eb) between conducting and insulating with small changes in voltage, (eb) they may be able to encode 1s and 0s at smaller scales and higher accuracy than silicon transistors.

'Initially, we were studying the structures for completely different reasons, namely to investigate the effects of inhomogeneities on (eb) superconductivity," Hilgenkamp said. "After discussing with Valerii Vinokur at Argonne, we looked more specifically into our data and were quite amazed to see that it revealed so nicely the details of the transition between the state of locked and (e&eb) moving vortices. There are many ideas for follow up studies, and we look forward to our continued collaboration." The results were printed in the study "Critical behavior at a dynamic vortex insulator-to-metal transition," released today in Science. Other co-authors are associated with the Siberian Branch of Russian Academy of Science, the Rome International Center for Materials Science Super stripes, Novosibirsk State University, the Moscow Institute of Physics and Technology and Queen Mary University of London.

G_{13} : Category one of they can be flipped between conducting and insulating with small changes in voltage, (eb) they may be able to encode 1s and 0s at smaller scales and higher accuracy than silicon transistors

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of Mott systems also offer a possible alternative to the silicon transistor

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

'Initially, we were studying the structures for completely different reasons, namely to investigate the effects of inhomogeneities on (eb) superconductivity," Hilgenkamp said.

"After discussing with Valerii Vinokur at Argonne, we looked more specifically into our data and were quite amazed to see that it revealed so nicely the details of the transition between the state of locked and (e&eb) moving vortices.

There are many ideas for follow up studies, and we look forward to our continued collaboration." The results were printed in the study "Critical behavior at a dynamic vortex insulator-to-metal transition," released today in Science. Other co-authors are associated with the Siberian Branch of Russian Academy of Science, the Rome International Center for Materials Science Super stripes, Novosibirsk State University, the Moscow Institute of Physics and Technology and Queen Mary University of London.

Effects of inhomogeneities on superconductivity

G_{16} : Category one of inhomogeneities; superconductivity

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of superconductivity; inhomogeneities

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Quantum entanglement is a fundamental resource for (e) secure information processing and communications,

hyperentanglement or high-dimensional entanglement has been separately proposed for (e) high data capacity and error resilience of quantum entanglement

G_{20} : Category one of secure information processing and communications

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Quantum entanglement is a fundamental resource

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

hyperentanglement or high-dimensional entanglement has been separately proposed for (e) high data capacity and error resilience of quantum entanglement

G_{24} : Category one of high data capacity and error resilience of quantum entanglement

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of hyperentanglement or high-dimensional entanglement has been separately proposed

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The continuous-variable nature of the energy–time entanglement makes it an ideal candidate for (e) efficient high-dimensional coding with minimal limitations

G_{28} : Category one of continuous-variable nature of the energy–time entanglement makes it an ideal candidate; efficient high-dimensional coding with minimal limitations

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of efficient high-dimensional coding with minimal limitations ;continuous-variable nature of the energy–time entanglement makes it an ideal candidate

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Here, authors demonstrate the first simultaneous high-dimensional hyperentanglement using (e) a biphoton frequency comb to harness (e) the full potential in (eb) both the **energy and time domain**

G_{32} : Category one of biphoton frequency comb to harness (e) the full potential in (eb) both the **energy and time domain**

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of simultaneous high-dimensional hyperentanglement

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Here, authors demonstrate the first simultaneous high-dimensional hyperentanglement using a biphoton frequency comb to harness (e) the full potential in (eb) both the **energy and time domain**

G_{36} : Category one of full potential in both the **energy and time domain**

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of high-dimensional hyperentanglement using a biphoton frequency comb

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

energy and time domain

G_{40} : Category one of **energy; time**

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of **time; energy**

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Long-postulated Hong–Ou–Mandel quantum revival is (=) exhibited, with up to 19 time-bins and 96.5% visibilities

G_{44} : Category one of Hong–Ou–Mandel quantum revival

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of exhibited, with up to 19 time-bins and 96.5% visibilities

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,

$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{ccc}(a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) \quad +(a_{20}'')^{(3,3,3)}(T_{21},t)\\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{ccc}(a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) \quad +(a_{21}'')^{(3,3,3)}(T_{21},t)\\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{ccc}(a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) \quad +(a_{22}'')^{(3,3,3)}(T_{21},t)\\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a_{44}'')^{(9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{array}{ccc}(b_{16}')^{(2)} & -(b_{16}'')^{(2)}(G_{19},t) & -(b_{13}'')^{(1,1,)}(G,t) \quad -(b_{20}'')^{(3,3,3,)}(G_{23},t)\\ -(b_{24}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)\\ -(b_{36}'')^{(7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9)}(G_{47},t)\end{array}\right]T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition*

coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}+(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t)\\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t)\\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{26} \quad (75)$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\,+(a''_{32})^{(6)}(T_{33},t)\;+(a''_{28})^{(5,5,5)}(T_{29},t)\;+(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\,+(a''_{33})^{(6)}(T_{33},t)\;+(a''_{29})^{(5,5,5)}(T_{29},t)\;+(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\,+(a''_{34})^{(6)}(T_{33},t)\;+(a''_{30})^{(5,5,5)}(T_{29},t)\;+(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{c} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(WW) \quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38 \qquad 131$$

(XX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(YY) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(ZZ)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(AAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(BBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(i) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

208

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

209

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

214

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}<1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t), t(t), t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, $\boldsymbol{if}\ (v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, $\boldsymbol{if}\ (v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$:-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(i) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*})=0$, T_{41}^{*} given by $f(T_{41}^{*})=0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\big((a_{30}')^{(5)} + (p_{30})^{(5)}\big)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\big((b_{28}')^{(5)} - (r_{28})^{(5)}\big)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(28)(j)}T_{28}^*\mathbb{G}_j\big) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\big((b_{29}')^{(5)} - (r_{29})^{(5)}\big)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\big(s_{(29)(j)}T_{29}^*\mathbb{G}_j\big) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\big((b_{30}')^{(5)} - (r_{30})^{(5)}\big)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(30)(j)}T_{30}^*\mathbb{G}_j\big) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586
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ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586 B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586 C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586 D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586 E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586 F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586 G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)})$$

$$\left[\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left((\lambda)^{(2)}\right)^{2} + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^{2} + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left((\lambda)^{(2)}\right)^{2} + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\} = 0$$

$$+$$

$$(\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)})\{((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)})$$

$$\left[\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21} + (a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20} + (a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left((\lambda)^{(3)}\right)^{2} + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left((\lambda)^{(3)}\right)^{2} + \left((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left((\lambda)^{(3)}\right)^{2} + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SEVEN B

Hong–Ou–Mandel Quantum Revival

INTRODUCTION—VARIABLES USED

Harnessing high-dimensional hyperentanglement through a biphoton frequency comb Zhenda Xie, Tian Zhong, Sajan Shrestha, XinAn Xu, Junlin Liang, Yan-Xiao Gong, Joshua C. Bienfang, Alessandro Restelli, Jeffrey H. Shapiro, Franco N. C. Wong & Chee Wei Wong AffiliationsContributionsCorresponding authors Nature Photonics (2015) doi:10.1038/nphoton.2015.110 Received 23 December 2014 Accepted 20 May 2015 Published online 29 June 2015

(1) Authors further witness the high-dimensional energy–time entanglement through (e&eb) Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility.
(2) This qudit state is observed to simultaneously violate (e) the generalized Bell inequality by up to (e) 10.95 standard deviations while observing recurrent Clauser–Horne–Shimony–Holt S-parameters violate (e) up to 2.76.
(3) Biphoton frequency comb provides (eb) a platform for (e) photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding.

Hong–Ou–Mandel effect from Wikipedia, the free encyclopedia

(1) The Hong–Ou–Mandel effect is a two-photon interference effect in (eb) quantum optics which was demonstrated by three physicists, C. K. Hong, Z. Y. Ou and Leonard Mandel in 1987 from the University of Rochester.[1]
(2) The effect occurs when (e) two identical single-photon waves enter a 50:50 beam splitter, one in each input port.
(3) When both photons are identical they will extinguish (e&eb) each other.
(4) If they become more distinguishable, (eb) the probability of detection will increase. In this way the interferometer can measure accurately bandwidth, path lengths and timing.

Quantum-mechanical description

Physical description

(5) When a photon enters a beam splitter, there are two possibilities: (eb) it will either be reflected or transmitted.
(6) The relative probabilities of transmission and reflection are determined by (e) the reflectivity of the beam splitter. Here, we assume a 50:50 beam splitter, in which a photon has equal probability of being reflected and transmitted.

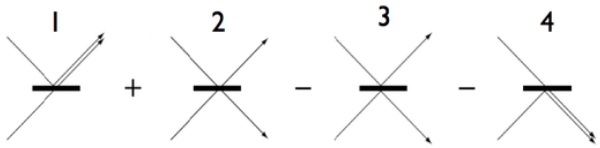


Figure 1: The four possibilities of two-photon reflection and transmission are added at the amplitude level.

(7) Next, consider two photons, one in each input mode of a 50:50 beam splitter (see figure 1). There are four possibilities for the photons to behave: 1) The photon coming in from above is (=) reflected

and the photon coming in from below is (=) transmitted; 2) both photons are (=) transmitted; 3) Both photons are (=) reflected; 4) The photon coming in from above is (=) transmitted and the photon coming in from below is reflected. We assume now that the two photons are identical in their physical properties (i.e., polarization, spatio-temporal mode structure, and frequency).

(8) Since the state of the beam splitter does not "record" which of the four possibilities actually happens, Feynman's rule dictates that we have to add all four possibilities at the amplitude level. In addition, reflection off the bottom side of the beam splitter introduces a relative phase shift of −1 in the associated term in the superposition. This is required by the reversibility (or unitarity) of the quantum evolution of the beam splitter. Since the two photons are identical, we cannot distinguish between the output states of possibilities 2 and 3 in figure 1, and their relative minus sign ensures that these two terms cancel. This can be interpreted as destructive interference.

Mathematical description

(1) Consider two optical modes a and b that carry **annihilation and creation operators** $\hat{a}$, $\hat{a}^\dagger$, and $\hat{b}$, $\hat{b}^\dagger$. Two identical photons in different modes can be described by the Fock states $\hat{a}^\dagger\hat{b}^\dagger|0,0\rangle_{ab} = |1,1\rangle_{ab}$, where $|1\rangle$ is a single-photon state. When the two modes a and b are mixed in a 50:50 beam splitter, they turn into new modes c and d, and the creation and annihilation operators transform accordingly:

$$\hat{a}^\dagger \rightarrow \frac{\hat{c}^\dagger + \hat{d}^\dagger}{\sqrt{2}} \quad \text{and} \quad \hat{b}^\dagger \rightarrow \frac{\hat{c}^\dagger - \hat{d}^\dagger}{\sqrt{2}}.$$

(2) The relative minus sign appears because the beam splitter is a unitary transformation. This can be seen most clearly when we write the two-mode beam splitter transformation in matrix form:

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}.$$

(3) Unitarity of the transformation now means unitarity of the matrix. Physically, this beam splitter transformation means that reflection off one surface induces a relative phase shift of −1 with respect to reflection off the other side of the beam splitter (see the Physical Description above). Similar transformations hold for the annihilation operators.

(4) When two photons enter the beam splitter, one on each side, the state of the two modes becomes

$$|1,1\rangle_{ab} = \hat{a}^\dagger\hat{b}^\dagger|0,0\rangle_{ab} \rightarrow \frac{1}{2}\left(\hat{c}^\dagger + \hat{d}^\dagger\right)\left(\hat{c}^\dagger - \hat{d}^\dagger\right)|0,0\rangle_{cd} = \frac{1}{2}\left(\hat{c}^{\dagger 2} - \hat{d}^{\dagger 2}\right)|0,0\rangle_{cd} = \frac{|2,0\rangle_{cd} - |0,2\rangle_{cd}}{\sqrt{2}}.$$

Since the commutator of the two creation operators $\hat{c}^\dagger$ and $\hat{d}^\dagger$ vanishes, the surviving terms in the superposition are $\hat{c}^{\dagger 2}$ and $\hat{d}^{\dagger 2}$. Therefore, when two identical photons enter a 50:50 beam splitter, they will always exit the beam splitter in the same (but random) output mode.

NOTATION

Module One

Authors further witness the high-dimensional energy–time entanglement through (e&eb) Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility

G_{13} : Category one of high-dimensional energy–time entanglement; Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility; high-dimensional energy–time entanglement

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Authors further witness the high-dimensional energy–time entanglement through Franson revivals, observed periodically at (eb) integer time-bins, with 97.8% visibility

G_{16} : Category one of high-dimensional energy–time entanglement through Franson revivals, observed periodically

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of integer time-bins, with 97.8% visibility

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

qudit state is observed to simultaneously violate (e) the generalized Bell inequality by up to (e) 10.95 standard deviations while observing recurrent Clauser–Horne–Shimony–Holt S-parameters violate (e) up to 2.76

G_{20} : Category one of generalized Bell inequality by up to 10.95 standard deviations while observing recurrent Clauser–Horne–Shimony–Holt S-parameters violate (e) up to 2.76

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of qudit state is observed to simultaneously violate

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Biphoton frequency comb provides (eb) a platform for (e) photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding

G_{24} : Category one of Biphoton frequency comb

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of platform for (e) photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Biphoton frequency comb provides a platform for (e) photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding

G_{28} : Category one of photon-efficient quantum communications towards (eb) the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Biphoton frequency comb provides a platform

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Biphoton frequency comb provides a platform for photon-efficient quantum communications towards the ultimate channel capacity through (e&eb) energy–time–polarization high-dimensional encoding

G_{32} : Category one of Biphoton frequency comb provides a platform for photon-efficient quantum communications towards the ultimate channel capacity; energy–time–polarization high-dimensional encoding

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of energy–time–polarization high-dimensional encoding; Biphoton frequency comb provides a platform for photon-efficient quantum communications towards the ultimate channel capacity

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Consider two optical modes a and b that carry **annihilation and creation operators** $\hat{a}$, $\hat{a}^{\dagger}$, and $\hat{b}$, $\hat{b}^{\dagger}$. Two identical photons in different modes can be described by the Fock states

$\hat{a}^{\dagger}\hat{b}^{\dagger}|0,0\rangle_{ab} = |1,1\rangle_{ab}$, where $|1\rangle$ is a single-photon state. When the two modes a and b are mixed in a 50:50 beam splitter, they turn into new modes c and d, and the creation and annihilation operators transform accordingly:

$$\hat{a}^{\dagger} \to \frac{\hat{c}^{\dagger} + \hat{d}^{\dagger}}{\sqrt{2}} \quad \text{and} \quad \hat{b}^{\dagger} \to \frac{\hat{c}^{\dagger} - \hat{d}^{\dagger}}{\sqrt{2}}.$$

Classification can be made by giving different values to the parameters for different experimentation.

G_{36} : Category one of $\hat{a}^{\dagger}\hat{b}^{\dagger}|0,0\rangle_{ab} = |1,1\rangle_{ab}$, ; $\hat{a}^{\dagger} \to \frac{\hat{c}^{\dagger} + \hat{d}^{\dagger}}{\sqrt{2}}$ and $\hat{b}^{\dagger} \to \frac{\hat{c}^{\dagger} - \hat{d}^{\dagger}}{\sqrt{2}}$.

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of $\hat{a}^{\dagger}\hat{b}^{\dagger}|0,0\rangle_{ab} = |1,1\rangle_{ab}$, ; $\hat{a}^{\dagger} \to \frac{\hat{c}^{\dagger} + \hat{d}^{\dagger}}{\sqrt{2}}$ and $\hat{b}^{\dagger} \to \frac{\hat{c}^{\dagger} - \hat{d}^{\dagger}}{\sqrt{2}}$.

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

$$|1,1\rangle_{ab} = \hat{a}^{\dagger}\hat{b}^{\dagger}|0,0\rangle_{ab} \to \frac{1}{2}\left(\hat{c}^{\dagger} + \hat{d}^{\dagger}\right)\left(\hat{c}^{\dagger} - \hat{d}^{\dagger}\right)|0,0\rangle_{cd} = \frac{1}{2}\left(\hat{c}^{\dagger 2} - \hat{d}^{\dagger 2}\right)|0,0\rangle_{cd} = \frac{|2,0\rangle_{cd} - |0,2\rangle_{cd}}{\sqrt{2}}.$$

G_{40} : Category one of LHS of the above transformation

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of RHS of the above transformation

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

When both photons are identical they will extinguish each other

G_{44} : Category one of photons are identical

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of they will extinguish each other

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 91$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 92$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,)}(G_{35},t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)} & +(a_{44}'')^{(9)}(T_{45},t) & +(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)} & +(a_{45}'')^{(9)}(T_{45},t) & +(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)} & +(a_{46}'')^{(9)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a_{44}'')^{(9)}(T_{45},t)$, $+(a_{45}'')^{(9)}(T_{45},t)$, $+(a_{46}'')^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}-(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}-(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}-(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(EEE) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(FFF)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(GGG) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(HHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\widehat{M}_{40})^{(8)}t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} \text{ , } T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} \text{ , } T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(j) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)}\left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} \left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)} (\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)} (\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} \mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad (208)$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad (209)$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$ 214

$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\ \ \text{for t}>0$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) \quad (226)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\;e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\;(G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}}G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}}G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}, \ \boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)} \quad 305$$

$$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}, \quad 306$$

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)} \quad 307$$

and analogously 308

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)} \quad 309$$

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad (326)$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad (327)$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- (328)

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right. \quad 334$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,

and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\le\frac{dv^{(1)}}{dt}\le-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\ge\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\le\nu^{(1)}(t)\le(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\le\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\le\nu^{(1)}(t)\le(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\le\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\le\nu^{(1)}(t)\le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\le(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\le(\bar{\nu}_1)^{(1)}\le\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\le\nu^{(1)}(t)\le\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\le(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\le\nu^{(1)}(t)\le(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21},t)\right) - (a_{21}'')^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(j) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION EIGHT

Efficient Compression Of Quantum Information

INTRODUCTION—VARIABLES USED

Focus: Saving Space with Quantum Information October 17, 2014• Physics 7, 106 A technique that packs the full content of three bits of quantum information into two may be a space saver for quantum computers

(1) If quantum computers are to become practical devices, they need to handle large amounts of data. Now researchers demonstrate the compression of quantum data by reducing the number of quantum bits (qubits) needed to (e) store a given amount of information. In this first experiment they compressed three qubits into (e&eb) two, but they say the results can lead to larger numbers of qubits in the future.

(2) To minimize data storage and transmission requirements for (e) traditional computers, computer scientists have devised data compression techniques for (e) the ones and zeros that make up digital data. Think of a data record containing a long string of identical bits—N zeros, for example. Rather than physically allocating (eb) the value "zero" to N distinct bits of memory, it's more economical to store a single zero, along with (e&eb) the instruction to read it N times. Both convey the same information, but the "compressed" statement takes up (e) less space.

(3) Quantum information is carried in (eb) the form of "qubits"—superpositions of (e) quantum states that can be (=) both one and zero in different proportions at the same time.

(4) Reading—that is, measuring—a qubit converts it into (e&eb) a classical bit with a definite one or zero value, no longer in a combination state.

(5) Reading a set of N identical qubits that are (=) equal parts one and zero, for example, would produce (eb) roughly N/2 ones and N/2 zeros.

(6) But taking a single such qubit and reading it N times would produce (eb) the same answer over and over, because (e) after the first reading, the qubit becomes (=) classical.

(7) In other words, the N-qubit state contains (e) more information than the classical reading procedure can convey, so the classical compression method will not work.

(8) However, a mathematical argument shows that the information contained in a set of N identical qubits can in principle be carried by (e&eb) many fewer qubits; the number is (=) approximately the logarithm of N [1]. Lee Rozema of the University of Toronto and his colleagues have now devised a way to put this method of qubit compression into practice. In the simplest case, the compression method puts the information contained in three identical qubits into (e&eb) two qubits that are not identical.

NOTATION

Module One

If quantum computers are to become practical devices, they need to handle large amounts of data.

Now researchers demonstrate the compression of quantum data by (e) reducing the number of quantum bits (qubits) needed to (e) store a given amount of information.

first experiment they compressed three qubits into (e&eb) two, but they say the results can lead to larger numbers of qubits in the future.

G_{13} : Category one of reducing the number of quantum bits (qubits) needed to (e) store a given amount of information

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of compression of quantum data

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

To minimize data storage and transmission requirements for (e) traditional computers, computer scientists have devised data compression techniques for (e) the ones and zeros that make up digital data.

Think of a data record containing a long string of identical bits—N zeros, for example.

Rather than physically allocating the value "zero" to N distinct bits of memory, it's more economical to store a single zero, along with (e&eb) the instruction to read it N times.

Both convey the same information, but the "compressed" statement takes up (e) less space

G_{16} : Category one of physically allocating the value "zero" to N distinct bits of memory; store a single zero, along with the instruction to read it N times.

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of store a single zero, along with the instruction to read it N times. ;physically allocating the value "zero" to N distinct bits of memory

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Quantum information is carried in the form of "qubits"—superpositions of quantum states that can be (=) both one and zero in different proportions at the same time

G_{20} : Category one of Quantum information is carried in the form of "qubits"

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of superpositions of quantum states that can be (=) both one and zero in different proportions at the same time

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

superpositions of quantum states that can be (=) both one and zero in different proportions at the same time

G_{24} : Category one of superpositions of quantum states

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of both one and zero in different proportions at the same time

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Reading—that is, measuring—a qubit converts it into (e&eb) a classical bit with a definite one or zero value, no longer in a combination state

G_{28} : Category one of Reading—that is, measuring—a qubit; classical bit with a definite one or zero value, no longer in a combination state

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of classical bit with a definite one or zero value, no longer in a combination state; Reading—that is, measuring—a qubit

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Reading a set of N identical qubits that are equal parts one and zero, for example, would produce (eb) roughly N/2 ones and N/2 zeros

G_{32} : Category one of Reading a set of N identical qubits that are equal parts one and zero

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of roughly N/2 ones and N/2 zeros

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

But taking a single such qubit and reading it N times would produce the same answer over and over, because (e) after the first reading, the qubit becomes classical

G_{36} : Category one of qubit that becomes classical after the first reading

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of single such qubit and reading it N times would produce the same answer over and over

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In other words, the N-qubit state contains (e) more information than the classical reading procedure can convey, so the classical compression method will not work

G_{40} : Category one of more information than the classical reading procedure can convey, so the classical compression method will not work

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of N-qubit state

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

However, a mathematical argument shows that the information contained in a set of N identical qubits can in principle be carried by (e&eb) many fewer qubits; the number is (=) approximately the logarithm of N [1].

Lee Rozema of the University of Toronto and his colleagues have now devised a way to put this method of qubit compression into practice. In the simplest case, the compression method puts the information contained in three identical qubits into (e&eb) two qubits that are not identical.

G_{44} : Category one of information contained in a set of N identical qubits; information carried by logarithm of N [1].

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of information contained in a set of N identical qubits; information carried by logarithm of N [1]

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}),t) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}), t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}), t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}), t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}), t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \\ +(a''_{20})^{(3,3,)}(T_{21},t) & +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) \\ +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) \\ +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \\ +(a''_{21})^{(3,3,)}(T_{21},t) & +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) \\ +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) \\ +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \\ +(a''_{22})^{(3,3,)}(T_{21},t) & +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) \\ +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) \\ +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{15} \quad (57)$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) \\ -(b''_{20})^{(3,3,)}(G_{23},t) & -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) \\ -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) \\ -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) \\ -(b''_{21})^{(3,3,)}(G_{23},t) & -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) \\ -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) \\ -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for

category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & \boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & \boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$ *are fourth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$ *are fifth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$ *are sixth detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients*

$for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients
$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients
$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients
$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients
$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition

coefficients for category 1, 2, and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{37}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$ *96*

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(III) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(JJJ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(KKK) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(LLL)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(MMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(NNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(k) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{36})^{(7)}|G_{37}^{(1)} - G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}|G_{36}^{(1)} - G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}(T_{37}^{(1)}, s_{(36)})|G_{36}^{(1)} - G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}|(a_{36}'')^{(7)}(T_{37}^{(1)}, s_{(36)}) - (a_{36}'')^{(7)}(T_{37}^{(2)}, s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$
Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\Big[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\big[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\big] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(k) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586

ASYMPTOTIC STABILITY ANALYSIS A

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION NINE

Saving Space With Quantum Information

INTRODUCTION—VARIABLES USED

Focus: Saving Space with Quantum Information October 17, 2014• Physics 7, 106 A technique that

packs the full content of three bits of quantum information into two may be a space saver for quantum computers

(1) Physically, compressing the qubits means making them interact to generate (eb) two new qubits with the properties prescribed by (e) the mathematical recipe.
(2) Figuring out how to do this was the hard part of the problem, says Rozema. As in any manipulation of quantum information, the challenge was to perform actions on (e&eb) the qubits without destroying (e) their quantum nature.
(3) Rozema and his colleagues used two independent properties of a photon to encode qubits, so in principle each photon could hold (e) two qubits.
(4) The two properties were polarization (horizontal or vertical orientation of the photon's electric field) and position in an interferometer (traveling along one path or the other through the experimental setup). The team began by creating three identical qubits with two photons, carried by (e) the polarization and path of one photon and by (e) the polarization of a second.
(5) They then compressed the three qubits into two by (e) passing both photons through (e&eb) a complex optical system that caused (ebb) the photons to interact.
(6) At the end of the process, one photon carried two nonidentical qubits containing (e) the information of the original three identical qubits.
(7) To read the compressed data, the researchers needed a classical bit that they obtained by (e) making a certain measurement on the second photon.
(8) The result of this measurement determined the procedure (measurements) that could extract the original three qubits from (e) the two compressed qubits.
(9) The extra classical bit from the second photon is akin to (e) the instruction, in classical compression, that says how many times to duplicate the single stored bit.
(10) Mark Wilde of Louisiana State University in Baton Rouge calls the work "a clever implementation of the quantum compression algorithm" and says it could be useful for "shuffling around quantum data in future quantum computation and communication devices." But scaling the process up to larger numbers of qubits will likely be hard, he says. Rozema agrees but says that the new work is a proof-of-principle demonstration, and besides, scaling up remains a challenge for all quantum **information operations.**

NOTATION

Module One

Physically, compressing the qubits means making them interact to generate (eb) two new qubits with the properties prescribed by (e) the mathematical recipe

G_{13} : Category one of compressing the qubits means making them interact

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of two new qubits with the properties prescribed by the mathematical recipe

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Figuring out how to do this was the hard part of the problem, says Rozema.

As in any manipulation of quantum information, the challenge was to perform actions on (e&eb) the qubits without destroying (e) their quantum nature

G_{16} : Category one of actions; qubits without destroying (e) their quantum nature

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of qubits without destroying (e) their quantum nature ;actions

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Performance of actions on the qubits without destroying their quantum nature

G_{20} : Category one of destroying their quantum nature

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Performance of actions on the qubits

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The two properties were polarization (horizontal or vertical orientation of the photon's electric field) and position in an interferometer (traveling along one path or the other through the experimental setup).

The team began by creating three identical qubits with two photons, carried by (e) the polarization and path of one photon and by (e) the polarization of a second

G_{24} : Category one of polarization and path of one photon and by the polarization of a second

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of three identical qubits with two photons

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They then compressed the three qubits into two by (e) passing both photons through (e&eb) a complex

optical system that caused (ebb) the photons to interact

G_{28} : Category one of photons through a complex optical system that caused the photons to interact

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of compression of three qubits into two

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

At the end of the process, one photon carried two nonidentical qubits containing (e) the information of the original three identical qubits

G_{32} : Category one of information of the original three identical qubits

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of one photon carried two nonidentical qubits

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

classical bit needed to read the compressed data was obtained by (e) making a certain measurement on the second photon

G_{36} : Category one of making a certain measurement on the second photon

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of classical bit needed to read the compressed data

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

result of this measurement determined the procedure (measurements) that could extract the original three qubits from (e) the two compressed qubits

G_{40} : Category one of two compressed qubits

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of result of this measurement determined the procedure (measurements) that could extract the original three qubits

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

extra classical bit from the second photon is akin to (e) the instruction, in classical compression, that says how many times to duplicate the single stored bit

G_{44} : Category one of extra classical bit from the second photon

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of instruction, in classical compression, that says how many times to duplicate the single stored bit

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{ccc}(a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) \quad +(a_{20}'')^{(3,3,3)}(T_{21},t)\\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{ccc}(a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) \quad +(a_{21}'')^{(3,3,3)}(T_{21},t)\\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{ccc}(a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) \quad +(a_{22}'')^{(3,3,3)}(T_{21},t)\\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a_{44}'')^{(9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{array}{ccc}(b_{16}')^{(2)} & -(b_{16}'')^{(2)}(G_{19},t) & -(b_{13}'')^{(1,1,)}(G,t) \quad -(b_{20}'')^{(3,3,3,)}(G_{23},t)\\ -(b_{24}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)\\ -(b_{36}'')^{(7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9)}(G_{47},t)\end{array}\right]T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition*

coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}+(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t)\\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t)\\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{26} \quad (75)$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\,+(a''_{32})^{(6)}(T_{33},t)\;+(a''_{28})^{(5,5,5)}(T_{29},t)\;+(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\,+(a''_{33})^{(6)}(T_{33},t)\;+(a''_{29})^{(5,5,5)}(T_{29},t)\;+(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\,+(a''_{34})^{(6)}(T_{33},t)\;+(a''_{30})^{(5,5,5)}(T_{29},t)\;+(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)} & -(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)} & -(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)} & -(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(OOO) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(PPP) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(QQQ) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(RRR)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(SSS) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(TTT) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - \ T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(l) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{16})^{(2)}|G_{17}^{(1)}-G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}|G_{16}^{(1)}-G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)|G_{16}^{(1)}-G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\;e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$|(G_{19})^{(1)}-(G_{19})^{(2)}|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\,(G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t})=0$ and $\mathrm{T}_i(\mathrm{t})=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$
Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$ 331

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$ 332

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(l) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 513$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ and $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586
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ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ and $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\big)\Big]$$

$$\big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\big)$$

$$+\big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\big)$$

$$\big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\big)$$

$$\big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\big)$$

$$\big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\big)$$

$$+\big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TEN

Efficient Quantum Circuits

INTRODUCTION—VARIABLES USED

Efficient Quantum Circuits for Schur and Clebsch-Gordan Transforms Dave Bacon, Isaac L. Chuang, and Aram W. Harrow Phys. Rev. Lett. 97, 170502 – Published 27 October 2006 Article has an altmetric score of 13

(1) The Schur basis on n d-dimensional quantum systems is (=) a generalization of the total angular momentum basis that is useful for (e) exploiting symmetry under permutations or collective unitary rotations.
(2) Authors present efficient {size poly [n, d, log(1/ϵ)] for accuracy ϵ} quantum circuits for (e) the Schur transform, which is the change of basis between the computational and (e&eb) the Schur bases.
(3) Circuits provide (erb) explicit efficient methods for solving such diverse problems as estimating the spectrum of a density operator, quantum hypothesis testing, and communicating without a shared reference frame.
(4) Authors thus render tractable a large series of methods for extracting resources from (e) quantum systems and for numerous quantum information protocols.

Efficient compression of quantum information Martin Plesch and Vladimír Bužek Phys. Rev. A 81, 032317 – Published 18 March 2010

(5) Authors propose a scheme for an exact efficient transformation of a tensor product state of many identically prepared qubits into (e&eb) a state of a logarithmically small number of qubits.
(6) Using a quadratic number of elementary quantum gates authors transform N identically prepared qubits into (e&eb) a state, which is nontrivial only on (eb) the first ⌈log2 (N+1) ⌉ qubits.
(7) This procedure might be useful for quantum memories, as only a small portion of the original qubits has to be (=) stored. Another possible application is in communicating a direction encoded in a set of quantum states, as the compressed state provides a high-effective method for such an encoding.

NOTATION

Module One

Schur basis on n d-dimensional quantum systems is (=) a generalization of the total angular momentum basis that is useful for (e) exploiting symmetry under permutations or collective unitary rotations

G_{13} : Category one of Schur basis on n d-dimensional quantum systems

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of generalization of the total angular momentum basis that is useful for (e) exploiting symmetry under permutations or collective unitary rotations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Schur basis on n d-dimensional quantum systems is a generalization of the total angular momentum basis that is useful for (e) exploiting symmetry under permutations or collective unitary rotations

G_{16} : Category one of Schur basis on n d-dimensional quantum systems is a generalization of the total angular momentum basis

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of exploiting symmetry under permutations or collective unitary rotations

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Authors present efficient {size poly [n, d, log(1/ϵ)] for accuracy ϵ} quantum circuits for (e) the Schur transform, which is the change of basis between the computational and (e&eb) the Schur bases

G_{20} : Category one of Schur transform, which is the change of basis between the computational and (e&eb) the Schur bases

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of efficient {size poly [n, d, log(1/ϵ)] for accuracy ϵ} quantum circuits

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Authors present efficient {size poly [n, d, log(1/ϵ)] for accuracy ϵ} quantum circuits for the Schur transform, which is the change of basis between the computational and (e&eb) the Schur bases

G_{24} : Category one of efficient {size poly [n, d, log(1/ϵ)] for accuracy ϵ} quantum circuits for the Schur transform, which is the change of basis between the computational; Schur bases

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Schur bases ;efficient {size poly [n, d, log(1/ϵ)] for accuracy ϵ} quantum circuits for the Schur transform, which is the change of basis between the computational

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Circuits provide (erb) explicit efficient methods for solving such diverse problems as estimating the spectrum of a density operator, quantum hypothesis testing, and communicating without a shared reference frame.

G_{28} : Category one of quantum circuits

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of estimating the spectrum of a density operator, quantum hypothesis testing, and communicating without a shared reference frame

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Authors propose a scheme for an exact efficient transformation of a tensor product state of many identically prepared qubits into (e&eb) a state of a logarithmically small number of qubits

G_{32} : Category one of exact efficient transformation of a tensor product state of many identically prepared qubits; state of a logarithmically small number of qubits

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of state of a logarithmically small number of qubits ;exact efficient transformation of a tensor product state of many identically prepared qubits

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Using a quadratic number of elementary quantum gates authors transform N identically prepared qubits into (e&eb) a state, which is nontrivial only on (eb) the first ⌈log2 (N+1) ⌉ qubits

G_{36} : Category one of quadratic number of elementary quantum gates

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of N identically prepared qubits into (e&eb) a state, which is nontrivial only on (eb) the first ⌈log2 (N+1) ⌉ qubits

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Using a quadratic number of elementary quantum gates authors transform N identically prepared qubits into (e&eb) a state, which is nontrivial only on (eb) the first ⌈log2 (N+1) ⌉ qubits

G_{40} : Category one of N identically prepared qubits; state, which is nontrivial only on the first ⌈log2 (N+1) ⌉ qubits

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of state, which is nontrivial only on the first ⌈log2 (N+1) ⌉ qubits; N identically prepared qubits

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

This procedure might be useful for quantum memories, as only a small portion of the original qubits has to be (=) stored.

Another possible application is in communicating a direction encoded in a set of quantum states, as the compressed state provides a high-effective method for such an encoding

G_{44} : Category one of small portion of the original qubits has to be stored

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of quantum memories

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients
Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth

augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a_{28}')^{(5)} & +(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad 79$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a_{29}')^{(5)} & +(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{29} \quad 80$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a_{30}')^{(5)} & +(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{30} \quad 81$$

$Where$ $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ $are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ $are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \left[\begin{array}{ccc} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\big((G_{19}), t\big) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\ ,\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)},\ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(UUU) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(VVV) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)},\ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le\ (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(WWW) $\lim_{T_2\to\infty}\ (a_i'')^{(7)}\,(T_{37}, t) = (p_i)^{(7)}$

(XXX)

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}), t\big) =\ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)},\ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(YYY) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(ZZZ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\widehat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\tilde{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\tilde{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le (\hat{P}_{13})^{(1)}\,, T_i^0 \le (\hat{Q}_{13})^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le (\hat{P}_{16})^{(2)}\,, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(m) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0 \quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$

$G_{20}^{(2)} \left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_{+}$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (239)

it results

$$G_i(t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0 e^{\left(-(b_i')^{(5)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\,(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}\ , if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}\ , if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}}G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}}G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)},(\nu_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},\nu^{(7)},u^{(7)}$: 361

By $(\nu_1)^{(7)}>0$, $(\nu_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0$, $(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{\nu}_1)^{(7)},,(\bar{\nu}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)}>0$, $(\bar{\nu}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0$, $(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)}, (v_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, v^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)}, , (\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, $\boldsymbol{if}\ (v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, $\boldsymbol{if}\ (v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\leq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\leq\nu^{(1)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\leq(\bar{\nu}_1)^{(1)}\leq\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\leq\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\leq\nu^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(m) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\Big]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\Big)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION ELEVEN

Quantum Data Compression, Quantum Information Generation, And The Density-Matrix Renormalization-Group Method

INTRODUCTION—VARIABLES USED

Limits for compression of quantum information carried by ensembles of mixed states Michał Horodecki Phys. Rev. A 57, 3364 – Published 1 May 1998

(1) **Michał Horodecki** considers the problem of compression of the quantum information carried by ensemble of mixed states. **Michał Horodecki** proves that for arbitrary coding schemes the least number of quantum bits (qubits) per message needed to (e) convey the signal states asymptotically faithfully is bounded from (e) below by the Holevo function S (ϱ) −∑ipiS (ϱi).

(2) He also shows that a compression protocol can be composed with (e&eb) another one, provided (e) that the latter offers perfect transmission.

(3) Such a compound protocol is applied to (e&eb) the case of binary source.

(4) It is conjectured to reach the obtained bound. Finally, it is pointed that in the case of mixed signal states there could be a difference between the maximal compression rates at the coding schemes that are "blind" to (e) the signal and (e&eb) the ones that assume (eb) the knowledge about the identities of the signal states.

Quantum data compression, quantum information generation, and the density-matrix renormalization-group method Ö. Legeza and J. Sólyom Phys. Rev. B 70, 205118 – Published 15 November 2004

(5) **Ö. Legeza and J. Sólyom** have studied quantum data compression for finite quantum systems where (e) the site density matrices are not independent, i.e., the density matrix cannot be given as direct product of (e) site density matrices and the von Neumann entropy is not (e) equal to the sum of site entropies.

(6) **(rrp)** Using the density-matrix renormalization-group (DMRG) method for the one-dimensional Hubbard model, authors have shown that a simple relationship exists between the entropy of the left or right block and dimension of the Hilbert space of that block as well as of (e&eb) the superblock for any fixed accuracy.

(7) The information loss during the RG procedure has been investigated and a more rigorous control of the relative error has been proposed based on (e) Kholevo's theory. Results are also supported by the quantum chemistry version of DMRG applied to (e&eb) various molecules with system lengths up to 60 lattice sites.

(8) A sum rule that relates site entropies and (e&eb) the total information generated by (e) the renormalization procedure has also been given, which serves as an alternative test of convergence of the DMRG method.

NOTATION

Module One

Michał Horodecki considers the problem of compression of the quantum information carried by ensemble of mixed states.

Michał Horodecki proves that for arbitrary coding schemes the least number of quantum bits (qubits) per message needed to convey the signal states asymptotically faithfully is bounded from (e) below by the Holevo function S (ϱ) −∑ipiS (ϱi)

G_{13} : Category one of compression of the quantum information; **ensemble** of mixed states

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of ensemble of mixed states; compression of the quantum information

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Michał Horodecki proves that for arbitrary coding schemes the least number of quantum bits (qubits) per message needed to convey the signal states asymptotically faithfully is bounded from (e) below by the Holevo function S (ϱ) −∑ipiS (ϱi)

We give a prediction model thereof.

G_{16} : Category one of below by the Holevo function S (ϱ) −∑ipiS (ϱi)

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of arbitrary coding schemes the least number of quantum bits (qubits) per message needed to convey the signal states asymptotically faithfully is bounded

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

He also shows that a compression protocol can be composed with (e&eb) another one, provided that the latter offers perfect transmission

G_{20} : Category one of compression protocol; another one, provided that the latter offers perfect transmission

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of compression protocols provided that the latter offers perfect transmission ;compression protocol

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Such a compound protocol is applied to (e&eb) the case of binary source

G_{24} : Category one of compound protocol; binary source

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of binary source ;compound protocol

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

It is conjectured to reach the obtained bound.

Finally, it is pointed that in the case of mixed signal states there could be a difference between the maximal compression rates at the coding schemes that are "blind" to (e) the signal and (e&eb) the ones that assume (eb) the knowledge about the identities of the signal states

G_{28} : Category one of mixed signal states

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of difference between the maximal compression rates at the coding schemes that are "blind" to (e) the signal and (e&eb) the ones that assume (eb) the knowledge about the identities of the signal states

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Finally, it is pointed that in the case of mixed signal states there could be a difference between the maximal compression rates at the coding schemes that are "blind" to (e) the signal and (e&eb) the ones that assume (eb) the knowledge about the identities of the signal states

G_{32} : Category one of signal and the ones that assume) the knowledge about the identities of the signal states

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of mixed signal states there could be a difference between the maximal compression rates at the coding schemes that are "blind"

T_{33} : Category two of

T_{34} : Category three of

Module seven

Ö. Legeza and J. Sólyom have studied quantum data compression for finite quantum systems where (e) the site density matrices are not independent, i.e., the density matrix cannot be given as direct product of (e) site density matrices and the von Neumann entropy is not (e) equal to the sum of site entropies

G_{36} : Category one of site density matrices are not independent, i.e., the density matrix cannot be given as direct product of site density matrices and the von Neumann entropy is not (e) equal to the sum of site entropies

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of quantum data compression for finite quantum systems

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

von Neumann entropy is not (e) equal to the sum of site entropies

G_{40} : Category one of von Neumann entropy; sum of site entropies

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of sum of site entropies ;von Neumann entropy

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Using the density-matrix renormalization-group (DMRG) method for the one-dimensional Hubbard model, authors have shown that a simple relationship exists between the entropy of the left or right block and dimension of the Hilbert space of that block as well as of (e&eb) the superblock for any fixed accuracy

G_{44} : Category one of entropy of the left or right block and dimension of the Hilbert space of that block; superblock for any fixed accuracy

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of superblock for any fixed accuracy ;entropy of the left or right block and dimension of the Hilbert space of that block

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWELVE

Quantum Source Compression with a Quantum Helper

INTRODUCTION—VARIABLES USED

Fully Quantum Source Compression with a Quantum Helper Min-Hsiu Hsieh, Shun Watanabe (Submitted on 20 Apr 2015)

(1) **Min-Hsiu Hsieh, Shun Watanabe** study source compression with a helper in (eb) the fully quantum regime, extending earlier result on classical source compression with a quantum helper [arXiv: 1501.04366, 2015].

(2) Authors characterise the quantum resources involved in this problem, and derive (eb) a single-letter expression of the achievable rate region when entanglement assistance is available.

(3) The direct coding proof is based on a combination of two fundamental protocols, namely the quantum state merging protocol and (e&eb) the quantum reverse Shannon theorem (QRST).

(4) This result demonstrates an unexpected connection between distributed source compressions with (e&eb) the QRST protocol, a quantum protocol that consumes (e) noiseless resources to simulate a noisy quantum channel. Cite as:arXiv: 1504.05227 [quant-ph] (or arXiv: 1504.05227v1 [quant-ph] for this version)

NOTATION

Module One

The information loss during the RG procedure has been investigated and a more rigorous control of the relative error has been proposed based on (e) Kholevo's theory.

Results are also supported by the quantum chemistry version of DMRG applied to (e&eb) various molecules with system lengths up to 60 lattice sites

G_{13} : Category one of Kholevo's theory

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of RG procedure investigated a more rigorous control of the relative error

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

A.S. Holevo made substantial contribution in mathematical foundations of quantum theory, quantum statistics and quantum information theory. In 1973 he obtained an upper bound for (e) amount of classical information which can be extracted from (e) ensemble of quantum states by quantum measurements (**this result is known as Holevo's theorem).** A.S. Holevo developed the mathematical theory of quantum communication channels, the noncommutative theory of statistical decisions; he proved coding theorems in quantum information theory and revealed the structure of quantum Markov semigroups and measurement processes. A.S. Holevo is the author of about a hundred and seventy published works, including five

monographs.[3]

Bounds for the Quantity of Information Transmitted by a Quantum Communication Channel
A. S. Holevo Probl. Peredachi Inf., 1973, Volume 9, Issue 3, Pages 3–11 (Mi ppi903)

Certain bounds are derived for the quantity of information transmitted by a quantum channel.

It is proved that if at least two of the set of density operators ρ0,…,ρn do not commute, then (eb) J(π)<H(∑απαρα)−∑απαH(ρα), where J(π) is the upper bound of the quantity of information with respect to all generalized measurements at the channel output for a fixed distribution π=(π0,…,πn) at the input. A sharper upper bound for J (π) is explicitly stated.

Model gives values for various parameters and concomitant results

G_{16} : Category one of two of the set of density operators ρ0,…,ρn do not commute

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of J(π)<H(∑απαρα)−∑απαH(ρα), where J(π) is the upper bound of the quantity of information with respect to all generalized measurements at the channel output for a fixed distribution π=(π0,…,πn) at the input. A sharper upper bound for J (π) is explicitly stated

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

A sum rule that relates site entropies and (e&eb) the total information generated by (e) the renormalization procedure has also been given, which serves as an alternative test of convergence of the DMRG method

G_{20} : Category one of site entropies; total information generated by (e) the renormalization procedure has also been given, which serves as an alternative test of convergence of the DMRG method

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of total information generated by (e) the renormalization procedure has also been given, which serves as an alternative test of convergence of the DMRG method ;site entropies

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

total information generated by (e) the renormalization procedure

G_{24} : Category one of renormalization procedure

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of total information generated

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Min-Hsiu Hsieh, Shun Watanabe study source compression with a helper in (eb) the fully quantum regime, extending earlier result on classical source compression with a quantum helper [arXiv: 1501.04366, 2015]

G_{28} : Category one of source compression with a helper

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of fully quantum regime

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Authors characterise the quantum resources involved in this problem, and derive (eb) a single-letter expression of the achievable rate region when entanglement assistance is available.

G_{32} : Category one of entanglement assistance is available

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of single-letter expression of the achievable rate region

T_{33} : Category two of

T_{34} : Category three of

Module seven

The direct coding proof is based on a combination of two fundamental protocols, namely the quantum state merging protocol and (e&eb) the quantum reverse Shannon theorem (QRST)

G_{36} : Category one of direct coding proof

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of quantum state merging protocol and (e&eb) the quantum reverse Shannon theorem (QRST)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

quantum state merging protocol and (e&eb) the quantum reverse Shannon theorem (QRST)

G_{40} : Category one of quantum state merging protocol; quantum reverse Shannon theorem (QRST)

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of quantum reverse Shannon theorem (QRST);quantum state merging protocol

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

This result demonstrates an unexpected connection between distributed source compressions with (e&eb) the QRST protocol, a quantum protocol that consumes (e) noiseless resources to simulate a noisy quantum channel.

Cite as:arXiv: 1504.05227 [quant-ph] (or arXiv: 1504.05227v1 [quant-ph] for this version)

G_{44} : Category one of distributed source compressions; QRST protocol, a quantum protocol that consumes (e) noiseless resources to simulate a noisy quantum channel

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of QRST protocol, a quantum protocol that consumes (e) noiseless resources to simulate a noisy quantum channel; distributed source compressions

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}),t)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}),t) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}), t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}), t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}), t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for

category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3
$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3
$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3
$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3
$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3
$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3
$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$ *are fourth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$ *are fifth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$ *are sixth detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients*

$for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\ \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\ \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\ \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition

coefficients for category 1, 2, and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition
coefficients for category 1, 2, and 3
$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition
coefficients for category 1, 2, and 3
$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition
coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3
$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$ *96*

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{c} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(AAAA) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(BBBB) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(CCCC) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(DDDD)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(EEEE) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(FFFF) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(n) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

208

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

209

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

214

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, $if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)} \quad 331$$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t} \quad 332$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)}$,, $(\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(n) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\big((a_{30}')^{(5)} + (p_{30})^{(5)}\big)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\big((b_{28}')^{(5)} - (r_{28})^{(5)}\big)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(28)(j)}T_{28}^*\mathbb{G}_j\big) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\big((b_{29}')^{(5)} - (r_{29})^{(5)}\big)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\big(s_{(29)(j)}T_{29}^*\mathbb{G}_j\big) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\big((b_{30}')^{(5)} - (r_{30})^{(5)}\big)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(30)(j)}T_{30}^*\mathbb{G}_j\big) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586\text{B})$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586\text{C})$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586\text{D})$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586\text{E})$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586\text{F})$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586\text{G})$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTEEN

Heralded Qubit Transfer

INTRODUCTION—VARIABLES USED

Synopsis: Floating Gates January 26, 2012 Semiconductor quantum dots connected by floating

metallic gates point the way to a scalable quantum computer– **Sami Mitra**

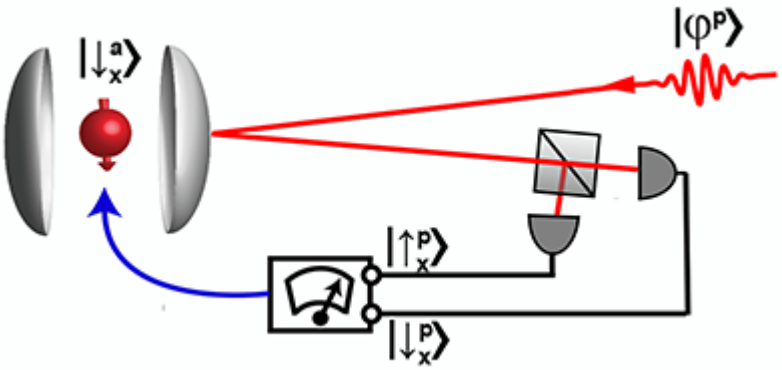
(1) The spin of an electron can act (e&eb) as a qubit—the on/off switch that is the building block of a quantum computer. In 1998, in a paper in Physical Review A, Daniel Loss and David DiVincenzo presented a scenario for implementing spin qubits in (eb) semiconductor quantum dots. Considerable experimental progress has since been made, evidenced in particular by longer decoherence times—the (=) time after which the spin's phase irreversibly changes (e&eb)—that are now around 270 microseconds, an improvement of 7 orders of magnitude.

(2) One barrier to building a scalable quantum computer remains: how to accommodate enough qubits in an arrangement without (e) overrunning it with wires and metallic gates. In a paper in Physical Review X, Luka Trifunovic at the University of Basel, Switzerland, and his coauthors propose a setup that would use a two-dimensional array of quantum dots to address this challenge by spacing dots far enough apart to provide (eb) sufficient gaps for wirings and gates, while bridging the distance with a mechanism to (e) enable long-range interdot tunnel coupling of (e) sufficient strength.

(3) Central to the proposal is an ingenious architecture that would connect dots in (eb) a two-dimensional electron gas with (e&eb) floating metallic gates (classical objects), leading to (eb) a robust entanglement (a quantum phenomenon) between spin qubits that are some distance apart. Authors claim that the technology to build this configuration, a seemingly substantive step towards a full-scale quantum computer, already exists. – **Sami Mitra**

Synopsis: Heralded Qubit Transfer June 2, 2015 Katherine Wright

(4) A scheme based on an atom trapped in a cavity faithfully transfers a qubit state between a photon and (e&eb) the atom.



N. Kalb et al., Phys. Rev. Lett. (2015 Optical photons would be ideal carriers to transfer quantum information over (e&eb) large distances.

Researchers envisage a network where information is processed (e&eb) in certain nodes and transferred between them via (e&eb) photons.

(1) However, inherent losses in long-distance networks mean (eb) that the information transfer is subject to (e&eb) probabilistic errors, making it hard to know whether the transfer of (e) a qubit of information has been successful.

(2) Now Gerhard Rempe and colleagues from the Max Planck Institute for Quantum Optics in

Germany have developed a new protocol that solves this problem through a strategy that "heralds" the accurate transfer of quantum information at (eb) a network node. The method developed by the researchers involves transferring a photonic qubit to (e&eb) an atomic qubit trapped inside (eb) an optical cavity.

NOTATION

Module One

The spin of an electron can act (e&eb) as a qubit—the on/off switch that is the building block of a quantum computer.

In 1998, in a paper in Physical Review A, Daniel Loss and David DiVincenzo presented a scenario for implementing spin qubits in (eb) semiconductor quantum dots. Considerable experimental progress has since been made, evidenced in particular by longer decoherence times—the (=) time after which the spin's phase irreversibly changes (e&eb)—that are now around 270 microseconds, an improvement of 7 orders of magnitude

G_{13} : Category one of spin of an electron on/off switch; quantum computer

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of quantum computer; spin of an electron on/off switch

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

In 1998, in a paper in Physical Review A, Daniel Loss and David DiVincenzo presented a scenario for implementing spin qubits in (eb) semiconductor quantum dots. Considerable experimental progress has since been made, evidenced in particular by longer decoherence times—the (=) time after which the spin's phase irreversibly changes (e&eb)—that are now around 270 microseconds, an improvement of 7 orders of magnitude

G_{16} : Category one of implementing spin qubits; semiconductor quantum dots

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of semiconductor quantum dots; implementing spin qubits

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

One barrier to building a scalable quantum computer remains: how to accommodate enough qubits in an arrangement without (e) overrunning it with wires and metallic gates

In a paper in Physical Review X, Luka Trifunovic at the University of Basel, Switzerland, and his coauthors propose a setup that would use a two-dimensional array of quantum dots to address this challenge by spacing

dots far enough apart to provide (eb) sufficient gaps for wirings and gates, while bridging the distance with a mechanism to (e) enable long-range interdot tunnel coupling of (e) sufficient strength.

G_{20} : Category one of overrunning it with wires and metallic gates

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of accommodate enough qubits in an arrangement

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In a paper in Physical Review X, Luka Trifunovic at the University of Basel, Switzerland, and his coauthors propose a setup that would use a two-dimensional array of quantum dots to address this challenge by spacing dots far enough apart to provide sufficient gaps for wirings and gates, while bridging the distance with a mechanism to (e) enable long-range interdot tunnel coupling of sufficient strength.

G_{24} : Category one of spacing dots far enough apart to provide) sufficient gaps for wirings and gates, while bridging the distance with a mechanism

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of long-range interdot tunnel coupling of sufficient strength

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Central to the proposal is an ingenious architecture that would connect dots in a two-dimensional electron gas with (e&eb) floating metallic gates (classical objects), leading to (eb) a robust entanglement (a quantum phenomenon) between spin qubits that are some distance apart.

Authors claim that the technology to build this configuration, a seemingly substantive step towards a full-scale quantum computer, already exists. – **Sami Mitra**

G_{28} : Category one of architecture that would connect dots in a two-dimensional electron gas; floating metallic gates (classical objects), leading to (eb) a robust entanglement (a quantum phenomenon) between spin qubits that are some distance apart

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of floating metallic gates (classical objects), leading to (eb) a robust entanglement (a quantum phenomenon) between spin qubits that are some distance apart; architecture that would connect

dots in a two-dimensional electron gas

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Researchers envisage a network where information is processed in certain nodes and transferred between them via (e&eb) photons

G_{32} : Category one of network where information is processed in certain nodes and transferred between them; photons

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of photons; network where information is processed in certain nodes and transferred between them

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

scheme based on an atom trapped in a cavity faithfully transfers a qubit state between a photon and (e&eb) the atom

G_{36} : Category one of scheme based on an atom trapped in a cavity transfers a qubit state between a photon; atom

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of atom; scheme based on an atom trapped in a cavity transfers a qubit state between a photon

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

However, inherent losses in long-distance networks mean (eb) that the information transfer is subject to (e&eb) probabilistic errors, making it hard to know whether the transfer of (e) a qubit of information has been successful

G_{40} : Category one of inherent losses in long-distance networks

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of information transfer is subject to (e&eb) probabilistic errors, making it hard to know whether the transfer of (e) a qubit of information has been successful

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

inherent losses in long-distance networks mean that the information transfer is subject to (e&eb) probabilistic errors, making it hard to know whether the transfer of a qubit of information has been successful

G_{44} : Category one of inherent losses in long-distance networks mean that the information transfer; probabilistic errors, making it hard to know whether the transfer of a qubit of information has been successful

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of probabilistic errors, making it hard to know whether the transfer of a qubit of information has been successful; inherent losses in long-distance networks mean that the information transfer

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category $1, 2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1,2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1,2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1,2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{array}{c}(a'_{32})^{(6)}\,+(a''_{32})^{(6)}(T_{33},t)\;+(a''_{28})^{(5,5,5)}(T_{29},t)\;+(a''_{24})^{(4,4,4,)}(T_{25},t)\\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)\end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c}(a'_{33})^{(6)}\,+(a''_{33})^{(6)}(T_{33},t)\;+(a''_{29})^{(5,5,5)}(T_{29},t)\;+(a''_{25})^{(4,4,4,)}(T_{25},t)\\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)\end{array}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c}(a'_{34})^{(6)}\,+(a''_{34})^{(6)}(T_{33},t)\;+(a''_{30})^{(5,5,5)}(T_{29},t)\;+(a''_{26})^{(4,4,4,)}(T_{25},t)\\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)\;+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)\;+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)\end{array}\right]G_{34} \qquad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{c} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(GGGG) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(HHHH)The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(IIII) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(JJJJ)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(KKKK) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(LLLL) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}$, $(r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0,t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(o) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right)=(b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}'')^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}((G_{47})(t),t)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}((G_{47})(t),t) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}((G_{47})(t),t) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$$

334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

335

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$$

336

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \le \frac{d\nu^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(o) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{*^*}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FOURTEEN

Roles Of The Atom And Photon Could Be Reversed

INTRODUCTION—VARIABLES USED

Synopsis: Heralded Qubit Transfer June 2, 2015 Katherine Wright

(1) The photon-atom quantum information transfer is initiated via (e&eb) a quantum "logic-gate" operation, performed by (e) reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a definite state.

(2) This state can be one of two possibilities, depending on the photonic state detected: Either the atom is in (eb) the initial qubit state encoded in the photon and the transfer process is complete, or the atom is in (eb) a rotated version of this state.

(3) The authors were able to show that the roles of the atom and (e&eb) photon could be reversed.

(4) Their method could thus be used as a quantum memory that stores (photon-to-atom state transfer) and recreates (atom-to-photon state transfer) (e&eb) a single-photon polarization qubit.

NOTATION

Module One

Now Gerhard Rempe and colleagues from the Max Planck Institute for Quantum Optics in Germany have developed a new protocol that solves this problem through a strategy that "heralds" the accurate transfer of quantum information at (eb) a network node.

The method developed by the researchers involves transferring a photonic qubit to (e&eb) an atomic qubit trapped inside (eb) an optical cavity.

G_{13} : Category one of transferring a photonic qubit; atomic qubit trapped inside (eb) an optical cavity

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of atomic qubit trapped inside (eb) an optical cavity; transferring a photonic qubit

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

transferring a photonic qubit to an atomic qubit trapped inside (eb) an optical cavity (ies)

G_{16} : Category one of transferring a photonic qubit to an atomic qubit trapped

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of optical cavity

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The photon-atom quantum information transfer is initiated via (e&eb) a quantum "logic-gate" operation, performed by (e) reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a definite state

G_{20} : Category one of photon-atom quantum information transfer is initiated; quantum "logic-gate" operation, performed by (e) reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a definite state

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of quantum "logic-gate" operation, performed by (e) reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a definite state; photon-atom quantum information transfer is initiated

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The photon-atom quantum information transfer is initiated via a quantum "logic-gate" operation, performed by (e) reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a definite state

G_{24} : Category one of reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a

definite state

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of photon-atom quantum information transfer is initiated via a quantum "logic-gate" operation, performed

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The photon-atom quantum information transfer is initiated via a quantum "logic-gate" operation, performed by reflecting the photon from (e) the atom-cavity system, which creates (eb) an entangled atom-photon state.

The detection of the reflected photon then collapses the atom into (eb) a definite state

G_{28} : Category one of atom-cavity system, which creates (eb) an entangled atom-photon state. The detection of the reflected photon then collapses the atom into (eb) a definite state

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of photon-atom quantum information transfer is initiated via a quantum "logic-gate" operation, performed by reflecting the photon

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The photon-atom quantum information transfer is initiated via a quantum "logic-gate" operation, performed by reflecting the photon from the atom-cavity system, which creates (eb) an entangled atom-photon state

G_{32} : Category one of photon-atom quantum information transfer is initiated via a quantum "logic-gate" operation, performed by reflecting the photon from the atom-cavity system

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of entangled atom-photon state

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

This state can be one of two possibilities, depending on the photonic state detected

Either the atom is in (eb) the initial qubit state encoded in the photon and the transfer process is complete,

or the atom is in (eb) a rotated version of this state.

G_{36} : Category one of atom

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of initial qubit state encoded in the photon and the transfer process is complete

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

atom is in (eb) a rotated version of this state

G_{40} : Category one of atom

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of rotated version of this state

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

roles of the atom and (e&eb) photon could be reversed

G_{44} : Category one of roles of the atom; photon could be reversed

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of photon could be reversed; roles of the atom

T_{45} : Category two of SAS

T_{46} : Category three of SAS

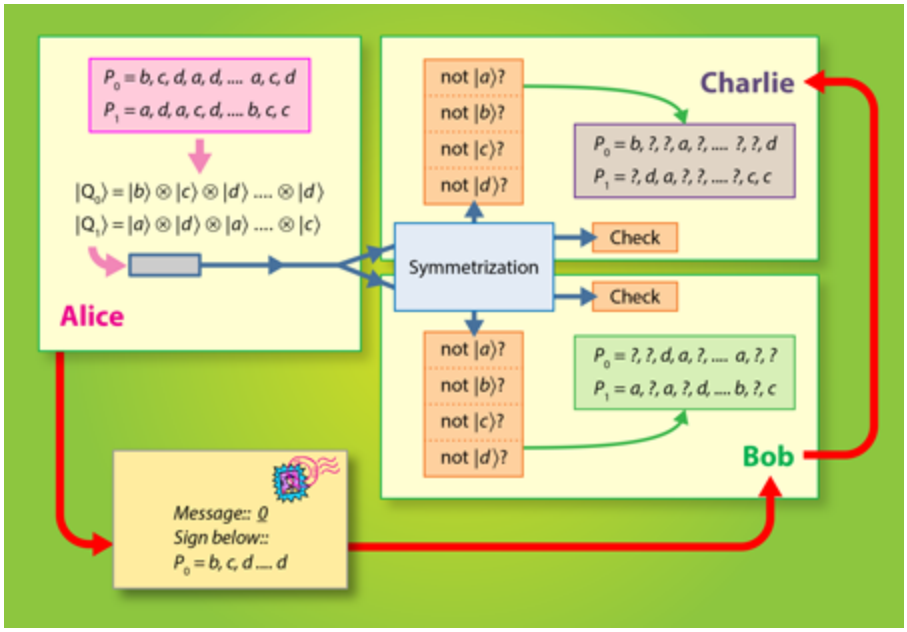
SECTION FIFTEEN

Messages With A Quantum Seal

INTRODUCTION—VARIABLES USED

Viewpoint: Sending Messages with a Quantum Seal Hermann Kampermann, Institute for Theoretical Physics III, Heinrich-Heine-University Düsseldorf, Germany July 21, 2014• Physics 7, 78. For references kindly see original article. References have been deleted attributed to spatial restraints.

(1) With a simple optical experiment, researchers have demonstrated a quantum digital signature scheme for (e) identifying the sender of an electronic message without (e) requiring the use of quantum memories.



APS/Alan Stonebraker

(2) Figure 1: In a quantum digital signature protocol, the sender Alice generates (eb) a private key Pk and a corresponding public key |Qk⟩, which is (=) a sequence of coherent states (laser pulses) drawn from (e) a set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch... Show more

(3) In today's information society, the transfer, processing, and protection of information has become an essential part of our daily lives. In many cases, a recipient must confirm the origin of a message, and this usually relies on (e) a so-called digital signature, which is a secret code sent along with the message that identifies the sender.

(4) To guard against forgery, classical digital signature schemes rely on (e) computationally intractable problems, like factoring extremely large numbers, but there's no proof that these codes couldn't be cracked.

(5) Quantum digital signatures (QDS) [1] could offer unconditional security by using (e) quantum states that cannot be fully characterized by someone hoping to forge a signature.

(6) The price to pay for going quantum is that these quantum states are (=) fragile and thus short-lived.

(7) Previous QDS schemes required using (e) quantum memories, which are currently not robust

enough.

(8) However, another scheme has recently been proposed that directly measures—rather than stores—quantum states in order to obtain (eb) partial information about them [2]. Reporting in Physical Review Letters, Robert Collins of Heriot-Watt University, UK, and his colleagues have successfully implemented a variant of this memory-free QDS protocol in (eb) an optical system [3].

(9) A digital signature is like a written signature in that it should be easily identified but not easily reproduced. In a simple example, Alice sends a plain text message "Meet me at 6 o'clock" to (e&eb) Bob and Charlie, and she signs it with some code, say Xa6OO.

NOTATION

Module One

Synopsis: Heralded Qubit Transfer June 2, 2015 Katherine Wright

Their method could thus be used as a quantum memory that stores (photon-to-atom state transfer) and recreates (atom-to-photon state transfer) (e&eb) a single-photon polarization qubit.

G_{13} : Category one of quantum memory that stores (photon-to-atom state transfer) and recreates (atom-to-photon state transfer); single-photon polarization qubit

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of single-photon polarization qubit ;quantum memory that stores (photon-to-atom state transfer) and recreates (atom-to-photon state transfer)

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

With a simple optical experiment, researchers have demonstrated a quantum digital signature scheme for (e) identifying the sender of an electronic message without (e) requiring the use of quantum memories

G_{16} : Category one of identifying the sender of an electronic message without (e) requiring the use of quantum memories

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantum digital signature scheme

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

quantum digital signature scheme for identifying the sender of an electronic message without (e) requiring the use of quantum memories

G_{20} : Category one of requiring the use of quantum memories

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of quantum digital signature scheme for identifying the sender of an electronic message

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Figure 1:

In a quantum digital signature protocol, the sender Alice generates (eb) a private key Pk and a corresponding public key |Qk⟩, which is (=) a sequence of coherent states (laser pulses) drawn from (e) a set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch

G_{24} : Category one of sender Alice (there could be many)

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of private key Pk and a corresponding public key |Qk⟩, which is (=) a sequence of coherent states (laser pulses) drawn from (e) a set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

In a quantum digital signature protocol, the sender Alice generates a private key Pk and a corresponding public key |Qk⟩, which is (=) a sequence of coherent states (laser pulses) drawn from (e) a set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch

G_{28} : Category one of sender Alice generates a private key Pk and a corresponding public key |Qk⟩

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of sequence of coherent states (laser pulses) drawn from (e) a set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

In a quantum digital signature protocol, the sender Alice generates a private key Pk and a corresponding public key |Qk⟩, which is a sequence of coherent states (laser pulses) drawn from (e) a set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch

G_{32} : Category one of set of four nonorthogonal states: |a⟩, |b⟩, |c⟩, |d⟩. Bob and Ch

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of sender Alice generates a private key Pk and a corresponding public key |Qk⟩, which is a sequence of coherent states (laser pulses) drawn

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

In today's information society, the transfer, processing, and protection of information has become an essential part of our daily lives.

In many cases, a recipient must confirm the origin of a message, and this usually relies on (e) a so-called digital signature, which is a secret code sent along with the message that identifies the sender

G_{36} : Category one of digital signature, which is a secret code sent along with the message that identifies the sender

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of recipient must confirm the origin of a message

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

To guard against forgery, classical digital signature schemes rely on (e) computationally intractable problems, like factoring extremely large numbers, but there's no proof that these codes couldn't be cracked

G_{40} : Category one of computationally intractable problems, like factoring extremely large numbers, but there's no proof that these codes couldn't be cracked

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of classical digital signature schemes to guard against forgery

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Quantum digital signatures (QDS) [1] could offer unconditional security by using (e) quantum states that cannot be fully characterized by someone hoping to forge a signature.

G_{44} : Category one of quantum states that cannot be fully characterized by someone hoping to forge a signature.

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Quantum digital signatures (QDS) [1] could offer unconditional security

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad (1)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad (2)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad (3)$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad (4)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad (5)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad (6)$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad (7)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad (8)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad (9)$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad (10)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad (11)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad (12)$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad (13)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad (14)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad (15)$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad (16)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad (17)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad (18)$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad (19)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (91)$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (92)$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}-(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}-(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}-(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(MMMM) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(NNNN) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(OOOO) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(PPPP)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(QQQQ) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(RRRR) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\widehat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''^{(9)}_{44} \left(T_{45}(s_{(44)}), s_{(44)} \right) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} \left(T_{45}(s_{(44)}), s_{(44)} \right) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} \left(T_{45}(s_{(44)}), s_{(44)} \right) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} \left(G_{47}(s_{(44)}), s_{(44)} \right) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} \left(G_{47}(s_{(44)}), s_{(44)} \right) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} \left(G_{47}(s_{(44)}), s_{(44)} \right) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}}\left(e^{(\tilde{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(p) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}}\left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)}\left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}}\left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)}\left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\ (G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} \left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)} (\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)} (\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} \mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (250)

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $((\widehat{M}_{36})^{(7)})_1, ((\widehat{M}_{36})^{(7)})_2$ *and* $((\widehat{M}_{36})^{(7)})_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)}((\widehat{M}_{36})^{(7)})_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)}((\widehat{M}_{36})^{(7)})_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}((G_{39})(t),t) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + ((\widehat{P}_{40})^{(8)} + G_j^0)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}}G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}}G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\leq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\leq\nu^{(1)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\leq(\bar{\nu}_1)^{(1)}\leq\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\leq\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\leq\nu^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

with $(p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(p) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)}\mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)}+(\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)}\mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)}+(\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 513$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)}\mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)}-(\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)}\mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)}-(\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SIXTEEN

"Private Key"

INTRODUCTION—VARIABLES USED

(1) The price to pay for going quantum is that these quantum states are (=) fragile and thus short-lived. Previous QDS schemes required using (e) quantum memories, which are currently not robust enough.

(2) However, another scheme has recently been proposed that directly measures—rather than stores—quantum states in order to obtain (eb) partial information about them [2]. Reporting in Physical Review Letters, Robert Collins of Heriot-Watt University, UK, and his colleagues have successfully implemented a variant of this memory-free QDS protocol in (eb) an optical system [3].

(3) A digital signature is like a written signature in that it should be easily identified but not easily reproduced. In a simple example, Alice sends a plain text message "Meet me at 6 o'clock" to (e&eb) Bob and Charlie, and she signs it with some code, say Xa6OO.

Viewpoint: Sending Messages with a Quantum Seal Hermann Kampermann, Institute for Theoretical Physics III, Heinrich-Heine-University Düsseldorf, Germany July 21, 2014• Physics 7, 78. For references kindly see original article. References have been deleted attributed to spatial restraints

(1) Only Alice would know the algorithm, or "private key," for generating (eb) this code.

(2) However, she would share (e&eb) a function or protocol that would allow (eb) anyone to validate (eb) that it's her signature (i.e., that the code and message match in some way).

(3) This system has to meet two security criteria. First, it must protect against (e) someone deciphering the private key and then writing (eb) fake messages with a forged Alice signature.

(4) Second, a digital signature scheme has to be secure against repudiation, i.e., all recipients of a message-signature pair must come to (eb) the same conclusion—either that the signature is valid or that it is not valid.

(5) In a rudimentary classical digital signature scheme [4], Alice wants to send a single bit message k, so she assigns (eb) a private key code Pk, for (e) each possible message k=0 or k=1.
(6) She then distributes (e&eb) a "one-way" function f and a "public key" Qk=f (Pk), which all parties can use to (e) validate her signature.
(7) When Alice later sends her message, say "0", she signs it with (e&eb) P0, and Bob and Charlie can verify the message by (e) calculating f (P0) and checking it against (e) the public key Q0.
(8) The principle behind the one-way function is that it is supposed to be (=) impossible to invert f and calculate (eb) the private key: f−1 (Qk) =Pk
(9) However, this computational bottleneck is not assured. Schemes based on the difficulty of factoring large numbers, for example, see Ref. [5], could be cracked by (e) a quantum computer.

NOTATION

Module One

The price to pay for going quantum is that these quantum states are (=) fragile and thus short-lived.

Previous QDS schemes required using (e) quantum memories, which are currently not robust enough

G_{13} : Category one of quantum memories, which are currently not robust enough

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of Previous QDS schemes

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

A digital signature is like a written signature in that it should be easily identified but not easily reproduced. In a simple example, Alice sends a plain text message "Meet me at 6 o'clock" to (e&eb) Bob and Charlie, and she signs it with some code, say Xa6OO

G_{16} : Category one of Alice sends a plain text message "Meet me at 6 o'clock"; Bob and Charlie, and she signs it with some code, say Xa6OO

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Bob and Charlie, and she signs it with some code, say Xa6OO ;Alice sends a plain text message "Meet me at 6 o'clock"

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Only Alice would know the algorithm, or "private key," for generating (eb) this code

G_{20} : Category one of Alice would know the algorithm, or "private key,"

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of this code (there shall be many such messages and many such private keys)

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

However, she would share a function or protocol that would allow (eb) anyone to validate (eb) that it's her signature (i.e., that the code and message match in some way)

G_{24} : Category one of she would share a function or protocol

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of anyone to validate (eb) that it's her signature (i.e., that the code and message match in some way)

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

This system has to meet two security criteria.

First, it must protect against (e) someone deciphering the private key and then writing (eb) fake messages with a forged Alice signature

G_{28} : Category one of someone deciphering the private key and then writing (eb) fake messages with a forged Alice signature

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of security criteria it must protect against

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Second, a digital signature scheme has to be secure against repudiation, i.e., all recipients of a message-

signature pair must come to (eb) the same conclusion—either that the signature is valid or that it is not valid

G_{32} : Category one of digital signature scheme has to be secure against repudiation, i.e., all recipients of a message-signature pair

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of same conclusion—either that the signature is valid or that it is not valid

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Alice in a rudimentary classical digital signature scheme [4], wants to send a single bit message k, so she assigns (eb) a private key code Pk, for (e) each possible message k=0 or k=1

G_{36} : Category one of Alice in a rudimentary classical digital signature scheme [4], wants to send a single bit message k, so she assigns

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of private key code Pk, for (e) each possible message k=0 or k=1

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

She then distributes a "one-way" function f and a "public key" Qk=f (Pk), which all parties can use to (e) validate her signature

G_{40} : Category one of validate her signature

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of She then distributes a "one-way" function f and a "public key" Qk=f (Pk), which all parties

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

When Alice later sends her message, say "0", she signs it with P0, and Bob and Charlie can verify the message by (e) calculating f (P0) and checking it against (e) the public key Q0

G_{44} : Category one of calculating f (P0) and checking it against (e) the public key Q0

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Alice later sends her message, say "0", she signs it with P0, and Bob and Charlie can verify the message

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15}$ 3

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$ 4

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$ 5

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**
Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**
Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\big((G_{35}),t\big)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\big((G_{35}),t\big)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\big((G_{35}),t\big)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\big((G_{39}),t\big)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\big((G_{39}),t\big)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\big((G_{39}),t\big)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for

1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) \quad +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) \quad +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) \quad -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) \quad -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) \quad -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{array}\right] T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation

coefficients for category 1, 2 and 3
$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3
$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3
$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*
$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*
$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*
$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*
$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*
$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*
$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 91$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 92$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}-(b_{44}'')^{(9)}(G_{47},t) & -(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}-(b_{45}'')^{(9)}(G_{47},t) & -(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}-(b_{46}'')^{(9)}(G_{47},t) & -(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \tag{97}$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \tag{98}$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(SSSS) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(TTTT) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(UUUU) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(VVVV)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(WWWW) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(XXXX) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\widehat{M}_{40})^{(8)}t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} \text{ , } T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} \text{ , } T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(q) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)}\left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\ (G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} \left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)} (\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)} (\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} \mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$ 214

$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\ \ \text{for t}>0$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (250)

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\;e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\;(G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)}-(v_0)^{(7)}}{(v_0)^{(7)}-(\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)}+(C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)} \le v^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq v^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$v^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq v^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq v^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$, we obtain 424

$$(\nu_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(q) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi(G_{39}{}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \ , \ \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SEVENTEEN

Unitarity And Intrinsic Randomness

INTRODUCTION—VARIABLES USED

Viewpoint: Sending Messages with a Quantum Seal Hermann Kampermann, Institute for Theoretical Physics III, Heinrich-Heine-University Düsseldorf, Germany July 21, 2014• Physics 7, 78. For references kindly see original article. References have been deleted attributed to spatial restraints

(1) In the original quantum digital signature scheme [1], the one-way function is replaced by (e&eb) a mapping of the private key Pk onto (e&eb) a quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie.

(2) The central concept here is that the public key state |Qk⟩ is composed of (e) quantum states from (e) a set, |a⟩,|b⟩,|c⟩…, that are, in general, not orthogonal.

(3) By the rules of quantum mechanics [6], i.e., unitarity and intrinsic randomness, it is not possible to make (eb) a measurement that identifies with certainty |Qk⟩.

(4) Instead, Bob and Charlie store (e) the public key and wait for Alice to send a message with (e&eb) the private key signature, at which point they can choose (e&eb) specific quantum operations and measurements to perform on (e&eb) the stored |Qk⟩.

(5) These measurements give them enough information to decide (e&eb) whether or not the signature is valid.

(6) A variation of the above scheme was developed on the basis of coherent states in Ref. [7] and experimentally demonstrated in Ref. [8]. However, such a scheme is not practicable, since the wait time before a message arrives may be (=) days or weeks, whereas currently available quantum memories can (=) reliably store quantum states for, at most, tens of minutes (after which decoherence occurs).

(7) Recent work has shown that long-lasting quantum memories are not (e) necessary in QDS [2].

(8) Instead of storing the public key |Qk⟩, receiving parties' measure it directly by (e) unambiguous state discrimination or quantum state elimination measurements, and the results of these measurements (the classical data) is stored until (e) a message is received.

NOTATION

Module One

(1) The principle behind the one-way function is that it is supposed to be (=) impossible to invert f and calculate (eb) the private key: f−1 (Qk) =Pk
(2) However, this computational bottleneck is not assured. Schemes based on the difficulty of factoring large numbers, for example, see Ref. [5], could be cracked by (e) a quantum computer.

The principle behind the one-way function is that it is supposed to be (=) impossible to invert f and calculate (eb) the private key: f−1 (Qk) =Pk

G_{13} : Category one of principle behind the one-way function

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of impossible to invert f and calculate (eb) the private key: f−1 (Qk) =Pk

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

one-way function in the original quantum digital signature scheme [1], is replaced by (e&eb) a mapping of the private key Pk onto (e&eb) a quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie

G_{16} : Category one of one-way function in the original quantum digital signature scheme [1]; mapping of the private key Pk onto (e&eb) a quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of mapping of the private key Pk onto (e&eb) a quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie; one-way function in the original quantum digital signature scheme [1],

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

one-way function in the original quantum digital signature scheme [1], is replaced by a mapping of the private key Pk onto (e&eb) a quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie

G_{20} : Category one of one-way function in the original quantum digital signature scheme [1], is replaced by a mapping of the private key Pk ; quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie ;one-way function in the original quantum digital signature scheme [1], is replaced by a mapping of the private key Pk

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

one-way function in the original quantum digital signature scheme [1], is replaced by a mapping of the private key Pk onto a quantum state |Qk⟩, which forms (eb) the public key that Alice sends to (e&eb) Bob and Charlie

G_{24} : Category one of one-way function in the original quantum digital signature scheme [1], is replaced by a mapping of the private key Pk onto a quantum state |Qk⟩

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of public key that Alice sends to (e&eb) Bob and Charlie

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The central concept here is that the public key state |Qk⟩ is composed of (e) quantum states from a set, |a⟩,|b⟩,|c⟩…, that are, in general, not orthogonal

G_{28} : Category one of quantum states from a set, |a⟩,|b⟩,|c⟩…, that are, in general, not orthogonal

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of central concept here is that the public key state |Qk⟩

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

By the rules of quantum mechanics [6], i.e., unitarity and intrinsic randomness, it is not possible to make (eb) a measurement that identifies with certainty |Qk⟩

G_{32} : Category one of unitarity and intrinsic randomness, it is not possible

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of measurement that identifies with certainty |Qk⟩

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

G_{36} : Category one of

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Recent work has shown that long-lasting quantum memories are not (e) necessary in QDS [2].

G_{40} : Category one of necessity to be in QDS [2].

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of long-lasting quantum memories

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Instead of storing the public key |Qk⟩, receiving parties' measure it directly by (e) unambiguous state discrimination or quantum state elimination measurements, and the results of these measurements (the classical data) is stored until (e) a message is received.

G_{44} : Category one of message is received

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of receiving parties' measure it directly by (e) unambiguous state discrimination or quantum state elimination measurements, and the results of these measurements (the classical data) is stored

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\;\;+(a''_{16})^{(2,2,2)}(T_{17},t)\;\;+(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\;\;+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\;\;+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\;\;+(a''_{40})^{(8,8,8,8)}(T_{41},t)\;\;+(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\;\;+(a''_{17})^{(2,2,2)}(T_{17},t)\;\;+(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\;\;+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\;\;+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\;\;+(a''_{41})^{(8,8,8,8)}(T_{41},t)\;\;+(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\;\;+(a''_{18})^{(2,2,2)}(T_{17},t)\;\;+(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\;\;+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\;\;+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\;\;+(a''_{42})^{(8,8,8,8)}(T_{41},t)\;\;+(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & \boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & \boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & \boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

$Where\ \boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}\ are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
$And\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ \ are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad (93)$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\big((G_{19}), t\big) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(YYYY) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(ZZZZ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(AAAAA) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(BBBBB)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(CCCCC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(DDDDD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\widehat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(r) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0 \quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$

$G_{20}^{(2)} \left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (239)

it results

$$G_i(t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0 e^{\left(-(b_i')^{(5)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 275

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\,((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}}G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}}G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\leq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\leq\nu^{(1)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\leq(\bar{\nu}_1)^{(1)}\leq\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\leq\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\leq\nu^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(r) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION EIGHTEEN

Quantum State Elimination Measurements On The Sequence Of Public Key States

INTRODUCTION—VARIABLES USED

Viewpoint: Sending Messages with a Quantum Seal Hermann Kampermann, Institute for Theoretical Physics III, Heinrich-Heine-University Düsseldorf, Germany July 21, 2014• Physics 7, 78. For references kindly see original article. References have been deleted attributed to spatial restraints

(1) Again, because the public key states are nonorthogonal, (eb) only partial information about the signature key is obtained.

(2) In a proof of principle experiment [3], Collins et al. show that with current linear optics hardware, it is possible to exchange quantum digital signatures without (e) relying on quantum memories.

(3) Here, the private key Pk is a list of L entries, where each entry is randomly chosen from (e) four possibilities: a, b, c, d (see Fig. 1).

(4) This private key is mapped to (e&eb) the public key |Qk⟩, which is (=) a time-ordered sequence of L coherent states: |Qk⟩=|qk1⟩⊗|qk2⟩⊗|qk3⟩…⊗|qkL⟩.

(5) In the optical experiment of Collins et al., each |qi⟩ is (=) a pulse fired from (e) a laser and then phase modulated so that its state (as determined by Pk) is (=) one of four nonorthogonal states, |a⟩, |b⟩, |c⟩, |d⟩.

(6) Alice sends one copy of |Qk⟩ to (e&eb) Bob and another copy to (e&eb) Charlie.

(7) To prevent Alice from repudiating the message, both copies are sent to (e) a symmetrization unit, which consists of (e) a set of beam splitters that nondestructively compare Bob and (e&eb) Charlie's copies.

(8) One output mode of the symmetrization unit comprises (e) a check measurement that outputs (eb) nothing (the vacuum state) when Bob and Charlie receive (e) identical public keys from Alice. Following the symmetrization procedure, both Bob and Charlie perform (e&eb) quantum state elimination measurements on the sequence of public key states.

NOTATION

Module One

Again, because the public key states are nonorthogonal, (eb) only partial information about the signature key is obtained

G_{13} : Category one of public key states are nonorthogonal

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of partial information about the signature key is obtained

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

In a proof of principle experiment [3], Collins et al. show that with current linear optics hardware, it is possible to exchange quantum digital signatures without (e) relying on quantum memories

Here, the private key Pk is a list of L entries, where each entry is randomly chosen from (e) four possibilities: a, b, c, d (see Fig. 1)

G_{16} : Category one of four possibilities: a, b, c, d (see Fig. 1)

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of private key Pk is a list of L entries, where each entry is randomly chosen

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

This private key is mapped to (e&eb) the public key |Qk⟩, which is (=) a time-ordered sequence of L coherent states: |Qk⟩=|qk1⟩⊗|qk2⟩⊗|qk3⟩…⊗|qkL⟩

G_{20} : Category one of private key is mapped; public key |Qk⟩, which is (=) a time-ordered sequence of L coherent states: |Qk⟩=|qk1⟩⊗|qk2⟩⊗|qk3⟩…⊗|qkL⟩

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of public key |Qk⟩, which is (=) a time-ordered sequence of L coherent states: |Qk⟩=|qk1⟩⊗|qk2⟩⊗|qk3⟩…⊗|qkL⟩;private key is mapped

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

private key is mapped to the public key |Qk⟩, which is (=) a time-ordered sequence of L coherent states: |Qk⟩=|qk1⟩⊗|qk2⟩⊗|qk3⟩…⊗|qkL⟩

G_{24} : Category one of private key is mapped to the public key |Qk⟩,

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of time-ordered sequence of L coherent states: |Qk⟩=|qk1⟩⊗|qk2⟩⊗|qk3⟩…⊗|qkL⟩

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

In the optical experiment of Collins et al., each |qi⟩ is (=) a pulse fired from a laser and then phase modulated so that its state (as determined by Pk) is (=) one of four nonorthogonal states, |a⟩, |b⟩, |c⟩, |d⟩

G_{28} : Category one of |qi⟩

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of pulse fired from (e) a laser and then phase modulated so that its state (as determined

by Pk) is (=) one of four nonorthogonal states, |a⟩, |b⟩, |c⟩, |d⟩

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

In the optical experiment of Collins et al., each |qi⟩ is a pulse fired from a laser and then phase modulated so that its state (as determined by Pk) is (=) one of four nonorthogonal states, |a⟩, |b⟩, |c⟩, |d⟩

G_{32} : Category one of each |qi⟩ is a pulse fired from a laser and then phase modulated so that its state (as determined by Pk)

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of one of four nonorthogonal states, |a⟩, |b⟩, |c⟩, |d⟩ (there may be man such quadruples for many experiments)

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Alice sends one copy of |Qk⟩ to (e&eb) Bob and another copy to (e&eb) Charlie

To prevent Alice from repudiating the message, both copies are sent to (e) a symmetrization unit, which consists of (e) a set of beam splitters that nondestructively compare Bob and (e&eb) Charlie's copies

G_{36} : Category one of Alice from repudiating the message, both copies

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of symmetrization unit, which consists of a set of beam splitters that nondestructively compare Bob and Charlie's copies

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

One output mode of the symmetrization unit comprises a check measurement that outputs (eb) nothing (the vacuum state) when Bob and Charlie receive (e) identical public keys from Alice.

Following the symmetrization procedure, both Bob and Charlie perform (e&eb) quantum state elimination

measurements on the sequence of public key states.

G_{40} : Category one of output mode of the symmetrization unit comprises a check measurement

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of nothing (the vacuum state) when Bob and Charlie receive (e) identical public keys from Alice

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

both Bob and Charlie following the symmetrization procedure perform (e&eb) quantum state elimination measurements on the sequence of public key states

G_{44} : Category one of both Bob and Charlie following the symmetrization procedure; quantum state elimination measurements on the sequence of public key states

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of quantum state elimination measurements on the sequence of public key states; both Bob and Charlie following the symmetrization procedure

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category $1, 2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1,2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1,2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1,2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \quad +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \quad +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) \quad +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(EEEEE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(FFFFF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(GGGGG) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(HHHHH)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(IIIII) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(JJJJJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0,t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(s) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t})=0$ and $\mathrm{T}_i(\mathrm{t})=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}<1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 250

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}\,((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436
45,46,27,28) A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(s) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{*^*}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586 B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586 C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586 D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586 E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586 F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586 G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION NINETEEN

Universal Quantum-Mechanical Manipulation

INTRODUCTION—VARIABLES USED

Viewpoint: Sending Messages with a Quantum Seal Hermann Kampermann, Institute for Theoretical Physics III, Heinrich-Heine-University Düsseldorf, Germany July 21, 2014• Physics 7, 78. For references kindly see original article. References have been deleted attributed to spatial restraints

(1) These measurements usually only eliminate (e) one possibility, such as |qi⟩ is not |c⟩.
(2) Still, this partial information can be used (e) to validate a signature.
(3) When Alice later sends a message k with her signature Pk, Bob and Charlie can compare (e&eb) the full key to (e&eb) their partial (uncorrelated) information about the key.
(4) To detect a forgery or repudiation attack, (e) the number L of coherent states for each public key |Qk⟩ must be on (eb) the order of 1013, but the authors have identified some future improvements that could reduce this number.
(5) Also this scheme straightforwardly generalizes to (e&eb with ac=dc=0: Diewart function) any number of verifying parties, but this further increases the technical demand. The level of security of the protocol depends on (=) the number L, the errors and imperfections in the measurements, imperfections in the state preparations, losses in the quantum channel, as well as on the overlap, e.g., |⟨a|b⟩|, between the coherent states in the set. One can also think of the interrelationships between these variables.
(6) This latest QDS scheme is a starting point to achieve the long-term goal of practically useful quantum digital signatures. However, several theoretical and practical items can still be addressed. The security of this scheme is proven for (e) individual as well as for collective quantum attacks, but for a universal quantum-mechanical manipulation—a so-called coherent attack—security is not yet proven.
(7) To lift these limitations, it could be promising to develop a discrete formulation in which the coherent states are replaced by (e&eb) single photon qubit states, as in the quantum cryptographic

BB84 protocol [9].

(8) This may allow us to adopt the exhaustive tools of (e) finite secret key analysis [10], which could prove (eb) unconditional security in general.

NOTATION

Module One

These measurements usually only eliminate (e) one possibility, such as |qi⟩ is not |c⟩

G_{13} : Category one of one possibility, such as |qi⟩ is not |c⟩ (model classifies based on million such experiments)

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of measurements

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Still, this partial information can be used (e) to validate a signature

G_{16} : Category one of partial information; validate a signature

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of validate a signature; partial information

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

When Alice later sends a message k with her signature Pk, Bob and Charlie can compare the full key to (e&eb) their partial (uncorrelated) information about the key

G_{20} : Category one of Alice later sends a message k with her signature Pk, Bob and Charlie can compare the full key; their partial (uncorrelated) information about the key

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of their partial (uncorrelated) information about the key; Alice later sends a message k with her signature Pk, Bob and Charlie can compare the full key

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

number L of coherent states to detect a forgery or repudiation attack for each public key |Qk⟩ must be on (eb) the order of 1013, but the authors have identified some future improvements that could reduce this number

G_{24} : Category one of number L of coherent states to detect a forgery or repudiation attack for each public key |Qk⟩

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of order of 1013, but the authors have identified some future improvements that could reduce this number

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Also this scheme straightforwardly generalizes to (e&eb with ac=dc=0: Diewart function) any number of verifying parties, but this further increases the technical demand.

The level of security of the protocol depends on (=) the number L, the errors and imperfections in the measurements, imperfections in the state preparations, losses in the quantum channel, as well as on the overlap, e.g., |⟨a|b⟩|, between the coherent states in the set. One can also think of the interrelationships between these variables.

Model generalises the formula

G_{28} : Category one of level of security of the protocol

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of (variable) X number L, the errors and imperfections in the measurements, imperfections in the state preparations, losses in the quantum channel, as well as on the overlap, e.g., |⟨a|b⟩|, between the coherent states in the set. One can also think of the interrelationships between these variables.

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

This latest QDS scheme is a starting point to achieve the long-term goal of practically useful quantum digital signatures.

However, several theoretical and practical items can still be addressed.

The security of this scheme is proven for (e) individual as well as for collective quantum attacks, but for a universal quantum-mechanical manipulation—a so-called coherent attack—security is not yet proven

G_{32} : Category one of individual as well as for collective quantum attacks, but for a universal quantum-mechanical manipulation—a so-called coherent attack—security is not yet proven

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of security of this scheme is proven

T_{33} : Category two of

T_{34} : Category three of

Module seven

individual as well as for collective quantum attacks, but for a universal quantum-mechanical manipulation—a so-called coherent attack—security is not yet proven

Model gives relation between universal quantum-mechanical manipulation—a so-called coherent attack and security

G_{36} : Category one of universal quantum-mechanical manipulation—a so-called coherent attack; security

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of security; universal quantum-mechanical manipulation—a so-called coherent attack

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

To lift these limitations, it could be promising to develop a discrete formulation in which the coherent states are replaced by (e&eb) single photon qubit states, as in the quantum cryptographic BB84 protocol [9]

G_{40} : Category one of discrete formulation in which the coherent states; single photon qubit states, as in the quantum cryptographic BB84 protocol [9]

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of single photon qubit states, as in the quantum cryptographic BB84 protocol [9]; discrete formulation in which the coherent states

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

This may allow us to adopt the exhaustive tools of finite secret key analysis [10], which could prove (eb) unconditional security in general.

G_{44} : Category one of finite secret key analysis [10],

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of unconditional security in general

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad (1)$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad (2)$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad (3)$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad (4)$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad (5)$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad (6)$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad (7)$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad (8)$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad (9)$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad (10)$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad (11)$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad (12)$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad (13)$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad (14)$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad (15)$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad (16)$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad (17)$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad (18)$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad (19)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (91)$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (92)$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}-(b_{44}'')^{(9)}(G_{47},t) & -(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}-(b_{45}'')^{(9)}(G_{47},t) & -(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}-(b_{46}'')^{(9)}(G_{47},t) & -(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KKKKK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(LLLLL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(MMMMM) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(NNNNN)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(OOOOO) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(PPPPP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty}(a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty}(b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\widehat{M}_{40})^{(8)}t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(t) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)}\left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)}\left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} \left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)} (\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} \mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$ 214

$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\ \ \text{for t}>0$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \qquad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad (326)$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad (327)$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- (328)

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(t) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\Big]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\Big)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY

Securely Distribution Of Quantum Public Keys

INTRODUCTION—VARIABLES USED

Viewpoint: Sending Messages with a Quantum Seal Hermann Kampermann, Institute for Theoretical Physics III, Heinrich-Heine-University Düsseldorf, Germany July 21, 2014• Physics 7, 78. For references kindly see original article. References have been deleted attributed to spatial restraints

(1) As a practical matter, photon losses may become significant when (e) sending quantum digital signatures over large distances.
(2) For such situations, sending quantum digital signatures over large distances repeater stations might be deployed (e&eb) in analogy to the case of quantum key distribution. It is fascinating to see that recent developments have promoted quantum digital signature schemes from a practically impossible model to the level of complexity similar to quantum key distribution.

Quantum Digital Signatures Daniel Gottesman, Isaac Chuang (Submitted on 8 May 2001 (v1), last revised 15 Nov 2001 (this version, v2))

(3) **Daniel Gottesman, Isaac Chuang present** a quantum digital signature scheme whose security is based on fundamental principles of quantum physics. It allows a sender (Alice) to sign a message in such a way that the signature can be validated by (e) a number of different people, and all will agree either that the message came from (e) Alice or that it has been tampered (e&eb) with.
(4) To accomplish this task, namely the consensus about the origination of the message from Alice or not each recipient of the message must have (e) a copy of Alice's "public key," which is (=) a set of quantum states whose exact identity is known only to Alice.
(5) Quantum public keys are more difficult to deal with than classical public keys: for instance, only a limited number of copies can be in circulation, (eb) or the scheme becomes insecure.
(6) However, in exchange for this price, namely limited number of copies can be in circulation, or the scheme becomes insecure (e) authors achieve unconditionally secure digital signatures. Sending an

m-bit message uses (e) up O (m) quantum bits for (e) each recipient of the public key.

(7) Authors briefly discuss how to securely distribute quantum public keys, and show the signature scheme is absolutely secure using (e) one method of key distribution.

(8) The protocol provides a model for importing the ideas of classical public key cryptography into (e&eb) the quantum world. Cite as:arXiv: quant-ph/0105032 (or arXiv: quant-ph/0105032v2 for this version)

NOTATION

Module One

As a practical matter, photon losses may become significant when (e) sending quantum digital signatures over large distances

G_{13} : Category one of sending quantum digital signatures over large distances

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of photon losses may become significant

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

For such situations, sending quantum digital signatures over large distances repeater stations might be deployed (e&eb) in analogy to the case of quantum key distribution.

It is fascinating to see that recent developments have promoted quantum digital signature schemes from a practically impossible model to the level of complexity similar to quantum key distribution.

G_{16} : Category one of sending quantum digital signatures over large distances repeater stations; analogy to the case of quantum key distribution

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of analogy to the case of quantum key distribution; sending quantum digital signatures over large distances repeater stations

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Daniel Gottesman, Isaac Chuang present a quantum digital signature scheme whose security is based on fundamental principles of quantum physics.

quantum digital signature scheme whose security is based on fundamental principles of quantum physics allows a sender (Alice) to sign a message in such a way that the signature can be validated by (e) a number of different people, and all will agree either that the message came from (e) Alice or that it has been

tampered (e&eb) with

G_{20} : Category one of number of different people, and all will agree either that the message came from (e) Alice or that it has been tampered with

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of quantum digital signature scheme whose security is based on fundamental principles of quantum physics allows a sender (Alice) to sign a message in such a way that the signature can be validated

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

To accomplish this task, namely the consensus about the origination of the message from Alice or not each recipient of the message must have (e) a copy of Alice's "public key," which is (=) a set of quantum states whose exact identity is known only to Alice

G_{24} : Category one of copy of Alice's "public key," which is a set of quantum states whose exact identity is known only to Alice

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of consensus about the origination of the message from Alice or not each recipient of the message

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Quantum public keys are more difficult to deal with than classical public keys: for instance, only a limited number of copies can be in circulation, (eb) or the scheme becomes insecure

G_{28} : Category one of Quantum public keys are more difficult to deal with than classical public keys: for instance, only a limited number of copies can be in circulation

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of security of the scheme

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

However, in exchange for this price, namely limited number of copies can be in circulation, or the scheme becomes insecure (e) authors achieve unconditionally secure digital signatures.

Sending an m-bit message uses (e) up O (m) quantum bits for each recipient of the public key

G_{32} : Category one of O (m) quantum bits for (e) each recipient of the public key

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Sending an m-bit message

T_{33} : Category two of

T_{34} : Category three of

Module seven

Authors briefly discuss how to securely distribute quantum public keys, and show the signature scheme is absolutely secure using (e) one method of key distribution

G_{36} : Category one of method of key distribution

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of signature scheme is absolutely secure

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The protocol provides a model for importing the ideas of classical public key cryptography into (e&eb) the quantum world.

Cite as:arXiv: quant-ph/0105032 (or arXiv: quant-ph/0105032v2 for this version)

G_{40} : Category one of protocol provides a model for importing the ideas of classical public key cryptography; quantum world

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of quantum world; protocol provides a model for importing the ideas of classical public key cryptography

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Dual Modules: Care

International Journal of Theoretical Physics First online: 19 July 2015 A Novel Quantum Blind Signature Scheme with Four-particle GHZ States Ling Fan , Ke-Jia Zhang, Su-Juan Qin, Fen-Zhuo Guo

In an arbitrated quantum signature scheme, the signer signs the message and the receiver verifies the signature's validity with the assistance of the arbitrator. **Ling Fan , Ke-Jia Zhang, Su-Juan Qin, Fen-Zhuo Guo** present an arbitrated quantum blind signature scheme by using (e) four-particle entangled Greenberger-Horne-Zeilinger (GHZ) states. By using the special relationship of four-particle GHZ states, we cannot only support the security of quantum signature, but also guarantee the anonymity of the message owner. It has a wide application to E-payment system, E-government, E-business, and etc.

Greenberger–Horne–Zeilinger state from Wikipedia, the free encyclopedia

In physics, in the area of quantum information theory, a Greenberger–Horne–Zeilinger state is a certain type of entangled quantum state which involves at least three subsystems (particles). It was first studied by Daniel Greenberger, Michael Horne and Anton Zeilinger in 1989.[1] They have noticed the extremely non-classical properties of the state.

Definition

The GHZ state is an entangled quantum state of M > 2 subsystems. In the case of each of the subsystems being two-dimensional, that is for qubits, it reads

$$|\mathrm{GHZ}\rangle = \frac{|0\rangle^{\otimes M} + |1\rangle^{\otimes M}}{\sqrt{2}}.$$

In simple words it is a quantum superposition of all subsystems being in state 0 with all of them being in state 1 (states 0 and 1 of a single subsystem are fully distinguishable).

The simplest one is the 3-qubit GHZ state: $$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}.$$

Properties

There is no standard measure of multi-partite entanglement because different types of multi-partite entanglement exist which are not mutually convertible. Nonetheless, many measures define the GHZ to be maximally entangled.

Another important property of the GHZ state is that when we trace over one of the three systems we get

$$\mathrm{Tr}_3\left((|000\rangle + |111\rangle)(\langle 000| + \langle 111|)\right) = \frac{(|00\rangle\langle 00| + |11\rangle\langle 11|)}{2}$$

This is an unentangled mixed state. It has certain two-particle (qubit) correlations, but these are of a classical nature.

On the other hand, if we were to measure one of the subsystems, in such a way that the measurement distinguishes between the states 0 and 1, we will leave behind either $|00\rangle$ or $|11\rangle$ which are unentangled pure states. This is unlike the W state which leaves bipartite entanglements even when we measure one of its subsystems.

The GHZ state leads to striking non-classical correlations (1989). Particles prepared in this state lead to a version of Bell's theorem, which shows the internal inconsistency of the notion of elements-of-reality introduced in the famous Einstein–Podolsky–Rosen paper. The first laboratory observation of GHZ correlations was by the group of Anton Zeilinger (1998). Many, more accurate observations followed. The correlations can be utilized in some quantum information tasks. These include multipartner quantum cryptography (1998) and communication complexity tasks (1997, 2004).

$$\mathrm{Tr}_3\left((|000\rangle + |111\rangle)(\langle 000| + \langle 111|)\right) = \frac{(|00\rangle\langle 00| + |11\rangle\langle 11|)}{2}$$

$$|\mathrm{GHZ}\rangle = \frac{|0\rangle^{\otimes M} + |1\rangle^{\otimes M}}{\sqrt{2}}.$$

G_{44} : Category one of four-particle entangled Greenberger-Horne-Zeilinger (GHZ) states; LHS of the above two equations

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of arbitrated quantum blind signature scheme; RHS of the above two equations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY ONE

Quantum Memory

INTRODUCTION—VARIABLES USED

Quantum Digital Signatures without Quantum Memory Vedran Dunjko, Petros Wallden, and Erika Andersson Phys. Rev. Lett 112, 040502 – Published 31 January 2014

(1) Quantum digital signatures (QDSs) allow the sending of messages from one sender to (e&eb) multiple recipients, with the guarantee that messages cannot be forged or tampered with.
(2) Additionally, messages cannot be (=) repudiated—if one recipient accepts a message, (eb) she is guaranteed that others will accept the same message as well.
(3) While messaging with these types of security guarantees are routinely performed in the modern digital world, current technologies only offer security under computational assumptions. QDSs, on the other hand, offer security guaranteed by (e) quantum mechanics.
(4) All thus far proposed variants of QDSs require (e) long-term, high quality quantum memory, making them unfeasible in (eb) the foreseeable future. Here, authors present a QDS scheme where no quantum memory is required, which also needs (e) just linear optics. This makes QDSs feasible with current technology.

Experimentally realizable quantum comparison of coherent states and its applications Erika Andersson, Marcos Curty, and Igor Jex Phys. Rev. A 74, 022304 – Published 3 August 2006

(5) When comparing quantum states to each other, (eb) it is possible to obtain an unambiguous answer, indicating that (eb) the states are definitely different, already after a single measurement.
(6) In this paper authors **Erika Andersson, Marcos Curty, and Igor Jex** investigate comparison of coherent states, which is (=) the simplest example of quantum state comparison for continuous variables.
(7) Method presented has a high success probability, and is experimentally feasible to realize as the only required components are (=) beam splitters and photon detectors. An easily realizable method for quantum state comparison could be important for real applications. As examples of such applications authors present a "lock and key" scheme and a simple scheme for quantum public key distribution

Quantum no-deleting theorem From Wikipedia, the free encyclopedia

(8) In physics, the no-deleting theorem of quantum information theory is a no-go theorem which states that, in general, given two copies of some arbitrary quantum state, it is (=) impossible to delete one of the copies.[1]
(9) It is a time-reversed dual to the no-cloning theorem,[2][3] which states that arbitrary states cannot (e) be copied. This theorem seems remarkable, because, in many senses, quantum states are fragile; the theorem asserts that, in a particular case, **they are also robust**
(10) The no-deleting theorem, together with the no-cloning theorem, underpin the interpretation of quantum mechanics in terms of category theory, and, in particular, as a dagger symmetric monoidal category.[4][5] This formulation, known as categorical quantum mechanics, in turn allows a connection to be made from quantum mechanics to (e&eb) linear logic as the logic of quantum information theory (in exact analogy to classical logic being founded on Cartesian closed categories.)

NOTATION

Module One

Quantum digital signatures (QDSs) allow the sending of messages from one sender to (e&eb) multiple recipients, with the guarantee that messages cannot be forged or tampered with

Additionally, messages cannot be (=) repudiated—if one recipient accepts a message, (eb) she is guaranteed that others will accept the same message as well

G_{13} : Category one of one recipient accepts a message

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of others will accept the same message as well

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

All thus far proposed variants of QDSs require (e) long-term, high quality quantum memory, making them unfeasible in (eb) the foreseeable future.

Here, authors present a QDS scheme where no quantum memory is required, which also needs (e) just linear optics.

This makes QDSs feasible with current technology.

G_{16} : Category one of quantum memory

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of QDS scheme

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

In physics, the no-deleting theorem of quantum information theory is a no-go theorem which states that, in general, given two copies of some arbitrary quantum state, it is (=) impossible to delete one of the copies.[1]

G_{20} : Category one of given two copies of some arbitrary quantum state

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of impossibility to delete one of the copies

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

It is a time-reversed dual to the no-cloning theorem,[2][3] which states that arbitrary states cannot (e) be copied.

This theorem seems remarkable, because, in many senses, quantum states are fragile; the theorem asserts that, in a particular case, **they are also robust**

G_{24} : Category one of copying

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of arbitrary states

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

NATURE PHOTONICS | LETTER Quantum teleportation from a telecom-wavelength photon to a solid-state quantum memory Félix Bussières, Christoph Clausen, Alexey Tiranov, Boris Korzh, Varun B. Verma, Ae Woo Nam, Francesco Marsili, Alban Ferrier, Philippe Goldner, Harald Herrmann, Christine Silberhorn, Wolfgang Sohler, Mikael Afzelius & Nicolas Gisin Nature Photonics 8, 775–778 (2014) doi:10.1038/nphoton.2014.215 Published online 21 September 2014

(1) Quantum teleportation1 is a cornerstone of quantum information science due to its essential role in important tasks such as the long-distance transmission of quantum information using (e) quantum repeaters2, 3.
(2) This requires the efficient distribution of entanglement between (e&eb) remote nodes of a network4. Here, **Félix Bussières, Christoph Clausen, Alexey Tiranov, Boris Korzh, Varun B. Verma, Ae Woo Nam, Francesco Marsili, Alban Ferrier, Philippe Goldner, Harald Herrmann, Christine Silberhorn, Wolfgang Sohler, Mikael Afzelius& Nicolas Gisin** demonstrate quantum teleportation of the polarization state of a telecom-wavelength photon onto (e&eb) the state of a solid-state quantum memory.
(3) Entanglement is established between a rare-earth-ion-doped crystal storing a single photon that is polarization-entangled with (e&eb) a flying telecom-wavelength photon5, 6.
(4) Latter is jointly measured with (e&eb) another flying polarization qubit to be teleported, which heralds the teleportation.
(5) The fidelity of the qubit retrieved from the memory is shown to be greater than (e) the maximum fidelity achievable without entanglement, even when the combined distances travelled by the two flying qubits is 25 km of standard optical fibre. Results demonstrate the possibility of long-distance quantum networks with solid-state resources.

Quantum teleportation1 is a cornerstone of quantum information science due to its essential role in important tasks such as the long-distance transmission of quantum information using (e) quantum repeaters2, 3

G_{28} : Category one of quantum repeaters2, 3

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Quantum teleportation1 is a cornerstone of quantum information science due to its essential role in important tasks such as the long-distance transmission of quantum information

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

This requires the efficient distribution of entanglement between (e&eb) remote nodes of a network4. Here, **Félix Bussières, Christoph Clausen, Alexey Tiranov, Boris Korzh, Varun B. Verma, Ae Woo Nam, Francesco Marsili, Alban Ferrier, Philippe Goldner, Harald Herrmann, Christine Silberhorn, Wolfgang Sohler, Mikael Afzelius& Nicolas Gisin** demonstrate quantum teleportation of the polarization state of a telecom-wavelength photon onto (e&eb) the state of a solid-state quantum memory

G_{32} : Category one of quantum teleportation of the polarization state of a telecom-wavelength photon; state of a solid-state quantum memory

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of state of a solid-state quantum memory ;quantum teleportation of the polarization state of a telecom-wavelength photon

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Entanglement is established between a rare-earth-ion-doped crystal storing a single photon that is polarization-entangled with (e&eb) a flying telecom-wavelength photon5, 6

G_{36} : Category one of rare-earth-ion-doped crystal storing a single photon that is polarization-entangled; flying telecom-wavelength photon5, 6

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of flying telecom-wavelength photon5, 6; rare-earth-ion-doped crystal storing a single photon that is polarization-entangled

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The fidelity of the qubit retrieved from the memory is shown to be greater than (e) the maximum fidelity achievable without entanglement, even when the combined distances travelled by the two flying qubits is 25 km of standard optical fibre. Results demonstrate the possibility of long-distance quantum networks with solid-state resources

G_{40} : Category one of maximum fidelity achievable without entanglement, even when the combined distances travelled by the two flying qubits is 25 km of standard optical fibre.

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of fidelity of the qubit retrieved from the memory

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

NATURE PHOTONICS | LETTER A quantum memory for orbital angular momentum photonic qubits A. Nicolas, L. Veissier, L. Giner, E. Giacobino, D. Maxein & J. Laurat Nature Photonics 8, 234–238 (2014) doi:10.1038/nphoton.2013.355 Published online 26 January 2014

(1) Among the optical degrees of freedom, the orbital angular momentum of light1 provides unique properties2, including mechanical torque action, which has applications for light manipulation3, enhanced sensitivity in imaging techniques4 and potential high-density information coding for optical communication systems5. Recent years have also seen a tremendous interest in exploiting orbital angular momentum at the single-photon level in quantum information technologies6, 7. In pursuing this endeavour, **Nicolas, L. Veissier, L. Giner, E. Giacobino, D. Maxein & J. Laurat** demonstrate here the implementation of a quantum memory8 for (e) quantum bits encoded in this optical degree of freedom.
(2) They generate various qubits with computer-controlled holograms, store and retrieve them on demand using (e) a dynamic electromagnetically induced transparency protocol.
(3) They further analyse the retrieved states by quantum tomography and thereby demonstrate fidelities exceeding (e) the classical benchmark, confirming (eb) the quantum functioning of our storage process
(4) Results provide an essential capability for future networks9 exploring the promises of orbital angular momentum of photons for (e) quantum information applications.

implementation of a quantum memory8 for (e) quantum bits encoded in this optical degree of freedom

G_{44} : Category one of quantum bits encoded in this optical degree of freedom

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of implementation of a quantum memory8

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) & \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a_{28}')^{(5)}+(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t)\\ +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)\\ +(a_{36}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad 79$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a_{29}')^{(5)}+(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t)\\ +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)\\ +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{29} \quad 80$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a_{30}')^{(5)}+(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t)\\ +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)\\ +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{30} \quad 81$$

$Where$ $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ $are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ $are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\big((G_{19}), t\big) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} ((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQQQQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(RRRRR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(SSSSS) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(TTTTT)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(UUUUU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(VVVVV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)$, $T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(u) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\left\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\right\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0 \quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$

$G_{20}^{(2)} \left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad (214)$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (216)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: (217)

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. (218)

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 239

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(5)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\,(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad (269)$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote (270)

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results (271)

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ (272)

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad (273)$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. (274)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ (275)

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\,((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}}G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}}G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)} \quad 293$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad 294$$

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad (326)$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad (327)$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- (328)

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$

$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,

and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

with $(p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(u) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27})=0$, T_{25}^{*} given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \ , \ \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY TWO

Quantum Digital Signatures

INTRODUCTION—VARIABLES USED

Nature 299, 802 - 803 (28 October 1982); doi: 10.1038/299802a0 A single quantum cannot be cloned w. k. Wootters*1 & W. H. Zurek2

(1) If a photon of definite polarization encounters an excited atom (eb) there is typically some

nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission.

(2) Such a photon is guaranteed to have (e) the same polarization as the original photon.

(3) But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? Assume the proposition and give the model

(4) If it were (eb) the amplifying process could be used to (e) ascertain the exact state of a quantum system

(5) One could determine its polarization in the case of a photon by (e) first producing a beam of identically polarized copies and then measuring (eb) the Stokes parameters1.

(6) Wootters & W. H. Zurek show here that the linearity of quantum mechanics forbids (e) such replication and that this conclusion holds for all quantum systems.

NOTATION

Module One

NATURE PHOTONICS | LETTER A quantum memory for orbital angular momentum photonic qubits A. Nicolas, L. Veissier, L. Giner, E. Giacobino, D. Maxein & J. Laurat Nature Photonics 8, 234–238 (2014) doi:10.1038/nphoton.2013.355 Published online 26 January 2014

(1) They generate various qubits with computer-controlled holograms, store and retrieve them on demand using (e) a dynamic electromagnetically induced transparency protocol.

(2) They further analyse the retrieved states by quantum tomography and thereby demonstrate fidelities exceeding (e) the classical benchmark, confirming (eb) the quantum functioning of our storage process

(3) Results provide an essential capability for future networks9 exploring the promises of orbital angular momentum of photons for (e) quantum information applications.

They generate various qubits with computer-controlled holograms, store and retrieve them on demand using (e) a dynamic electromagnetically induced transparency protocol

G_{13} : Category one of dynamic electromagnetically induced transparency protocol

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of various qubits with computer-controlled holograms, store and retrieve them on demand

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

If a photon of definite polarization encounters an excited atom (eb) there is typically some nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission

G_{16} : Category one of photon of definite polarization; excited atom (eb) there is typically some nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of excited atom (eb) there is typically some nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission; photon of definite polarization

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

If a photon of definite polarization encounters an excited atom (eb) there is typically some nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission

G_{20} : Category one of photon of definite polarization encounters an excited atom

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of typically some nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

If a photon of definite polarization encounters an excited atom there is typically some nonvanishing probability that the atom will emit (eb) a second photon by stimulated emission

G_{24} : Category one of photon of definite polarization encounters an excited atom there is typically some nonvanishing probability that the atom

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of second photon by stimulated emission

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Such a photon is guaranteed to have (e) the same polarization as the original photon

G_{28} : Category one of same polarization as the original photon

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Such a photon (many such photons exist)

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original?
Assume the proposition and give the model

G_{32} : Category one of several copies of a quantum system (the polarized photon in the present case) each having the same state as the original ; process X

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of process X; several copies of a quantum system (the polarized photon in the present case) each having the same state as the original

T_{33} : Category two of

T_{34} : Category three of

Module seven

If it were (eb) the amplifying process could be used to (e) ascertain the exact state of a quantum system

Having assumed the existent of the process, we give the model for amplification process usage for obtention of the exact state of the quantum system.

G_{36} : Category one of amplifying process

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of ascertain the exact state of a quantum system

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

One could determine its polarization in the case of a photon by (e) first producing a beam of identically polarized copies and then measuring (eb) the Stokes parameters1

G_{40} : Category one of producing a beam of identically polarized copies and then measuring (eb) the Stokes parameters1

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of polarization in the case of a photon

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Wootters & W. H. Zurek show here that the linearity of quantum mechanics forbids (e) such replication and that this conclusion holds for all quantum systems

G_{44} : Category one of replication and that this conclusion holds for all quantum systems

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of linearity of quantum mechanics

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,

$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$
$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{ccc}(a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) \quad +(a_{20}'')^{(3,3,3)}(T_{21},t)\\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{ccc}(a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) \quad +(a_{21}'')^{(3,3,3)}(T_{21},t)\\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{ccc}(a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) \quad +(a_{22}'')^{(3,3,3)}(T_{21},t)\\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a_{44}'')^{(9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{array}{ccc}(b_{16}')^{(2)} & -(b_{16}'')^{(2)}(G_{19},t) & -(b_{13}'')^{(1,1,)}(G,t) \quad -(b_{20}'')^{(3,3,3,)}(G_{23},t)\\ -(b_{24}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)\\ -(b_{36}'')^{(7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9)}(G_{47},t)\end{array}\right]T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition*

coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}+(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t)\\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t)\\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{26} \quad (75)$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \quad +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \quad +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) \quad +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)} & -(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)} & -(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)} & -(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41}$$

$$- \begin{bmatrix} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38 \qquad 131$$ (WWWWW)

(XXXXX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(YYYYY) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(ZZZZZ)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(AAAAAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(BBBBBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - \ T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(v) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

208

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

209

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

214

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right| \; e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}} \left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)}; \; (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t} \; and \; (\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and \; not \; on \; \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2 \; and \; \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45} \; and \; G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$$

334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

335

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$$

336

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations
$(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)}$,, $(\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(v) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 513$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\big((a_{30}')^{(5)} + (p_{30})^{(5)}\big)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\big((b_{28}')^{(5)} - (r_{28})^{(5)}\big)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(28)(j)}T_{28}^*\mathbb{G}_j\big) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\big((b_{29}')^{(5)} - (r_{29})^{(5)}\big)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\big(s_{(29)(j)}T_{29}^*\mathbb{G}_j\big) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\big((b_{30}')^{(5)} - (r_{30})^{(5)}\big)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(30)(j)}T_{30}^*\mathbb{G}_j\big) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586

ASYMPTOTIC STABILITY ANALYSIS A

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586 B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586 C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586 D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586 E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586 F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586 G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY THREE

Quantum Digital Signatures Offer Information-Theoretic Security Based On Laws Of Quantum Mechanics

INTRODUCTION—VARIABLES USED

Collins, Ross J. Donaldson, Vedran Dunjko, Petros Wallden, Patrick J. Clarke, Erika Andersson, John Jeffers, and Gerald S. Buller Phys. Rev. Lett. 113, 040502 – Published 21 July 2014

(1) Digital signatures are widely used to provide (eb) security for electronic communications, for example, in financial transactions and electronic mail.
(2) Currently used classical digital signature schemes, however, only offer security relying on (e) unproven computational assumptions.
(3) In contrast, quantum digital signatures offer information-theoretic security based on (e) laws of quantum mechanics.
(4) Here, security against forging relies on (e) the impossibility of perfectly distinguishing between nonorthogonal quantum states.
(5) A serious drawback of previous quantum digital signature schemes is that they require (e) long-term quantum memory, making them impractical at present.
(6) **Robert J. Collins, Ross J. Donaldson, Vedran Dunjko, Petros Wallden, Patrick J. Clarke, Erika Andersson, John Jeffers, and Gerald S. Buller** present the first realization of a scheme that does not need quantum memory and which also uses (e) only standard linear optical components and photodetectors.
(7) In realization, the recipients measure (eb) the distributed quantum signature states using (e) a new type of quantum measurement, quantum state elimination. This significantly advances quantum digital signatures as a quantum technology with potential for real applications.

NOTATION

Module One

Digital signatures are widely used to provide (eb) security for electronic communications, for example, in financial transactions and electronic mail

G_{13} : Category one of Digital signatures

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of security for electronic communications, for example, in financial transactions and electronic mail

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

In contrast, quantum digital signatures offer information-theoretic security based on (e) laws of quantum mechanics

G_{16} : Category one of laws of quantum mechanics

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantum digital signatures offer information-theoretic security

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Here, security against forging relies on (e) the impossibility of perfectly distinguishing between nonorthogonal quantum states

G_{20} : Category one of impossibility of perfectly distinguishing between nonorthogonal quantum states

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of security against forging

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

A serious drawback of previous quantum digital signature schemes is that they require (e) long-term quantum memory, making them impractical at present

G_{24} : Category one of long-term quantum memory, making them impractical at present

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of serious drawback of previous quantum digital signature schemes

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Here, security against forging relies on (e) the impossibility of perfectly distinguishing between nonorthogonal quantum states

G_{28} : Category one of impossibility of perfectly distinguishing between nonorthogonal quantum states

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of security against forging

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Robert J. Collins, Ross J. Donaldson, Vedran Dunjko, Petros Wallden, Patrick J. Clarke, Erika Andersson, John Jeffers, and Gerald S. Buller present the first realization of a scheme that does not need quantum memory and which also uses (e) only standard linear optical components and photodetectors

G_{32} : Category one of standard linear optical components and photodetectors

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of scheme that does not need quantum memory

T_{33} : Category two of

T_{34} : Category three of

Module seven

In realization, the recipients measure (eb) the distributed quantum signature states using (e) a new type of quantum measurement, quantum state elimination.

This significantly advances quantum digital signatures as a quantum technology with potential for real applications.

G_{36} : Category one of recipients

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of distributed quantum signature states using (e) a new type of quantum measurement, quantum state elimination.

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In realization, the recipients measure the distributed quantum signature states using (e) a new type of quantum measurement, quantum state elimination

G_{40} : Category one of new type of quantum measurement, quantum state elimination

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of recipients measure the distributed quantum signature states

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Convergence and adiabatic elimination for a driven dissipative quantum harmonic oscillator

Rémi Azouit, Alain Sarlette, Pierre Rouchon

(Submitted on 21 Mar 2015)

Authors prove that a harmonic oscillator driven by Lindblad dynamics where the typical drive and loss channels are two-photon processes instead of single-photon ones, converges to a protected subspace spanned by two coherent states of opposite amplitude.

They then characterize the slow dynamics induced by a perturbative single-photon loss on this protected subspace, by performing (e&eb) adiabatic elimination in the Lindbladian dynamics.

Cite as: arXiv:1503.06324 [quant-ph]

(or arXiv:1503.06324v1 [quant-ph] for this version)

G_{44} : Category one of harmonic oscillator driven by Lindblad dynamics where the typical drive and loss channels are two-photon processes instead of single-photon ones; protected subspace spanned by two coherent states of opposite amplitude.

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of protected subspace spanned by two coherent states of opposite amplitude; harmonic oscillator driven by Lindblad dynamics where the typical drive and loss channels are two-photon processes instead of single-photon ones

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY FOUR

Single Dot Confabulates With Single Ion

INTRODUCTION—VARIABLES USED

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark March 23, 2015• Physics 8, 27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion.

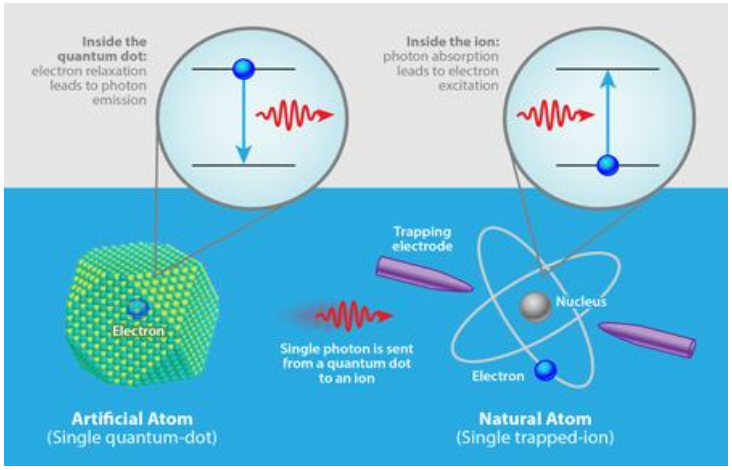


Figure caption APS/Alan Stonebraker

(1) Figure 1: Researchers have shown that a single photon emitted by (e) a quantum dot can excite (eb) an ion 25 meters away.
(2) (Left) A laser excites (e&eb) an electron in the quantum dot.
(3) When the electron relaxes (eb) it emits a photon, which travels through (e&eb) a fiber (not shown) to t... Show more
(4) Researchers have made many advances in controlling atomic, solid-state, and other types of quantum systems. Hybrid quantum systems bring together (e&eb) two or more such systems and allow (eb) them to interact.
(5) Hybrid systems combine the advantages of each component and as such, are of interest for a number of proposed technologies, including a hybrid quantum network. One hybrid system of particular interest is (e) a single artificial atom (e.g., a quantum dot) coupled to (e&eb) a natural atom (e.g., a trapped ion).
(6) Combining these two components is, however, a challenge: specialized techniques are needed to control (e&eb) single artificial atoms and single natural atoms, while experimentalists have so far mastered only one system or the other. Research groups at the University of Cambridge, UK, led by Mete Atatüre, an expert on quantum dots, and Michael Köhl, an expert on trapped ions, now overcome this challenge to demonstrate a photonic link between a single ion and (e&eb) a single quantum dot—a nanoscale semiconductor that behaves (e&eb) like an atom [1].
(7) Artificial and (=) natural atoms are conceptually the same physical systems.
(8) Both artificial and (=) natural atoms have (e) confined electrons that can only occupy (e) well-defined discrete energy levels.

About the Author

Jungsang Kim is a professor of Electrical and Computer Engineering, Physics, and Computer Science at Duke University in North Carolina. He leads the Multifunctional Integrated Systems Technology group, concentrating on experimental implementation of novel computing and sensing systems such as scalable modular quantum computers and quantum networks using trapped ions, high-performance single-photon detectors and free-space quantum key distribution systems, and gigapixel-scale cameras. Jungsang Kim received his Ph.D. in physics from Stanford University and worked as a Member of Technical Staff and a Technical Manager at Bell Laboratories in Murray Hill, New Jersey.

NOTATION

Module One

Figure 1: Researchers have shown that a single photon emitted by a quantum dot can excite (eb) an ion 25 meters away

G_{13} : Category one of single photon emitted by a quantum dot can excite

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of ion 25 meters away

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

A laser excites (e&eb) an electron in the quantum dot

G_{16} : Category one of laser excites; electron in the quantum dot

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of electron in the quantum dot; laser excites

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

When the electron relaxes it emits a photon, which travels through a fiber (not shown) to t...

G_{20} : Category one of electron relaxes

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of photon is emitted

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

When the electron relaxes it emits a photon, which travels through a fiber

Many such experiments could be conducted

G_{24} : Category one of electron relaxes it emits a photon; fiber

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of fiber ;electron relaxes it emits a photon

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Researchers have made many advances in controlling atomic, solid-state, and other types of quantum systems

Hybrid quantum systems bring together (e&eb) two or more such systems and allow (eb) them to interact

G_{28} : Category one of Hybrid quantum systems; two or more such systems and allow (eb) them to interact

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of two or more such systems and allow (eb) them to interact ;Hybrid quantum systems

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Hybrid quantum systems bring together two or more such systems and allow (eb) them to interact

G_{32} : Category one of Hybrid quantum systems bring together two or more such systems

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of interation between them

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Combining these two components is, however, a challenge: specialized techniques are needed to control (e&eb) single artificial atoms and single natural atoms, while experimentalists have so far mastered only one system or the other.

Research groups at the University of Cambridge, UK, led by Mete Atatüre, an expert on quantum dots, and Michael Köhl, an expert on trapped ions, now overcome this challenge to demonstrate a photonic link between a single ion and (e&eb) a single quantum dot—a nanoscale semiconductor that behaves (e&eb) like an atom [1]

G_{36} : Category one of photonic link between a single ion; single quantum dot—a nanoscale semiconductor that behaves (e&eb) like an atom [1]

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of single quantum dot—a nanoscale semiconductor that behaves (e&eb) like an atom [1]; photonic link between a single ion

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

single quantum dot—a nanoscale semiconductor that behaves (e&eb) like an atom [1]

G_{40} : Category one of single quantum dot (there are many such); atom [1]

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of atom [1]; single quantum dot (there are many such)

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Both artificial and natural atoms have (e) confined electrons that can only occupy (e) well-defined discrete energy levels

G_{44} : Category one of confined electrons that can only occupy (e) well-defined discrete energy levels

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of artificial and natural atoms

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$

$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\begin{bmatrix}(a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t)\end{bmatrix}G_{16} \quad 61$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\begin{bmatrix}(a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t)\\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t)\end{bmatrix}G_{17} \quad 62$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\begin{bmatrix}(a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t)\\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t)\end{bmatrix}G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for

category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{16}'')^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) & \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{30} \quad (81)$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)} & -(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)} & -(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)} & -(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38 \qquad 131$$

(DDDDDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(EEEEEE) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(FFFFFF)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(GGGGGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(HHHHHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}$, $(r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(w) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\;e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\;(G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ and $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations
$(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},, (\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,

and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(w) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\big((a_{30}')^{(5)} + (p_{30})^{(5)}\big)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\big((b_{28}')^{(5)} - (r_{28})^{(5)}\big)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(28)(j)}T_{28}^*\mathbb{G}_j\big) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\big((b_{29}')^{(5)} - (r_{29})^{(5)}\big)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\big(s_{(29)(j)}T_{29}^*\mathbb{G}_j\big) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\big((b_{30}')^{(5)} - (r_{30})^{(5)}\big)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(30)(j)}T_{30}^*\mathbb{G}_j\big) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{**}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY FIVE

Natural Atoms Are "Cleaner" Systems And Their Electrons Can Be Prepared In Quantum States That Have Long Coherence Times

INTRODUCTION—VARIABLES USED

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark March 23, 2015• Physics 8, 27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion.

(1) And in both systems, an external field (such as the optical field from a laser) can be used to (e) "move" the electrons between the levels to generate (eb) single and entangled photons, prepare an electron spin in a quantum superposition, and implement quantum memories.
(2) For particular applications, however, each system has its own unique advantages and limitations. A key difference is that a natural atom is an isolated system while (e) an artificial atom is not.
(3) A single trapped atom typically sits in (eb) a vacuum, while a single artificial atom is usually embedded in (eb) a host material.
(4) As a result, natural atoms are (=) "cleaner" systems and their electrons can be prepared in (eb) quantum states that have (e) long coherence times.
(5) Artificial atoms are, in contrast, easier to manipulate. For instance, contacts can be fabricated around (e&eb) the host material and used to (e) controls the electrons in (eb) the artificial atom electrically.
(6) But electron quantum states in artificial atoms tend to have shorter coherence times than (eb) those associated with ions because of (e) their interaction with the host material.
(7) The first hybrid semiconductor-atomic system [2, 3] was an interface between a single GaAs quantum dot and (e&eb) a vapor of rubidium atoms.
(8) In one experiment, the atomic system was used as a slow light medium [2] for single photons generated by (e) an artificial atom.
(9) In another experiment, the frequency of single photons emitted by the artificial atoms was locked to (e&eb) the frequency of an atomic optical transition [3].

NOTATION

Module One

artificial and natural atoms have confined electrons that can only occupy (e) well-defined discrete energy levels

G_{13} : Category one of well-defined discrete energy levels

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of artificial and natural atoms have confined electrons that can only occupy

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

And in both systems, an external field (such as the optical field from a laser) can be used to (e) "move" the electrons between the levels to generate (eb) single and entangled photons, prepare an electron spin in a quantum superposition, and implement quantum memories

G_{16} : Category one of "move" the electrons between the levels to generate (eb) single and entangled

photons, prepare an electron spin in a quantum superposition, and implement quantum memories

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of external field (such as the optical field from a laser) can be used

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

And in both systems, an external field (such as the optical field from a laser) can be used to "move" the electrons between the levels to generate (eb) single and entangled photons, prepare an electron spin in a quantum superposition, and implement quantum memories

G_{20} : Category one of external field (such as the optical field from a laser) can be used to "move" the electrons between the levels

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of single and entangled photons, prepare an electron spin in a quantum superposition, and implement quantum memories

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

And in both systems, an external field (such as the optical field from a laser) can be used to "move" the electrons between the levels to generate single and entangled photons, prepare an electron spin in a quantum superposition, and implement quantum memories

G_{24} : Category one of external field (such as the optical field from a laser) can be used to "move" the electrons between the levels to generate single and entangled photons

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of electron spin in a quantum superposition, and implement quantum memories

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

And in both systems, an external field (such as the optical field from a laser) can be used to "move" the electrons between the levels to generate single and entangled photons, prepare an electron spin in a quantum

superposition, and implement quantum memories

G_{28} : Category one of external field (such as the optical field from a laser) can be used to "move" the electrons between the levels to generate single and entangled photons, prepare an electron spin in a quantum superposition

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of quantum memories

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

first hybrid semiconductor-atomic system [2, 3] was an interface between a single GaAs quantum dot and (e&eb) a vapor of rubidium atoms

G_{32} : Category one of single GaAs quantum dot; vapor of rubidium atoms

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of vapor of rubidium atoms; single GaAs quantum dot

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

But electron quantum states in artificial atoms tend to have shorter coherence times than) those associated with ions because of (e) their interaction with the host material

G_{36} : Category one of their interaction with the host material

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of electron quantum states in artificial atoms tend to have shorter coherence times than) those associated with ions

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In one experiment, the atomic system was used as a slow light medium [2] for single photons generated by (e) an artificial atom

G_{40} : Category one of atomic system was used as a slow light medium [2] for single photons generated; artificial atom

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of artificial atom ;atomic system was used as a slow light medium [2] for single photons generated

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

In another experiment, the frequency of single photons emitted by the artificial atoms was locked to (e&eb) the frequency of an atomic optical transition [3].

G_{44} : Category one of frequency of single photons emitted by the artificial atoms; frequency of an atomic optical transition [3].

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of frequency of an atomic optical transition [3].;frequency of single photons emitted by the artificial atoms

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$

$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{16})^{(2)}\,+(a''_{16})^{(2)}(T_{17},t)\;+(a''_{13})^{(1,1,)}(T_{14},t)\;+(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{c}(a'_{17})^{(2)}\,+(a''_{17})^{(2)}(T_{17},t)\;+(a''_{14})^{(1,1,)}(T_{14},t)\;+(a''_{21})^{(3,3,3)}(T_{21},t)\\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{18})^{(2)}\,+(a''_{18})^{(2)}(T_{17},t)\;+(a''_{15})^{(1,1,)}(T_{14},t)\;+(a''_{22})^{(3,3,3)}(T_{21},t)\\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for

category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) & \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{30} \quad (81)$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \quad +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \quad +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) \quad +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(IIIIII) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(JJJJJJ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(KKKKKK) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(LLLLLL)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(MMMMMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(NNNNNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(x) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0$ *and* $T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)}, , (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(x) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY SIX

"Single-To-Single" Interface That Couples A Single Artificial Atom To A Single Natural Atom

INTRODUCTION—VARIABLES USED

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark March 23, 2015• Physics 8, 27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion.

(1) Ultimately, the nodes in a hybrid quantum network have to be able to exchange (e&eb) quantum information.
(2) Research so far suggests single artificial and natural atoms could, if placed in the nodes of a network, perform (e&eb) this function.
(3) The next significant milestone is therefore a "single-to-single" interface that couples a single artificial atom to (e&eb) a single natural atom.
(4) This is exactly what the researchers from the University of Cambridge have demonstrated. The natural atom in their experiment is a single Yb+ ion trapped between (e&eb) two needle-shaped electrodes.
(5) The quantum dot is surrounded by (e) a host material (GaAs), which is electrically contacted.
(6) An electric field applied through (e&eb) the contacts serves two purposes: it controls (e&eb) the number of electrons present in the dot, and it can be used to (e) tune an optical transition of the quantum dot so it is in resonance with (e&eb) a transition of the Yb+ ion.
(7) When the frequencies of the optical transitions are matched, (eb) the quantum dot and (e&eb) the ion can exchange single photons.
(8) With a laser, **Atatüre, Köhl, and their colleagues** excite an electron in the quantum dot, which emits (eb) a single photon when the electron relaxes (Fig. 1). (deleted due to spatial restrictions)

Nature 438, 833-836 (8 December 2005) | doi: 10.1038/nature04315; Received 23 August 2005;

Accepted 13 October 2005 Storage and retrieval of single photons transmitted between remote quantum memories T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1

(9) An elementary quantum network operation involves (e&eb) storing a qubit state in an atomic quantum memory node, and then retrieving and transporting (e&eb) the information through (e&eb) a single photon excitation to a remote quantum memory node for further storage or analysis.

(10) Implementations of quantum network operations are thus conditioned on the ability to (e) realize matter-to-light and/or light-to-matter quantum state mappings

(11) Here **T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1** report the generation, transmission, storage and retrieval of single quanta using (e) two remote atomic ensembles.

(12) A single photon is generated from (e) a cold atomic ensemble at one site 1, and is directed to (e&eb) another site through 100 metres of optical fibre.

(13) The photon is then converted into (e&eb) a single collective atomic excitation using (e) a dark-state polariton approach2.

(14) After a programmable storage time, the atomic excitation is converted back into (e&eb) a single photon.

(15) This is demonstrated experimentally, for a storage time of 0.5 microseconds, by measurement of an anti-correlation parameter. Storage times exceeding ten microseconds are observed by intensity cross-correlation measurements. This storage period is two orders of magnitude longer than the time required (e) for achieving conversion between photonic and (e&eb) atomic quanta.

(16) The controlled transfer of single quanta between remote quantum memories constitutes (e) an important step towards distributed quantum networks.

NOTATION

Module One

Ultimately, the nodes in a hybrid quantum network have to be able to exchange (e&eb) quantum information

G_{13} : Category one of nodes in a hybrid quantum network have to be able to exchange; quantum information

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of quantum information ;nodes in a hybrid quantum network have to be able to exchange

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Research so far suggests single artificial and natural atoms could, if placed in the nodes of a network, perform (e&eb) this function

G_{16} : Category one of single artificial and natural atoms could, if placed in the nodes of a network; function

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of function ;single artificial and natural atoms could, if placed in the nodes of a network

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The next significant milestone is therefore a "single-to-single" interface that couples a single artificial atom to (e&eb) a single natural atom

G_{20} : Category one of single artificial atom; single natural atom

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of single natural atom; single artificial atom

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

This is exactly what the researchers from the University of Cambridge have demonstrated. The natural atom in their experiment is a single Yb+ ion trapped between (e&eb) two needle-shaped electrodes

Whenever classification is not possible we can take all the three categories as equivalent This holds for the entire monograph.

G_{24} : Category one of single Yb+ ion; two needle-shaped electrodes

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of two needle-shaped electrodes ;single Yb+ ion

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

An electric field applied through the contacts serves two purposes: it controls the number of electrons present in the dot, and it can be used to (e) tune an optical transition of the quantum dot so it is in resonance with (e&eb) a transition of the Yb+ ion

G_{28} : Category one of electric field applied through the contacts

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of controls the number of electrons present in the dot, and it can be used to (e) tune an optical transition of the quantum dot so it is in resonance with (e&eb) a transition of the Yb+ ion

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

G_{32} : Category one of number of electrons present in the dot; tune an optical transition of the quantum dot so it is in resonance with (e&eb) a transition of the Yb+ ion

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of tune an optical transition of the quantum dot so it is in resonance with (e&eb) a transition of the Yb+ ion; number of electrons present in the dot

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

When the frequencies of the optical transitions are matched, (eb) the quantum dot and (e&eb) the ion can exchange single photons

G_{36} : Category one of frequencies of the optical transitions are matched

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of quantum dot and (e&eb) the ion can exchange single photons

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

quantum dot and (e&eb) the ion can exchange single photons

G_{40} : Category one of quantum dot; ion

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of ion ;quantum dot

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

With a laser, **Atatüre, Köhl, and their colleagues** excite an electron in the quantum dot, which emits (eb) a single photon when the electron relaxes (Fig. 1) (deleted due to spatial restrictions)

G_{44} : Category one of laser excites an electron in the quantum dot

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of single photon when the electron relaxes

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ are first augmentation coefficients for category 1, 2 and 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 91$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 92$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ *94*

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b_{40}')^{(8)}\left[-(b_{40}'')^{(8)}(G_{43},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b_{41}')^{(8)}\left[-(b_{41}'')^{(8)}(G_{43},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b_{42}')^{(8)}\left[-(b_{42}'')^{(8)}(G_{43},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{36}'')^{(7)}(G_{39},t)$, $-(b_{37}'')^{(7)}(G_{39},t)$, $-(b_{38}'')^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}-(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}-(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}-(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \tag{97}$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \tag{98}$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: (99)

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: (100)

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: (101)

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(OOOOOO) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(PPPPPP) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(QQQQQQ) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(RRRRRR)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(SSSSSS) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(TTTTTT) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\widehat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} \text{ , } T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} \text{ , } T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left(e^{(\tilde{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(y) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)}\left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)} \left(T_{21}^{(1)}, s_{(20)}\right) \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left| (a''_{20})^{(3)} \left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)} \left(T_{21}^{(2)}, s_{(20)}\right) \right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| G_{23}{}^{(1)} - G_{23}{}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d \left(((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}) \right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[- \int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)} \right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (250)

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \qquad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\;e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\;(G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously 331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities 332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,

and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\le T_{38}(t)\le$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\le-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\le-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\le-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\le-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(y) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY SEVEN

Excitation In A Single Quantum Dot Can Be Transferred To An Excitation In A Single Ion By Means Of A Single Photon

INTRODUCTION—VARIABLES USED

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark March 23, 2015• Physics 8, 27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion.

(1) A periodic excitation of the quantum dot generates (eb) a stream of single photons that are channeled into (e&eb) an optical fiber and sent 25 meters away to (e&eb) the trapped ion.
(2) An electron in the ion is prepared in a state such that it can absorb (e) a single photon from (e) the quantum dot. This absorption leads to (eb) an electron excitation that the researchers are able to detect.
(3) The experiment demonstrates that an excitation in a single quantum dot can be transferred to (e&eb) an excitation in a single ion by means of (e) a single photon.
(4) A great advantage of this type of "single-to-single" interface is that it provides (eb) the starting infrastructure for exchanging quantum information between a single quantum dot and (e&eb) a single ion.
(5) This, however, is yet to be demonstrated. **Atatüre, Köhl, and their colleagues** have made a preliminary step forward towards this goal by showing that the spin of an electron in the quantum dot is correlated with (e&eb) the excitation transfer.
(6) In this experiment, the quantum dot is first prepared to have (e) one excess electron, and the spin of this electron is controlled to be (=) either up or down.
(7) When an excitation laser is turned on (eb) it can only excite the dot if (e) the electron has its spin up.
(8) This excitation is then transferred to (e&eb) an ion and detected, exactly as in the first experiment.
(9) If the electron's spin is down, however, (eb) the laser can't excite the dot, (eb) so no photon is

transferred to the ion.

NOTATION

Module One

A periodic excitation of the quantum dot generates (eb) a stream of single photons that are channeled into (e&eb) an optical fiber and sent 25 meters away to (e&eb) the trapped ion

G_{13} : Category one of periodic excitation of the quantum dot

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of stream of single photons that are channeled into (e&eb) an optical fiber and sent 25 meters away to (e&eb) the trapped ion

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

periodic excitation of the quantum dot generates a stream of single photons that are channeled into (e&eb) an optical fiber and sent 25 meters away to (e&eb) the trapped ion

G_{16} : Category one of periodic excitation of the quantum dot generates a stream of single photons that are channeled; optical fiber and sent 25 meters away to (e&eb) the trapped ion

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of optical fiber and sent 25 meters away to (e&eb) the trapped ion; periodic excitation of the quantum dot generates a stream of single photons that are channeled

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

periodic excitation of the quantum dot generates a stream of single photons that are channeled into an optical fiber and sent 25 meters away to (e&eb) the trapped ion

G_{20} : Category one of periodic excitation of the quantum dot generates a stream of single photons that are channeled into an optical fiber and sent 25 meters; trapped ion

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of trapped ion ; periodic excitation of the quantum dot generates a stream of single photons that are channeled into an optical fiber and sent 25 meters

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

An electron in the ion is prepared in a state such that it can absorb (e) a single photon from (e) the quantum dot.

absorption leads to (eb) an electron excitation that the researchers are able to detect

G_{24} : Category one of single photon from the quantum dot

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of electron in the ion is prepared in a state

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

absorption leads to (eb) an electron excitation that the researchers are able to detect

G_{28} : Category one of absorption

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of electron excitation that the researchers are able to detect

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The experiment demonstrates that an excitation in a single quantum dot can be transferred to (e&eb) an excitation in a single ion by means of (e) a single photon

G_{32} : Category one of excitation in a single quantum dot; excitation in a single ion by means of (e) a single photon

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of excitation in a single ion by means of (e) a single photon; excitation in a single quantum dot

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

excitation in a single quantum dot can be transferred to an excitation in a single ion by means of (e) a single photon

G_{36} : Category one of single photon; excitation in a single quantum dot can be transferred to an excitation in a single ion

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of excitation in a single quantum dot can be transferred to an excitation in a single ion; single photon

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

A great advantage of this type of "single-to-single" interface is that it provides (eb) the starting infrastructure for exchanging quantum information between a single quantum dot and (e&eb) a single ion

G_{40} : Category one of "single-to-single" interface

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of starting infrastructure for exchanging quantum information between a single quantum dot and (e&eb) a single ion

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

great advantage of this type of "single-to-single" interface is that it provides the starting infrastructure for exchanging quantum information between a single quantum dot and (e&eb) a single ion

G_{44} : Category one of great advantage of this type of "single-to-single" interface is that it provides the starting infrastructure for exchanging quantum information between a single quantum dot; single ion

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of single ion ;great advantage of this type of "single-to-single" interface is that it provides the starting infrastructure for exchanging quantum information between a single quantum dot

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients
Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a_{28}')^{(5)} & +(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad 79$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a_{29}')^{(5)} & +(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{29} \quad 80$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a_{30}')^{(5)} & +(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{30} \quad 81$$

$Where$ $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ $are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ $are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad (93)$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{matrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{matrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{matrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\big((G_{19}), t\big) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(UUUUUU) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(VVVVVV) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(WWWWWW) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(XXXXXX)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(YYYYYY) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(ZZZZZZ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\widehat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(z) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0 \quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$

$G_{20}^{(2)}|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t} \leq \frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)})\right) \quad (214)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (216)

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\}ds_{(20)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(3)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: (217)

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)}G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a_{22}')^{(3)}$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. (218)

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ (239)

it results

$$G_i(t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0 e^{\left(-(b_i')^{(5)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 275

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\,((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21},t)\right) - (a_{21}'')^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(z) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY EIGHT

Quantum State Can Be Transferred Between The Quantum Dot And The Ion

INTRODUCTION—VARIABLES USED

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark March 23, 2015• Physics 8, 27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion.

(1) In other words, if the researchers detect the ion in (eb) an excited state, they know the spin state of the electron in (eb) the quantum dot.
(2) The photonic link between a quantum dot and (e&eb) an ion, developed in this work, can, in principle, be extended to (eb)_ hybrid quantum networks [4], where the strength of each system can be combined to gain (eb) new functionalities.
(3) For instance, solid-state systems can be used as (=) scalable and fast processing units, while atomic systems can serve as (=) long-lived quantum memories.
(4) But physicists are still in the early stages of developing a hybrid network technology. The first challenge to overcome is (=) the low transfer efficiency between the dot and the ion. In the experiments of Atatüre, Köhl, and their colleagues, only 5 out of 1 million photons from the quantum dot excite (e&eb) the ion. Photon loss occurs at (eb) every step in the transfer, but various techniques exist to reduce it.
(5) Another challenge is demonstrating that a quantum state can be transferred between the quantum dot and (e&eb) the ion, and that the two objects can be entangled [5]. We should also be able to send information in the reverse direction, from an ion to (e&eb) a quantum dot.

NOTATION

Module One

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark March 23, 2015• Physics 8, 27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion.

(1) This, however, is yet to be demonstrated. **Atatüre, Köhl, and their colleagues** have made a preliminary step forward towards this goal by showing that the spin of an electron in the quantum dot is correlated with (e&eb) the excitation transfer.
(2) When an excitation laser is turned on (eb) it can only excite the dot if (e) the electron has its spin up.
(3) This excitation is then transferred to (e&eb) an ion and detected, exactly as in the first experiment.
(4) If the electron's spin is down, however, (eb) the laser can't excite the dot, (eb) so no photon is transferred to the ion.

This, however, is yet to be demonstrated. **Atatüre, Köhl, and their colleagues** have made a preliminary step forward towards this goal by showing that the spin of an electron in the quantum dot is correlated with (e&eb) the excitation transfer

G_{13} : Category one of spin of an electron in the quantum dot; excitation transfer

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of excitation transfer; spin of an electron in the quantum dot

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

In this experiment, the quantum dot is first prepared to have one excess electron, and the spin of this electron is controlled to be (=) either up or down

G_{16} : Category one of quantum dot is first prepared to have one excess electron, and the spin of this electron is controlled

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of either up or down

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

When an excitation laser is turned on it can only excite the dot if (e) the electron has its spin up

G_{20} : Category one of electron has its spin up

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of excitation laser is turned on it can only excite the dot

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

excitation is then transferred to (e&eb) an ion and detected, exactly as in the first experiment

G_{24} : Category one of excitation is then transferred; ion and detected, exactly as in the first experiment

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of ion and detected, exactly as in the first experiment ;excitation is then transferred

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

In other words, if the researchers detect the ion in an excited state, they know the spin state of the electron in (eb) the quantum dot

G_{28} : Category one of researchers detect the ion in an excited state

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of they know the spin state of the electron in the quantum dot

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The photonic link between a quantum dot and (e&eb) an ion, developed in this work, can, in principle, be extended to (eb)_ hybrid quantum networks [4], where the strength of each system can be combined to gain (eb) new functionalities

G_{32} : Category one of photonic link between a quantum dot in hybrid quantum networks where the strength of each system can be combined to gain (eb) new functionalities; ion in hybrid quantum networks

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of in hybrid quantum networks; photonic link between a quantum dot in hybrid quantum networks where the strength of each system can be combined to gain (eb) new functionalities

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

For instance, solid-state systems can be used as (=) scalable and fast processing units, while

atomic systems can serve as (=) long-lived quantum memories

G_{36} : Category one of solid-state systems can be used

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of scalable and fast processing units

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

atomic systems can serve as (=) long-lived quantum memories

G_{40} : Category one of atomic systems can serve

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of long-lived quantum memories

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

But physicists are still in the early stages of developing a hybrid network technology. The first challenge to overcome is (=) the low transfer efficiency between the dot and the ion. In the experiments of Atatüre, Köhl, and their colleagues, only 5 out of 1 million photons from the quantum dot excite (e&eb) the ion.

Photon loss occurs at (eb) every step in the transfer, but various techniques exist to reduce it

G_{44} : Category one of Photon loss occurs

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of every step in the transfer, but various techniques exist to reduce it

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$

$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for

category 1, 2 and 3

$+(a_{16}'')^{(2,2,)}(T_{17},t)$, $+(a_{17}'')^{(2,2,)}(T_{17},t)$, $+(a_{18}'')^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,)}(T_{21},t)$, $+(a_{21}'')^{(3,3,)}(T_{21},t)$, $+(a_{22}'')^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8)}(T_{41},t)$ $+(a_{41}'')^{(8,8)}(T_{41},t)$ $+(a_{42}'')^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b_{13}')^{(1)}\left[-(b_{13}'')^{(1)}(G,t)\right] & \left[-(b_{16}'')^{(2,2,)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b_{36}'')^{(7,7,)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8)}(G_{43},t)\right] & \left[-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b_{14}')^{(1)}\left[-(b_{14}'')^{(1)}(G,t)\right] & \left[-(b_{17}'')^{(2,2,)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b_{37}'')^{(7,7,)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8)}(G_{43},t)\right] & \left[-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b_{15}')^{(1)}\left[-(b_{15}'')^{(1)}(G,t)\right] & \left[-(b_{18}'')^{(2,2,)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b_{38}'')^{(7,7,)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8)}(G_{43},t)\right] & \left[-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad (60)$$

Where $-(b_{13}'')^{(1)}(G,t)$, $-(b_{14}'')^{(1)}(G,t)$, $-(b_{15}'')^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,)}(G_{19},t)$, $-(b_{17}'')^{(2,2,)}(G_{19},t)$, $-(b_{18}'')^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for

category 1, 2 and 3
$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)}\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{c}(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)}\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)}\end{array}\right]G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category $1, 2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

are fourth detrition coefficients for category $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

are fifth detrition coefficients for category $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category $1,2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

are seventh detrition coefficients for category $1,2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

are eighth detrition coefficients for category $1,2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \quad +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \quad +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) \quad +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \qquad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)} & -(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)} & -(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)} & -(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{c} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: (129)

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(AAAAAAA) \qquad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38 \quad (131)$$

(BBBBBBB) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(CCCCCCC) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(DDDDDDD)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(EEEEEEE) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(FFFFFFF) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(aa) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 250

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t), t(t), t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right| \; e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}} \left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)}; \; (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t} \; and \; (\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2 \; and \; \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45} \; and \; G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t} \right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right.$ 312

$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} [e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}] + G_{18}^0 e^{-(a'_{18})^{(2)} t} \right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t} \right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(aa) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{*^*}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY NINE

Mølmer–Sørensen-Type Gate Operation Entangling Ions

INTRODUCTION—VARIABLES USED

Letter Nature Physics 4, 463 - 466 (2008) Published online: 27 April 2008 | doi: 10.1038/nphys961: Towards fault-tolerant quantum computing with trapped ions Jan Benhelm, Gerhard Kirchmair, Christian F. Roos & Rainer Blatt

(1) Today, ion traps are among the most promising physical systems for constructing a quantum device harnessing the computing power inherent in (eb) the laws of quantum physics1, 2. For the implementation of arbitrary operations, a quantum computer requires (e) a universal set of quantum logic gates.

(2) As in classical models of computation, quantum error correction techniques3, 4 enable (eb) rectification of small imperfections in gate operations, thus enabling (eb) perfect computation in the presence of noise.

(3) For fault-tolerant computation5, it is believed that error thresholds ranging between 10−4 and 10−2 will be required—depending on the noise model and the computational overhead for (e) realizing the quantum gates6, 7, 8—but so far all experimental implementations have fallen short of these requirements.

(4) Here, authors report on a Mølmer–Sørensen-type gate operation9, 10 entangling ions with (e&eb) fidelity of 99.3(1) %.

(5) The gate is carried out on (e&eb) a pair of qubits encoded in (eb) two trapped calcium ions using (e) an amplitude-modulated laser beam interacting with (e&eb) both ions at the same time.

(6) A robust gate operation, mapping separable states onto (e&eb) maximally entangled states is achieved by (e) adiabatically switching the laser–ion coupling on and off.

(7) Authors analyse the performance of a single gate and concatenations of (e) up to 21 gate operations

NOTATION

Module One

Another challenge is demonstrating that a quantum state can be transferred between the quantum dot and (e&eb) the ion, and that the two objects can be entangled [5].

We should also be able to send information in the reverse direction, from an ion to (e&eb) a quantum dot.

G_{13} : Category one of quantum state can be transferred between the quantum dot; ion, and that the two objects can be entangled [5].

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of ion, and that the two objects can be entangled [5]. ;quantum state can be transferred between the quantum dot

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Today, ion traps are among the most promising physical systems for constructing a quantum device harnessing the computing power inherent in (eb) the laws of quantum physics1, 2.

For the implementation of arbitrary operations, a quantum computer requires (e) a universal set of quantum logic gates

G_{16} : Category one of universal set of quantum logic gates

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantum computer

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

As in classical models of computation, quantum error correction techniques3, 4 enable (eb) rectification of small imperfections in gate operations, thus enabling (eb) perfect computation in the presence of noise

G_{20} : Category one of quantum error correction techniques3

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of rectification of small imperfections in gate operations, thus enabling (eb) perfect computation in the presence of noise

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

As in classical models of computation, quantum error correction techniques3, 4 enable rectification of small imperfections in gate operations, thus enabling (eb) perfect computation in the presence of noise

G_{24} : Category one of quantum error correction techniques3, 4 enable rectification of small imperfections in gate operations,

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of perfect computation in the presence of noise

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Here, authors report on a Mølmer–Sørensen-type gate operation9, 10 entangling ions with (e&eb) fidelity of 99.3(1) %.

G_{28} : Category one of Mølmer–Sørensen-type gate operation9, 10 entangling ions; fidelity of 99.3(1) %.

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of fidelity of 99.3(1) %.; Mølmer–Sørensen-type gate operation9, 10 entangling ions

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The gate is carried out on (e&eb) a pair of qubits encoded in (eb) two trapped calcium ions using (e) an amplitude-modulated laser beam interacting with (e&eb) both ions at the same time

G_{32} : Category one of gate is carried out; pair of qubits encoded in (eb) two trapped calcium ions using (e) an amplitude-modulated laser beam interacting with (e&eb) both ions at the same time

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of pair of qubits encoded in (eb) two trapped calcium ions using (e) an amplitude-modulated laser beam interacting with (e&eb) both ions at the same time; gate is carried out

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

gate is carried out on a pair of qubits encoded in two trapped calcium ions using (e) an amplitude-modulated laser beam interacting with (e&eb) both ions at the same time

G_{36} : Category one of amplitude-modulated laser beam interacting with (e&eb) both ions at the same time

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of gate is carried out on a pair of qubits encoded in two trapped calcium ions

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

gate is carried out on a pair of qubits encoded in two trapped calcium ions using an amplitude-modulated laser beam interacting with (e&eb) both ions at the same time

G_{40} : Category one of gate is carried out on a pair of qubits encoded in two trapped calcium ions using an amplitude-modulated laser beam ; both ions at the same time

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of both ions at the same time; gate is carried out on a pair of qubits encoded in two trapped calcium ions using an amplitude-modulated laser beam

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

A robust gate operation, mapping separable states onto (e&eb) maximally entangled states is achieved by (e) adiabatically switching the laser–ion coupling on and off

G_{44} : Category one of robust gate operation, mapping separable states; maximally entangled states is achieved by (e) adiabatically switching the laser–ion coupling on and off

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of maximally entangled states is achieved by (e) adiabatically switching the laser–ion coupling on and off ;robust gate operation, mapping separable states

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{matrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{matrix}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{matrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{matrix}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{matrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{matrix}\right]G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{matrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{matrix}\right]T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients

for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{28}')^{(5)} & +(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) \quad +(a_{32}'')^{(6,6,6)}(T_{33},t) \\ +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t) \\ +(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a_{29}')^{(5)} & +(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) \quad +(a_{33}'')^{(6,6,6)}(T_{33},t) \\ +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t) \\ +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{30}')^{(5)} & +(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) \quad +(a_{34}'')^{(6,6,6)}(T_{33},t) \\ +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t) \\ +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{30} \quad (81)$$

Where $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation

coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}-(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad 82$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)}-(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad 83$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)}-(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt}=(a_{32})^{(6)}G_{33} \quad 85$$

$$-\begin{bmatrix}(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t)\\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad 90$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,5)}(G_{31},t)}$ $are\ second\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{24}'')^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4,4,4,)}(G_{27},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}, \boxed{(a_{37}'')^{(7)}(T_{37},t)}, \boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad (97)$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \quad (98)$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: (99)

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: (100)

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: (101)

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\left((G_{19}), t\right)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \qquad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty} (b_i'')^{(4)}\big((G_{27}), t\big) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}\big[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}\big] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\big((G_{35}), t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(GGGGGGG) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(HHHHHHH) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(IIIIIII) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(JJJJJJJ)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(KKKKKKK) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(LLLLLLL) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}, (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(bb) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\widehat{P}_{13})^{(1)}$ and $(\widehat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)} - G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t} \leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ and $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ and $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205

The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right)=(b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}<1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a_{20}')^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a_{20}'')^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a_{20}')^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t} \; and \; (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2 \; and \; \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21} \; and \; G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21} \; or \; G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20} \; is$ bounded, from below, the same property holds for $G_{21} \; and \; G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t), t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}), (\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}), (\widetilde{T_{27}})\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)}\left|G_{25}^{(1)} - G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)} +$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t} \leq \tag{226}$$

$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ and $(b''_i)^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 228

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(4)}-(a''_i)^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(4)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1-e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}), (\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a_{28}')^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right) - (a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)}t} \leq \frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right) d\left(((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)})\right) \quad (237)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(5)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a'_{30})^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. (241)

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. (242)

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}})\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{32})^{(6)}\left|G_{33}^{(1)} - G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)} +$

$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)} - G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} +$

$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}} +$

$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\} ds_{(32)}$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}t} \leq \frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right) \quad (248)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. (252)

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\big((G_{35})(t),t\big) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t),t(t),t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\tilde{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})}$: $\left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)}-G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)}-G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on (G_{39})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)} - G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)} - G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)} (\widehat{k}_{40})^{(8)}\right) d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ *and* $(\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)} - (a''_i)^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

279 A

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(\left((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$$G_{45}\le\left((\widehat{M}_{44})^{(9)}\right)_2=G_{45}^0+2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$

$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t} \right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right.$ 286

$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})} [e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}] + G_{15}^0 e^{-(a'_{15})^{(1)} t} \right)$

$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$ 287

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$ 288

$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le$ 289

$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t} \right] + T_{15}^0 e^{-(R_2)^{(1)} t}$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t} \right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right.$ 312

$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} [e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}] + G_{18}^0 e^{-(a'_{18})^{(2)} t} \right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t} \right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t}-e^{-(a'_{22})^{(3)}t}\right]+\ G_{22}^{0}e^{-(a'_{22})^{(3)}t})$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le\frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t}-e^{-(b'_{22})^{(3)}t}\right]+T_{22}^{0}e^{-(b'_{22})^{(3)}t}\le T_{22}(t)\le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t}-e^{-(R_2)^{(3)}t}\right]+T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)},(S_2)^{(3)},(R_1)^{(3)},(R_2)^{(3)}$:- 328

Where $(S_1)^{(3)}=(a_{20})^{(3)}(m_2)^{(3)}-(a'_{20})^{(3)}$

$(S_2)^{(3)}=(a_{22})^{(3)}-(p_{22})^{(3)}$

$(R_1)^{(3)}=(b_{20})^{(3)}(\mu_2)^{(3)}-(b'_{20})^{(3)}$

$(R_2)^{(3)}=(b'_{22})^{(3)}-(r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$:

$(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)}\le-(a'_{24})^{(4)}+(a'_{25})^{(4)}-(a''_{24})^{(4)}(T_{25},t)+(a''_{25})^{(4)}(T_{25},t)\le-(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)}\le-(b'_{24})^{(4)}+(b'_{25})^{(4)}-(b''_{24})^{(4)}\left((G_{27}),t\right)-(b''_{25})^{(4)}\left((G_{27}),t\right)\le-(\tau_1)^{(4)}$

Definition of $(v_1)^{(4)},(v_2)^{(4)},(u_1)^{(4)},(u_2)^{(4)},v^{(4)},u^{(4)}$: 329

By $(v_1)^{(4)}>0$, $(v_2)^{(4)}<0$ and respectively $(u_1)^{(4)}>0$, $(u_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2+(\sigma_1)^{(4)}v^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_1)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)}>0$, $(\bar{v}_2)^{(4)}<0$ and respectively $(\bar{u}_1)^{(4)}>0$, $(\bar{u}_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2+(\sigma_2)^{(4)}v^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_2)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$

Definition of $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)},\ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t} \tag{332}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \tag{333}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \tag{334}$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \tag{335}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \tag{336}$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$

$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, ***if*** $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, ***if*** $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, ***if*** $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}\ , \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}\ \text{where}\ (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{40}(t) \leq G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{41}(t) \leq \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \leq G_{42}(t) \leq \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \leq T_{40}(t) \leq T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \leq T_{40}(t) \leq \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \leq T_{42}(t) \leq \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G^0_{44} e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t}\Big] + G^0_{46} e^{-(S_2)^{(9)} t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G_{44}^{0}}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+\ G_{46}^{0}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^{0}e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^{0}e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)}=(a_{44})^{(9)}(m_2)^{(9)}-(a'_{44})^{(9)}$

$(S_2)^{(9)}=(a_{46})^{(9)}-(p_{46})^{(9)}$

$(R_1)^{(9)}=(b_{44})^{(9)}(\mu_2)^{(9)}-(b'_{44})^{(9)}$

$(R_2)^{(9)}=(b'_{46})^{(9)}-(r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a'_{13})^{(1)}-(a'_{14})^{(1)}+(a''_{13})^{(1)}(T_{14},t)\right)-\ (a''_{14})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\le\frac{dv^{(1)}}{dt}\le-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\ge\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)}\le\nu^{(1)}(t)\le(\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$, we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)\le\frac{d\nu^{(6)}}{dt}\le-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)},(\nu_0)^{(6)}$:-

For $0<\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}<(\nu_1)^{(6)}<(\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t)\ge\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}\quad,\quad\boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)}\le\nu^{(6)}(t)\le(\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t)\le\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)}\le\nu^{(6)}(t)\le(\bar{\nu}_1)^{(6)}$

If $0<(\nu_1)^{(6)}<(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}<(\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)}\le\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}\le\nu^{(6)}(t)\le$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\le(\bar{\nu}_1)^{(6)}$$

If $0<(\nu_1)^{(6)}\le(\bar{\nu}_1)^{(6)}\le\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)}\le\nu^{(6)}(t)\le\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\le(\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)}\le\nu^{(6)}(t)\le(m_1)^{(6)},\quad\boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)}\le u^{(6)}(t)\le(\mu_1)^{(6)},\quad\boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .
Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then
$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \le \frac{d\nu^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \le \frac{d\nu^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \qquad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \qquad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \qquad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \qquad 430$$

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$$

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0\ ,$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$$

$$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$$

$$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0\ ,$$

$$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$$

$$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$$

$$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0\ ,$$

$$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \qquad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \qquad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \qquad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \qquad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(bb) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T_{14}^* :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^{*} :- 494

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^{*} for which $f(\mathrm{T}_{17}^{*})=0$. With this value , we obtain from the three first equations

$$G_{16}=\frac{(a_{16})^{(2)}\mathrm{G}_{17}}{[(a_{16}')^{(2)}+(a_{16}'')^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad G_{18}=\frac{(a_{18})^{(2)}\mathrm{G}_{17}}{[(a_{18}')^{(2)}+(a_{18}'')^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 495$$

Definition and uniqueness of T_{21}^{*} :- 496

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^{*} for which $f(T_{21}^{*})=0$. With this value , we obtain from the three first equations

$$G_{20}=\frac{(a_{20})^{(3)}G_{21}}{[(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}=\frac{(a_{22})^{(3)}G_{21}}{[(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21}^{*})]}$$

Definition and uniqueness of T_{25}^{*} :- 497

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^{*} for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a_{24}')^{(4)}+(a_{24}'')^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a_{26}')^{(4)}+(a_{26}'')^{(4)}(T_{25}^{*})]}$$

Definition and uniqueness of T_{29}^{*} :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^{*} for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a_{28}')^{(5)}+(a_{28}'')^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a_{30}')^{(5)}+(a_{30}'')^{(5)}(T_{29}^{*})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a_{32}')^{(6)}+(a_{32}'')^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a_{34}')^{(6)}+(a_{34}'')^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]}$, $G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T^*_{41} for which $f(T^*_{41}) = 0$. With this value , we obtain from the three first equations

$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]}$, $G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]}$, $G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$

$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$

$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$

$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is

a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G^*_{14} given by $\varphi(G^*) = 0$, T^*_{14} given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G^*_{17} given by $\varphi((G_{19})^*) = 0$, T^*_{17} given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G^*_{21} given by $\varphi((G_{23})^*) = 0$, T^*_{21} given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G^*_{25} given by $\varphi(G_{27}) = 0$, T^*_{25} given by $f(T^*_{25}) = 0$ and

$$G^*_{24} = \frac{(a_{24})^{(4)}G^*_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G^*_{26} = \frac{(a_{26})^{(4)}G^*_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*})=0$, T_{41}^{*} given by $f(T_{41}^{*})=0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^* \mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}} (T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j} ((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} G_{16}^* \mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)} G_{17}^* \mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)} G_{18}^* \mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)} T_{16}^* \mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)} T_{17}^* \mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)} T_{18}^* \mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}} (T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j} ((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)} G_{20}^* \mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)} G_{21}^* \mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)} G_{22}^* \mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)} T_{20}^* \mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)} T_{21}^* \mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}')^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)} T_{22}^* \mathbb{G}_j\right) \tag{546}$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \ , \ \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(9)}$ *and* $(b''_i)^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\,(G_{47})^*\,) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\,(a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left((\lambda)^{(1)}\right)^2+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\Big)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\Big)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\Big)$$

$$\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

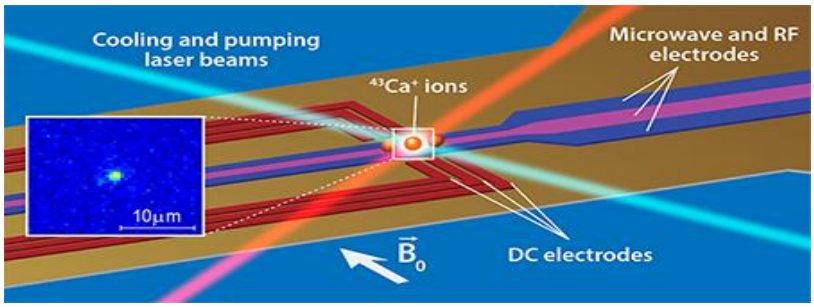
SECTION THIRTY

Trapped Ions Make Impeccable Qubits

INTRODUCTION—VARIABLES USED

Viewpoint: Trapped Ions Make Impeccable Qubits Jungsang Kim, Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA November 24, 2014• Physics 7, 119

(1) Qubits based on trapped ions can be prepared and manipulated with (e&eb) record-breaking accuracy, offering (eb) a promising scalable platform for quantum computing.



APS/Alan Stonebraker

(2) Figure 1: Qubits made of a trapped 43Ca+ ion. RF and dc electrodes provide (eb) a trapping field for the ions, which are cooled by (e) laser beams (blue) to microkelvin temperatures. A combination of laser pumping and microwave signals can deterministically prepare the

(3) The realization, two decades ago, that quantum mechanics can be a powerful resource to speed up important computational tasks [1] led to intense research efforts to find (eb) adequate physical systems for quantum computation. One of the hurdles to a viable technology is the requirement to prepare, manipulate, and measure quantum bits (qubits) with near perfect accuracy: Imperfect control leads to (eb) errors that can accumulate over the computation process.

(4) Techniques like quantum error correction and fault-tolerant designs can, in principle, overcome (e) these errors.

(5) But these strategies can be successful only if (e) the error probabilities are lower than (eb) a threshold value.

(6) Quantum error correction and fault-tolerant designs also increase the complexity of the required quantum hardware, since (e) they require additional qubits.

(7) Recent calculations [2] suggest that an error probability of less than 1% would enable (eb) fault-tolerant codes, and that lower error probabilities dramatically decrease (e) the number of qubits required for such codes.

Nature 438, 833-836 (8 December 2005) | doi: 10.1038/nature04315; Received 23 August 2005; Accepted 13 October 2005 Storage and retrieval of single photons transmitted between remote quantum memories T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1

(1) An elementary quantum network operation involves (e&eb) storing a qubit state in an atomic quantum memory node, and then retrieving and transporting (e&eb) the information through (e&eb) a single photon excitation to a remote quantum memory node for further storage or analysis.

(2) Implementations of quantum network operations are thus conditioned on the ability to (e) realize matter-to-light and/or light-to-matter quantum state mappings

(3) Here **T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1** report the generation, transmission, storage and retrieval of single quanta using (e) two remote atomic ensembles.

(4) A single photon is generated from (e) a cold atomic ensemble at one site 1, and is directed to (e&eb) another site through 100 metres of optical fibre.

(5) The photon is then converted into (e&eb) a single collective atomic excitation using (e) a dark-state polariton approach2.

(6) After a programmable storage time, the atomic excitation is converted back into (e&eb) a single photon.

(7) This is demonstrated experimentally, for a storage time of 0.5 microseconds, by measurement of an anti-correlation parameter. Storage times exceeding ten microseconds are observed by intensity cross-correlation measurements. This storage period is two orders of magnitude longer than the time required (e) for achieving conversion between photonic and (e&eb) atomic quanta.

(8) The controlled transfer of single quanta between remote quantum memories constitutes (e) an important step towards distributed quantum networks.

NOTATION

Module One

A robust gate operation, mapping separable states onto maximally entangled states is achieved by (e) adiabatically switching the laser–ion coupling on and off

G_{13} : Category one of adiabatically switching the laser–ion coupling on and off

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of robust gate operation, mapping separable states onto maximally entangled states is achieved

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Qubits made of a trapped 43Ca+ ion. RF and dc electrodes provide (eb) a trapping field for the ions, which are cooled by (e) laser beams (blue) to microkelvin temperatures.

A combination of laser pumping and microwave signals can deterministically prepare the

G_{16} : Category one of Qubits made of a trapped 43Ca+ ion. RF and dc electrodes

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of trapping field for the ions, which are cooled by laser beams (blue) to microkelvin temperatures

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The realization, two decades ago, that quantum mechanics can be a powerful resource to speed up important computational tasks [1] led to intense research efforts to find (eb) adequate physical systems for quantum computation.

One of the hurdles to a viable technology is the requirement to prepare, manipulate, and measure quantum bits (qubits) with near perfect accuracy:

Imperfect control leads to (eb) errors that can accumulate over the computation process

G_{20} : Category one of Imperfect control

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of errors that can accumulate over the computation process

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Techniques like quantum error correction and fault-tolerant designs can, in principle, overcome (e) these errors

G_{24} : Category one of errors

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of quantum error correction and fault-tolerant designs

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

But these strategies can be successful only if (e) the error probabilities are lower than (eb) a threshold value

G_{28} : Category one of error probabilities; threshold value

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of threshold value; error probabilities

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Quantum error correction and fault-tolerant designs also increase the complexity of the required quantum hardware, since (e) they require additional qubits

G_{32} : Category one of requirement of quantum hardware of extra qubits

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Quantum error correction and fault-tolerant designs also increase the complexity of the required quantum hardware

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Recent calculations [2] suggest that an error probability of less than 1% would enable (eb) fault-tolerant codes, and that lower error probabilities dramatically decrease (e) the number of qubits required for such code

G_{36} : Category one of number of qubits required for such code

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of error probability of less than 1% would enable fault-tolerant codes, and that lower error probabilities

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

An elementary quantum network operation involves (e&eb) storing a qubit state in an atomic quantum memory node, and then retrieving and transporting (e&eb) the information through (e&eb) a single photon excitation to a remote quantum memory node for further storage or analysis

G_{40} : Category one of storing a qubit state in an atomic quantum memory node, and then retrieving and transporting the information; single photon excitation to a remote quantum memory node for further storage or analysis

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of single photon excitation to a remote quantum memory node for further storage or analysis; storing a qubit state in an atomic quantum memory node, and then retrieving and transporting the information

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

quantum network operations implementations are thus conditioned on the ability to realize matter-to-light and/or light-to-matter quantum state mappings

G_{44} : Category one of quantum network operations implementations; ability to realize matter-to-light and/or light-to-matter quantum state mappings

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of ability to realize matter-to-light and/or light-to-matter quantum state mappings; quantum network operations implementations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14}$ 5

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \quad +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \quad +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{15} \quad 57$$

Where $+(a''_{13})^{(1)}(T_{14},t)$, $+(a''_{14})^{(1)}(T_{14},t)$, $+(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) \quad +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) \quad +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) \quad -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) \quad -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) \quad -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{array}\right] T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21},t)+(a''_{16})^{(2,2,2)}(T_{17},t)+(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)+(a''_{40})^{(8,8,8,8)}(T_{41},t)+(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}+(a''_{21})^{(3)}(T_{21},t)+(a''_{17})^{(2,2,2)}(T_{17},t)+(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)+(a''_{41})^{(8,8,8,8)}(T_{41},t)+(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21},t)+(a''_{18})^{(2,2,2)}(T_{17},t)+(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)+(a''_{42})^{(8,8,8,8)}(T_{41},t)+(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficients for category 1, 2 and 3
$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3
$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3
$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*
$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*
$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*
$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*
$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*
$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*
$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 91$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 92$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b_{40}')^{(8)}\left[-(b_{40}'')^{(8)}(G_{43},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b_{41}')^{(8)}\left[-(b_{41}'')^{(8)}(G_{43},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b_{42}')^{(8)}\left[-(b_{42}'')^{(8)}(G_{43},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{36}'')^{(7)}(G_{39},t)$, $-(b_{37}'')^{(7)}(G_{39},t)$, $-(b_{38}'')^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}-(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}-(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}-(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \le \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \le (\hat{k}_{32})^{(6)}|T_{33} - \ T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(MMMMMMM) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(NNNNNNN) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(OOOOOOO) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(PPPPPPP)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(QQQQQQQ) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(RRRRRRR) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\widehat{M}_{40})^{(8)}t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}}\left(e^{(\tilde{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(cc) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}}\left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)}\left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}}\left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\ \ \text{for t}>0$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$

$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$ 214

$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\ \ \text{for t}>0$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\;e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\;(G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)} \quad 293$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad 294$$

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}$$ 335

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le$$ 336

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)} - (v_2)^{(6)}\right)t\right]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}\right)t\right]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(cc) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi(G_{39}{}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27})=0$, T_{25}^{*} given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY ONE

Laser-Pumping Techniques That Are Well Established For Atomic Systems

INTRODUCTION—VARIABLES USED

Viewpoint: Trapped Ions Make Impeccable Qubits Jungsang Kim, Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA November 24, 2014• Physics 7, 119

(1) The quality of qubit manipulation in a number of physical systems has dramatically improved in the

past few years [3, 4], raising hopes that a quantum computer, at a large enough scale to carry out meaningful computations, might be within reach. Now, Thomas Harty at the University of Oxford, UK, and colleagues [5] are reporting an important contribution to this goal with the demonstration that qubits consisting of trapped 43Ca+ ions can be manipulated with (e&eb) record high fidelities (in quantum information theory, fidelity is a measure of (e) the "closeness" of two quantum states). Their experiments suggest trapped-ion schemes could potentially provide (eb) the basic fundamental building blocks of a universal quantum computer.

(2) Trapped atomic ions are one of the leading candidate systems to construct a robust quantum computer: they provide a stable and well-isolated quantum system and the strong Coulomb forces between the ions can be used to realize logical gate operations by (e) coupling different qubits

(3) In the last decade, researchers have demonstrated trapped-ion qubits with long coherence times [6], high-fidelity state preparation and readout [7], and single- and two-qubit logic gate operations with low error rates [3, 8]. Yet each of these properties was demonstrated individually in different systems. The work of Harty and colleagues now poses a combined improvement on all of these fronts in a single experimental system. In the authors' scheme (see Fig. 1), a 43Ca+ ion is confined by (e) radiofrequency electric fields (a so-called "Paul trap") on (eb) the surface of a sapphire substrate.

(4) Electrodes connected to the structure provide (eb) the signals necessary for trapping (e&eb) the ions and driving changes to (e&eb) the qubit states.

(5) The choice of the 43Ca+ was crucial to the authors' results: With a modest applied magnetic field (146 gauss), the energy-level separation between the two hyperfine ground-state sublevels of the ion is sufficiently large to become (=) insensitive to small magnetic field fluctuations, abundant in (eb) a laboratory environment, that affected (e&eb) the performance of previous trapped-ion schemes.

(6) The two hyperfine states of 43Ca+ are thus ideal for representing (eb) the $|0\rangle$ and $|1\rangle$ qubit states (analogous to "0" and "1" in classical computing systems).

(7) A crucial property of a qubit is (=) its coherence time (how long a quantum superposition of $|0\rangle$and $|1\rangle$ states can be maintained).

(8) Unlike previous experiments with 40Ca+ ions, here the hyperfine levels' stability led to (eb) a measured coherence time of about 50 seconds, marking a record for an atomic-ion qubit unshielded from (e) fluctuating background magnetic fields.

(9) To "prepare" the ion's electron in a well-defined initial state, the authors use (e) **laser-pumping techniques that are well established for atomic systems.**

NOTATION

Module One

The quality of qubit manipulation in a number of physical systems has dramatically improved in the past few years [3, 4], raising hopes that a quantum computer, at a large enough scale to carry out meaningful computations, might be within reach.

Now, Thomas Harty at the University of Oxford, UK, and colleagues [5] are reporting an important contribution to this goal with the demonstration that qubits consisting of trapped 43Ca+ ions can be manipulated with record high fidelities (in quantum information theory, fidelity is (=) a measure of the "closeness" of two quantum states).

Their experiments suggest trapped-ion schemes could potentially provide (eb) the basic fundamental

building blocks of a universal quantum computer

G_{13} : Category one of, fidelity

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of measure of the "closeness" of two quantum states).

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Nature 438, 833-836 (8 December 2005) | doi: 10.1038/nature04315; Received 23 August 2005; Accepted 13 October 2005 Storage and retrieval of single photons transmitted between remote quantum memories T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1

(1) Here **T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1** report the generation, transmission, storage and retrieval of single quanta using (e) two remote atomic ensembles. A single photon is generated from (e) a cold atomic ensemble at one site 1, and is directed to (e&eb) another site through 100 metres of optical fibre.

(2) The photon is then converted into (e&eb) a single collective atomic excitation using (e) a dark-state polariton approach2.

(3) After a programmable storage time, the atomic excitation is converted back into (e&eb) a single photon.

(4) This is demonstrated experimentally, for a storage time of 0.5 microseconds, by measurement of an anti-correlation parameter. Storage times exceeding ten microseconds are observed by intensity cross-correlation measurements. This storage period is two orders of magnitude longer than the time required (e) for achieving conversion between photonic and (e&eb) atomic quanta.

(5) The controlled transfer of single quanta between remote quantum memories constitutes (e) an important step towards distributed quantum networks.

Here **T. Chanelière1, D. N. Matsukevich1, S. D. Jenkins1, S.-Y. Lan1, T. A. B. Kennedy1 & A. Kuzmich1** report the generation, transmission, storage and retrieval of single quanta using (e) two remote atomic ensembles. A single photon is generated from (e) a cold atomic ensemble at one site 1, and is directed to (e&eb) another site through 100 metres of optical fibre

The photon is then converted into (e&eb) a single collective atomic excitation using (e) a dark-state polariton approach2

G_{16} : Category one of photon; single collective atomic excitation using (e) a dark-state polariton approach2

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of single collective atomic excitation using (e) a dark-state polariton approach2;photon

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

atomic excitation after a programmable storage time is converted back into (e&eb) a single photon

G_{20} : Category one of atomic excitation after a programmable storage time; single photon (many photons could be there for many such experiments)

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of single photon; atomic excitation after a programmable storage time

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Electrodes connected to the structure the signals necessary for trapping the ions and driving changes to (e&eb) the qubit states.

G_{24} : Category one of electrodes connected to the structure the signals necessary for trapping the ions; qubit states

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of qubit states; electrodes connected to the structure the signals necessary for trapping the ions

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The two hyperfine states of 43Ca+ are thus ideal for representing (eb) the |0⟩ and |1⟩ qubit states (analogous to “0” and “1” in classical computing systems)

G_{28} : Category one of two hyperfine states of 43Ca+ are thus ideal

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of |0⟩ and |1⟩ qubit states (analogous to “0” and “1” in classical computing systems

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

(1) A crucial property of a qubit is (=) its coherence time (how long a quantum superposition of |0⟩and |1⟩ states can be maintained).
(2) Unlike previous experiments with 40Ca+ ions, here the hyperfine levels' stability led to (eb) a measured coherence time of about 50 seconds, marking a record for an atomic-ion qubit unshielded from (e) fluctuating background magnetic fields.
(3) To "prepare" the ion's electron in a well-defined initial state, the authors use (e) **laser-pumping techniques that are well established for atomic systems.**

A crucial property of a qubit is (=) its coherence time (how long a quantum superposition of |0⟩and |1⟩ states can be maintained)

G_{32} : Category one of coherence time

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of quantum superposition of |0⟩and |1⟩ states can be maintained

T_{33} : Category two of

T_{34} : Category three of

Module seven

Unlike previous experiments with 40Ca+ ions, here the hyperfine levels' stability led to (eb) a measured coherence time of about 50 seconds, marking a record for an atomic-ion qubit unshielded from (e) fluctuating background magnetic fields

G_{36} : Category one of hyperfine levels' stability

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of measured coherence time of about 50 seconds, marking a record for an atomic-ion qubit unshielded from (e) fluctuating background magnetic fields

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Unlike previous experiments with 40Ca+ ions, here the hyperfine levels' stability led to a measured coherence time of about 50 seconds, marking a record for an atomic-ion qubit unshielded from (e) fluctuating background magnetic fields

G_{40} : Category one of fluctuating background magnetic fields

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of hyperfine levels' stability led to a measured coherence time of about 50 seconds, marking a record for an atomic-ion qubit unshielded

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

"prepare" the ion's electron in a well-defined initial state, the authors use (e) **laser-pumping techniques that are well established for atomic systems**

G_{44} : Category one of **laser-pumping techniques that are well established for atomic systems**

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of "prepare" the ion's electron in a well-defined initial state

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$

$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad (19)$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad (20)$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad (21)$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad (22)$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad (23)$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad (24)$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad (25)$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad (26)$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad (27)$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad (28)$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad (29)$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad (30)$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad (31)$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad (32)$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad (33)$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad (34)$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad (35)$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad (36)$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad (37)$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}),t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for

category 1, 2 and 3

$+(a_{16}'')^{(2,2,)}(T_{17},t)$, $+(a_{17}'')^{(2,2,)}(T_{17},t)$, $+(a_{18}'')^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,)}(T_{21},t)$, $+(a_{21}'')^{(3,3,)}(T_{21},t)$, $+(a_{22}'')^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8)}(T_{41},t)$ $+(a_{41}'')^{(8,8)}(T_{41},t)$ $+(a_{42}'')^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b_{13}')^{(1)}\left[-(b_{13}'')^{(1)}(G,t)\right] & \left[-(b_{16}'')^{(2,2,)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b_{36}'')^{(7,7,)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8)}(G_{43},t)\right] & \left[-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b_{14}')^{(1)}\left[-(b_{14}'')^{(1)}(G,t)\right] & \left[-(b_{17}'')^{(2,2,)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b_{37}'')^{(7,7,)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8)}(G_{43},t)\right] & \left[-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b_{15}')^{(1)}\left[-(b_{15}'')^{(1)}(G,t)\right] & \left[-(b_{18}'')^{(2,2,)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b_{38}'')^{(7,7,)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8)}(G_{43},t)\right] & \left[-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad (60)$$

Where $-(b_{13}'')^{(1)}(G,t)$, $-(b_{14}'')^{(1)}(G,t)$, $-(b_{15}'')^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,)}(G_{19},t)$, $-(b_{17}'')^{(2,2,)}(G_{19},t)$, $-(b_{18}'')^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for

category 1, 2 and 3
$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)}\end{array}\right]G_{16} \quad 61$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{c}(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)}\end{array}\right]G_{17} \quad 62$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{c}(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)}\end{array}\right]G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \quad \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category $1, 2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1,2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1,2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1,2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\,,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \quad +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \quad +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) \quad +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \qquad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$
Eighth augmentation coefficients
$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(SSSSSSS) \qquad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38 \qquad 131$$

(TTTTTTT) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(UUUUUUU) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(VVVVVVV)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(WWWWWWW) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(XXXXXXX) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - \ T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0,t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(dd) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$
Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$ 331

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$ 332

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(dd) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{*^*}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY TWO

Optical Excitation Would Bring Qubit To A Metastable State

INTRODUCTION—VARIABLES USED

Viewpoint: Trapped Ions Make Impeccable Qubits Jungsang Kim, Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA November 24, 2014• Physics 7, 119

(1) They first drive (eb) the electron to one of the hyperfine ground states of 43Ca+ by (e) shining a laser beam resonant with the atomic transition.
(2) From such a ground state, either the |0⟩ or the |1⟩ qubit state can be prepared using (e) adequate microwave pulses applied to (e&eb) the ion through the on-chip microwave electrodes.
(3) In order to "read" the qubit states with high accuracy, the researchers exploit the fact that, when excited by a sequence of optical pulses, (eb) only one of the two qubit states would fluoresce.
(4) If the qubit was a |0⟩ (eb) optical excitation would bring qubit to a metastable state, which would not (e) emit fluorescence upon application of (e&eb) a resonant laser beam.
(5) If instead the qubit was a |1⟩, (eb) the ion would not be transferred to the metastable state and resonant laser excitation would induce (eb) a fluorescence signal that can be imaged by (e) a photon detector (see inset of Fig. 1). The authors repeated the qubit preparation and readout 150,000 times, reporting (eb) an average error rate of only 0.07%.
(6) To carry out logic-gate operations, the qubit state can be coherently manipulated by (e&eb) exposing the ion to a 3.2-gigahertz microwave pulse that toggles (e&eb) the qubit between its two levels.
(7) The authors show that the phase and duration of the microwave pulse can be precisely controlled to achieve (eb) the desired operations with near-perfect accuracy.

NOTATION

Module One

They first drive (eb) the electron to one of the hyperfine ground states of 43Ca+ by (e) shining a laser beam resonant with the atomic transition

G_{13} : Category one of shining a laser beam resonant with the atomic transition

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of electron to one of the hyperfine ground states of 43Ca+

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

From such a ground state, either the |0⟩ or the |1⟩ qubit state can be prepared using (e) adequate microwave pulses applied to (e&eb) the ion through the on-chip microwave electrodes

G_{16} : Category one of adequate microwave pulses applied to (e&eb) the ion through the on-chip microwave electrodes

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of either the |0⟩ or the |1⟩ qubit state can be prepared

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

From such a ground state, either the |0⟩ or the |1⟩ qubit state can be prepared using adequate microwave pulses applied to (e&eb) the ion through the on-chip microwave electrodes

G_{20} : Category one of either the |0⟩ or the |1⟩ qubit state can be prepared using adequate microwave pulses; ion through the on-chip microwave electrodes

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of ion through the on-chip microwave electrodes; either the |0⟩ or the |1⟩ qubit state can be prepared using adequate microwave pulses

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In order to "read" the qubit states with high accuracy, the researchers exploit the fact that, when excited by a sequence of optical pulses, (eb) only one of the two qubit states would fluoresce.

G_{24} : Category one of excited by a sequence of optical pulses

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of only one of the two qubit states would fluoresce

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

If the qubit was a |0⟩ (eb) optical excitation would bring qubit to a metastable state, which would not (e) emit fluorescence upon application of (e&eb) a resonant laser beam.

G_{28} : Category one of qubit was a |0⟩

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of optical excitation would bring qubit to a metastable state, which would not (e) emit fluorescence upon application of (e&eb) a resonant laser beam.

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

If the qubit was a |0⟩ optical excitation would bring qubit to a metastable state, which would not (e) emit fluorescence upon application of (e&eb) a resonant laser beam

G_{32} : Category one of emit fluorescence upon application of a resonant laser beam

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of qubit was a |0⟩ optical excitation would bring qubit to a metastable state

T_{33} : Category two of

T_{34} : Category three of

Module seven

If instead the qubit was a |1⟩, (eb) the ion would not be transferred to the metastable state and resonant laser excitation would induce (eb) a fluorescence signal that can be imaged by (e) a photon detector (see inset of Fig. 1).

The authors repeated the qubit preparation and readout 150,000 times, reporting (eb) an average error rate of

only 0.07%.

All categories could be taken as equivalent

G_{36} : Category one of qubit was a |1⟩

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of ion would not be transferred to the metastable state and resonant laser excitation would induce (eb) a fluorescence signal that can be imaged by (e) a photon detector (see inset of Fig. 1)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

If instead the qubit was a |1⟩, the ion would not be transferred to the metastable state and resonant laser excitation would induce (eb) a fluorescence signal that can be imaged by (e) a photon detector (see inset of Fig. 1)

G_{40} : Category one of qubit was a |1⟩, the ion would not be transferred to the metastable state and resonant laser excitation

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of fluorescence signal that can be imaged by a photon detector (see inset of Fig. 1)

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

qubit state can be coherently manipulated by (e&eb) exposing the ion to a 3.2-gigahertz microwave pulse that toggles (e&eb) the qubit between its two levels

G_{44} : Category one of qubit state; exposing the ion to a 3.2-gigahertz microwave pulse that toggles the qubit between its two levels

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of exposing the ion to a 3.2-gigahertz microwave pulse that toggles the qubit between its two levels; qubit state

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a_{28}')^{(5)}+(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t)\\ +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)\\ +(a_{36}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad 79$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a_{29}')^{(5)}+(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t)\\ +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)\\ +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{29} \quad 80$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a_{30}')^{(5)}+(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t)\\ +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)\\ +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{30} \quad 81$$

$Where\ +(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ $are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
$And\ +(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ $are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad (93)$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\big((G_{19}), t\big) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(YYYYYYY) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(ZZZZZZZ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(AAAAAAAA) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(BBBBBBBB)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(CCCCCCCC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(DDDDDDDD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\widehat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)$, $T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(ee) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0\quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$

$G_{20}^{(2)} \left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_{+}$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ (239)

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{\left(-(b_i')^{(5)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t)=0$ *and* $T_i\,(t)=0$ 250

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0\quad\text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le$$ 259
$$\frac{1}{(\widehat{M}_{36})^{(7)}}((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)})d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad (269)$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote (270)

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results (271)

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ (272)

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad (273)$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. (274)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ (275)

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\,((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, $\boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, $\boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, $\boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, $\boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right. \quad 334$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,

and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \le \nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \le \nu^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(ee) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27})=0$, T_{25}^{*} given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY THREE

"Randomized Benchmarking"

INTRODUCTION—VARIABLES USED

Viewpoint: Trapped Ions Make Impeccable Qubits Jungsang Kim, Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA November 24, 2014• Physics 7, 119

(1) Following a gate-testing protocol known as "randomized benchmarking," the authors use the

microwave electrodes to drive (eb) a long sequence of single-qubit gate operations, comparing the final qubit state with (e&eb) the expected outcome.

(2) They report an average error probability of 1×10−6 for a single-qubit gate operation, which is more than an order of magnitude better than previous demonstrations [8]. These results, combined with the high-fidelity two-qubit gate operation reported by the same authors in a separate paper [9], make a convincing demonstration of a universal set of qubit operations that represent (eb) a high-fidelity record in any physical platform so far investigated.

(3) The reported combined accuracy in preparation, readout, and logical-gate operation satisfies (eb) the important benchmark of an error rate lower than the 1% threshold, beating it by (e) over an order of magnitude.

(4) Along with the impressive recent progress made in superconducting qubits [4], these results show that scientists seem to be homing in on achieving the qubit quality necessary for (e) constructing and operating a large-scale, fault-tolerant quantum information processing system.

(5) The next set of challenges lies in the ability to integrate all of these functionalities into a single experimental setup and to scale up the system to a larger number of qubits [10]. At such a pace of technology development, quantum computers, which seemed a remote possibility only a decade ago, might become a reality sooner than most of us anticipated. This research is published in Physical Review Letters.

References: (Please see the original article)

P. W. Shor, "Algorithms for Quantum Computation: Discrete Logarithms and Factoring," in Proceedings of the 35th Annual Symposium on Foundations of Computer Science (IEEE Computer Society Press, Los Alamitos, 1994); L. K. Grover, "Quantum Mechanics Helps in Searching for a Needle in a Haystack," Phys. Rev. Lett. 79, 325 (1997)

NOTATION

Module One

Following a gate-testing protocol known as "randomized benchmarking," the authors use the microwave electrodes to drive (eb) a long sequence of single-qubit gate operations, comparing the final qubit state with (e&eb) the expected outcome.

G_{13} : Category one of microwave electrodes **f**ollowing a gate-testing protocol known as "randomized benchmarking

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of long sequence of single-qubit gate operations, comparing the final qubit state with the expected outcome

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Randomized benchmarking of quantum gates E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland Phys. Rev. A 77, 012307 –

Published 8 January 2008

(1) A key requirement for scalable quantum computing is that elementary quantum gates can be implemented with sufficiently low error. One method for determining the error behavior of a gate implementation is to perform (e&eb) process tomography. However, standard process tomography is limited by errors in state preparation, measurement and one-qubit gates. Process tomography suffers from inefficient scaling with number of qubits and does not (e) detect adverse error-compounding when (e) gates are composed in long sequences.

(2) An additional problem is due to the fact that desirable error probabilities for scalable quantum computing are of the order of 0.0001 or lower. Experimentally proving such low errors is challenging. **E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland** describe a randomized benchmarking method that yields (eb) estimates of the computationally relevant errors without (e) relying on accurate state preparation and measurement. Since it involves long sequences of randomly chosen gates, it also verifies that error behavior is stable when used in long computations. They implemented randomized benchmarking on (e&eb) trapped atomic ion qubits, establishing a one-qubit error probability per randomized $\pi/2$ pulse of 0.00482(17) in a particular experiment. Authors expect this error probability to be readily improved with straightforward technical modifications.

Scalable and Robust Randomized Benchmarking of Quantum Processes Easwar Magesan, J. M. Gambetta, and Joseph Emerson Phys. Rev. Lett 106, 180504 – Published 6 May 2011

(3) **Easwar Magesan, J. M. Gambetta, and Joseph Emerson** propose a fully scalable randomized benchmarking protocol for (e) quantum information processors.

(4) They prove that the protocol provides an efficient and reliable estimate of the average error-rate for (e) set operations (gates) under a very general noise model that allows (eb) for both time and gate-dependent errors. In particular they obtain a sequence of fitting models for the observable fidelity decay as (=) a function of a (convergent) perturbative expansion of the gate errors about the mean error. They illustrate the protocol through numerical examples.

Randomized Benchmarking of Multiqubit Gates J. P. Gaebler, A. M. Meier, T. R. Tan, R. Bowler, Y. Lin, D. Hanneke, J. D. Jost, J. P. Home, E. Knill, D. Leibfried, and D. J. Wineland Phys. Rev. Lett. 108, 260503 – Published 26 June 2012; Erratum Phys. Rev. Lett. 109, 179902 (2012) http://dx.doi.org/10.1103/PhysRevLett.108.260503

(5) **J. P. Gaebler, A. M. Meier, T. R. Tan, R. Bowler, Y. Lin, D. Hanneke, J. D. Jost, J. P. Home, E. Knill, D. Leibfried, and D. J. Wineland** describe an extension of single-qubit gate randomized benchmarking that measures (eb) the error of multiqubit gates in a quantum information processor.

(6) This platform-independent protocol evaluates (e&eb) the performance of Clifford unitaries, which form a basis of (e) fault-tolerant quantum computing.

(7) They implemented the benchmarking protocol with trapped ions and found an error per random two-qubit Clifford unitary of 0.162±0.008, thus setting the first benchmark for such unitaries. By implementing a second set of sequences with an extra two-qubit phase gate inserted after each step, they extracted an error per phase gate of 0.069±0.017. Authors conducted these experiments with transported, sympathetically cooled ions in a multizone Paul trap—a system that can in principle be scaled to larger numbers of ions.

Characterization of Addressability by Simultaneous Randomized Benchmarking Jay M. Gambetta, A. D. Córcoles, S. T. Merkel, B. R. Johnson, John A. Smolin, Jerry M. Chow, Colm A. Ryan, Chad

Rigetti, S. Poletto, Thomas A. Ohki, Mark B. Ketchen, and M. Steffen Phys. Rev. Lett. 109, 240504 – Published 11 December 2012 http://dx.doi.org/10.1103/PhysRevLett.109.240504

(8) The control and handling of errors arising from cross talk and unwanted interactions in multiqubit systems is an important issue in quantum information processing architectures. **Jay M. Gambetta, A. D. Córcoles, S. T. Merkel, B. R. Johnson, John A. Smolin, Jerry M. Chow, Colm A. Ryan, Chad Rigetti, S. Poletto, Thomas A. Ohki, Mark B. Ketchen, and M. Steffen** introduce a benchmarking protocol that provides information about the amount of addressability present in the system and implement it on (e&eb) coupled superconducting qubits.

(9) The protocol consists of randomized benchmarking experiments run both individually and simultaneously on pairs of qubits. A relevant figure of merit for the addressability is then related to (e&eb) the differences in the measured average gate fidelities in the two experiments. They present results from two similar samples with differing cross talk and unwanted qubit-qubit interactions. The results agree with predictions based on simple models of the classical cross talk and Stark shifts.

Characterizing errors on qubit operations via iterative randomized benchmarking Sarah Sheldon, Lev S. Bishop, Easwar Magesan, Stefan Filipp, Jerry M. Chow, Jay M. Gambetta (Submitted on 24 Apr 2015)

(10) With improved gate calibrations reducing unitary errors, **Sarah Sheldon, Lev S. Bishop, Easwar Magesan, Stefan Filipp, Jerry M. Chow, Jay M. Gambetta** achieve benchmarked single-qubit gate fidelity of 99.95% with superconducting qubits in a circuit quantum electrodynamics system. They present a method for distinguishing between unitary and non-unitary errors in quantum gates by interleaving repetitions of (e) a target gate within (eb) a randomized benchmarking sequence.

(11) Benchmarking fidelity decays quadratically with (e&eb) the number of interleaved gates for unitary errors but linearly for non-unitary, allowing to separate systematic coherent errors from (e) decoherent effects. With this protocol they show that the fidelity of the gates is not limited by (e) unitary errors, but by another **drive-activated source of decoherence such as amplitude fluctuations**. **Cite as: arXiv: 1504.06597 [quant-ph] (or arXiv: 1504.06597v1 [quant-ph] for this version)**

(12) A key requirement for scalable quantum computing is that elementary quantum gates can be implemented with sufficiently low error. One method for determining the error behavior of a gate implementation is to perform (e&eb) process tomography. However, standard process tomography is limited by errors in state preparation, measurement and one-qubit gates.

Process tomography suffers from inefficient scaling with number of qubits and does not detect adverse error-compounding when (e) gates are composed in long sequences

G_{16} : Category one of gates are composed in long sequences

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Process tomography suffers from inefficient scaling with number of qubits and does not detect adverse error-compounding

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

An additional problem is due to the fact that desirable error probabilities for scalable quantum computing are of the order of 0.0001 or lower. Experimentally proving such low errors is challenging. **E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland** describe a randomized benchmarking method that yields (eb) estimates of the computationally relevant errors without (e) relying on accurate state preparation and measurement

. Since it involves long sequences of randomly chosen gates, it also verifies that error behavior is stable when used in long computations.

They implemented randomized benchmarking on (e&eb) trapped atomic ion qubits, establishing a one-qubit error probability per randomized π/2 pulse of 0.00482(17) in a particular experiment. Authors expect this error probability to be readily improved with straightforward technical modifications

G_{20} : Category one of randomized benchmarking method

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of estimates of the computationally relevant errors without (e) relying on accurate state preparation and measurement

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

randomized benchmarking method that yields estimates of the computationally relevant errors without (e) relying on accurate state preparation and measurement

G_{24} : Category one of relying on accurate state preparation and measurement

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of randomized benchmarking method that yields estimates of the computationally relevant errors

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

J. P. Gaebler, A. M. Meier, T. R. Tan, R. Bowler, Y. Lin, D. Hanneke, J. D. Jost, J. P. Home, E. Knill, D. Leibfried, and D. J. Wineland describe an extension of single-qubit gate randomized benchmarking that measures (eb) the error of multiqubit gates in a quantum information processor.

This platform-independent protocol evaluates (e&eb) the performance of Clifford unitaries, which form a

basis of (e) fault-tolerant quantum computing

G_{28} : Category one of extension of single-qubit gate randomized benchmarking

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of error of multiqubit gates in a quantum information processor

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

platform-independent protocol evaluates (e&eb) the performance of Clifford unitaries, which form a basis of (e) fault-tolerant quantum computing

G_{32} : Category one of platform-independent protocol; performance of Clifford unitaries, which form a basis of (e) fault-tolerant quantum computing

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of performance of Clifford unitaries, which form a basis of (e) fault-tolerant quantum computing ;platform-independent protocol

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

platform-independent protocol evaluates the performance of Clifford unitaries, which form a basis of (e) fault-tolerant quantum computing

G_{36} : Category one of platform-independent protocol evaluates the performance of Clifford unitaries; fault-tolerant quantum computing

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of fault-tolerant quantum computing; platform-independent protocol evaluates the performance of Clifford unitaries

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The control and handling of errors arising from cross talk and unwanted interactions in multiqubit systems is an important issue in quantum information processing architectures. **Jay M. Gambetta, A. D. Córcoles, S. T. Merkel, B. R. Johnson, John A. Smolin, Jerry M. Chow, Colm A. Ryan, Chad Rigetti, S. Poletto, Thomas A. Ohki, Mark B. Ketchen, and M. Steffen** introduce a benchmarking protocol that provides information about the amount of addressability present in the system and implement it on (e&eb) coupled superconducting qubits

G_{40} : Category one of benchmarking protocol that provides information about the amount of addressability present in the system; coupled superconducting qubits

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of coupled superconducting qubits; benchmarking protocol that provides information about the amount of addressability present in the system

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Benchmarking fidelity decays quadratically with (e&eb) the number of interleaved gates for unitary errors but linearly for non-unitary, allowing to separate systematic coherent errors from (e) decoherent effects.

With this protocol they show that the fidelity of the gates is not limited by (e) unitary errors, but by another **drive-activated source of decoherence such as amplitude fluctuations**. **Cite as: arXiv: 1504.06597 [quant-ph] (or arXiv: 1504.06597v1 [quant-ph] for this version)**

G_{44} : Category one of Benchmarking fidelity decays quadratically; number of interleaved gates for unitary errors but linearly for non-unitary, allowing to separate systematic coherent errors from (e) decoherent effects

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of number of interleaved gates for unitary errors but linearly for non-unitary, allowing to separate systematic coherent errors from (e) decoherent effects; Benchmarking fidelity decays quadratically

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}),t) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}), t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}), t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}), t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}), t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & \boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for

category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a_{20}')^{(3)} & +(a_{20}'')^{(3)}(T_{21},t) & +(a_{16}'')^{(2,2,2)}(T_{17},t) & +(a_{13}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a_{21}')^{(3)} & +(a_{21}'')^{(3)}(T_{21},t) & +(a_{17}'')^{(2,2,2)}(T_{17},t) & +(a_{14}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a_{22}')^{(3)} & +(a_{22}'')^{(3)}(T_{21},t) & +(a_{18}'')^{(2,2,2)}(T_{17},t) & +(a_{15}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$ *are fourth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$ *are fifth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$ *are sixth detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients*

$for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\ \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\ \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\ \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition

coefficients for category 1, 2, and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{37}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(EEEEEEEE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(FFFFFFFF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(GGGGGGGG) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(HHHHHHHH)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(IIIIIIII) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(JJJJJJJJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}$, $(r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(ff) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0$ *and* $T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right)=(b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}'')^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations
$(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{32}'')^{(6)} = (a_{33}'')^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b_{32}'')^{(6)} = (b_{33}'')^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$:-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(ff) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY FOUR

Semiconductor-Atomic Interface

INTRODUCTION—VARIABLES USED

NATURE PHOTONICS | LETTER Hybrid semiconductor-atomic interface: slowing down single

photons from a quantum dot N. Akopian, L. Wang, A. Rastelli, O. G. Schmidt & V. Zwiller Nature Photonics 5, 230–233 (2011) doi:10.1038/nphoton.2011.16 Received 17 October 2010 Accepted 13 January 2011 Published online 20 February 2011

(1) Hybrid interfaces between semiconductor quantum dots and (e&eb) atomic systems could be of potential fundamental and technological interest, because they can combine the advantages of both constituents.
(2) Semiconductor quantum dots are tunable and deterministic sources of (e) single1 and entangled photons2.
(3) Atomic vapours are widely used as slow-light media3, 4 and quantum memories5, 6. Merging both systems could enable (eb) the storage of quantum dot emission—an important step towards the implementation of quantum memories and quantum repeaters 7.
(4) Here, authors show a hybrid semiconductor–atomic interface for (e) slowing down single photons emitted from a single quantum dot.
(5) Authors use a double absorption resonance4 in rubidium vapour to create (eb) a slow-light medium in which a single photon is stored for (e) 15 times its temporal width.
(6) Result is the first demonstration of non-classical light storage, where (e) single photons are generated on demand from (e) a semiconductor source.

Letters to Nature 430, 654-657 (5 August 2004) | doi:10.1038/nature02772; Received 25 February 2004; Accepted 16 June 2004 Controlling the dynamics of spontaneous emission from quantum dots by photonic crystals Peter Lodahl1, A. Floris van Driel2, Ivan S. Nikolaev1, Arie Irman1, Karin Overgaag1, Daniël Vanmaekelbergh2 & Willem L. Vos1

(1) Control of spontaneously emitted light lies at the heart of quantum optics. It is essential for diverse applications ranging from miniature lasers and light-emitting diodes1, 2, 3, 4, 5, to single-photon sources for quantum information6, 7, 8, and to solar energy harvesting9. To explore such new quantum optics applications, a suitably tailored dielectric environment is required in which the vacuum fluctuations that control (e&eb) spontaneous emission can be manipulated10, 11
(2) Photonic crystals provide such an environment: they strongly modify (e&eb) the vacuum fluctuations, causing (erb) the decay of emitted light to be accelerated or slowed down12, 13, to reveal (eb) unusual statistics14, or to be completely inhibited in the ideal case of a photonic bandgap1, 15.
(3) **Peter Lodahl1, A. Floris van Driel2, Ivan S. Nikolaev1, Arie Irman1, Karin Overgaag1, Daniël Vanmaekelbergh2 & Willem L. Vos1** study spontaneous emission from semiconductor quantum dots embedded in (eb) inverse opal photonic crystals16.
(4) They show that the spectral distribution and time-dependent decay of light emitted from excitons confined in the quantum dots are controlled by (e&eb) the host photonic crystal.
(5) Modified emission is observed over large frequency bandwidths of 10%, orders of magnitude larger than reported for resonant optical microcavities17. Both inhibited and enhanced decay rates are observed depending on the optical emission frequency, and they are controlled by (e&eb) the crystals' lattice parameter. Experimental results provide a basis for all-solid-state dynamic control of optical quantum systems18

NOTATION

Module One

Benchmarking fidelity decays quadratically with the number of interleaved gates for unitary errors but

linearly for non-unitary, allowing separating systematic coherent errors from (e) decoherent effects.

With this protocol they show that the fidelity of the gates is not limited by (e) unitary errors, but by another **drive-activated source of decoherence such as amplitude fluctuations**. **Cite as: arXiv: 1504.06597 [quant-ph] (or arXiv: 1504.06597v1 [quant-ph] for this version)**

G_{13} : Category one of Benchmarking fidelity decays quadratically with the number of interleaved gates for unitary errors but linearly for non-unitary, allowing separating systematic coherent errors ; decoherent effects

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of decoherent effects; Benchmarking fidelity decays quadratically with the number of interleaved gates for unitary errors but linearly for non-unitary, allowing separating systematic coherent errors

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Hybrid interfaces between semiconductor quantum dots and (e&eb) atomic systems could be of potential fundamental and technological interest, because they can combine the advantages of both constituents

G_{16} : Category one of semiconductor quantum dots; atomic systems

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of atomic systems ; semiconductor quantum dots

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Semiconductor quantum dots are tunable and deterministic sources of (e) single1 and entangled photons2

G_{20} : Category one of single1 and entangled photons2

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Semiconductor quantum dots are tunable and deterministic sources

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Atomic vapours are widely used as slow-light media3, 4 and quantum memories5, 6. Merging both systems could enable (eb) the storage of quantum dot emission—an important step towards the implementation of quantum memories and quantum repeaters 7

G_{24} : Category one of Atomic vapours; quantum memories5, 6

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of quantum memories5, 6; Atomic vapours

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Atomic vapours are widely used as slow-light media3, 4 and quantum memories5, 6. Merging both systems could enable (eb) the storage of quantum dot emission—an important step towards the implementation of quantum memories and quantum repeaters 7

G_{28} : Category one of Atomic vapours are widely used as slow-light media3, 4 and quantum memories5, 6

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of storage of quantum dot emission—an important step towards the implementation of quantum memories and quantum repeaters 7

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Here, authors show a hybrid semiconductor–atomic interface for (e) slowing down single photons emitted from a single quantum dot

G_{32} : Category one of slowing down single photons emitted from a single quantum dot

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of hybrid semiconductor–atomic interface

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Authors use a double absorption resonance4 in rubidium vapour to create (eb) a slow-light medium in which a single photon is stored for (e) 15 times its temporal width

G_{36} : Category one of double absorption resonance4 in rubidium vapour

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of slow-light medium in which a single photon is stored for (e) 15 times its temporal width

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Result is the first demonstration of non-classical light storage, where single photons are generated on demand from (e) a semiconductor source

G_{40} : Category one of semiconductor source

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of non-classical light storage, where single photons are generated on demand

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Photonic crystals provide such an environment: they strongly modify (e&eb) the vacuum fluctuations, causing (erb) the decay of emitted light to be accelerated or slowed down12, 13, to reveal (eb) unusual statistics14, or to be completely inhibited in the ideal case of a photonic bandgap1, 15

G_{44} : Category one of Photonic crystals; vacuum fluctuations, causing (erb) the decay of emitted light to be accelerated or slowed down12, 13, to reveal (eb) unusual statistics14, or to be completely inhibited in the ideal case of a photonic bandgap1, 15

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of vacuum fluctuations, causing (erb) the decay of emitted light to be accelerated or slowed down12, 13, to reveal (eb) unusual statistics14, or to be completely inhibited in the ideal case of a photonic bandgap1, 15; Photonic crystals

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}\end{array}\right]G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients

for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$
$for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a_{28}')^{(5)} & +(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad 79$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a_{29}')^{(5)} & +(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{29} \quad 80$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a_{30}')^{(5)} & +(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t)\\ & +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)\\ & +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{30} \quad 81$$

$Where$ $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ $are\ first\ augmentation$
$coefficients\ for\ category\ 1,2\ and\ 3$
And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ $are\ second\ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for

category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$
are eighth augmentation coefficient for 1,2,3
$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

96

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ 97

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\big((G_{19}), t\big) = \ (r_i)^{(2)} \qquad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KKKKKKKK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(LLLLLLLL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(MMMMMMMM) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(NNNNNNNN)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(OOOOOOOO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(PPPPPPPP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)$, $T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$: B

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \qquad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \qquad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{16}\right)^{(2)}, T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\tilde{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\tilde{M}_{16})^{(2)}} \left(e^{(\tilde{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(- \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(gg) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left(- \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left(- \frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}} \right) \right] ds_{(44)} =$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a_{16}')^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b_i')^{(2)}t)}>0\quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t))=(b_{17}')^{(2)}$ then $T_{17}\to\infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left((\widetilde{G_{23}}), (\widetilde{T_{23}})\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)}|G_{21}^{(1)} - G_{21}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}|G_{20}^{(1)} - G_{20}^{(2)}|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$

$G_{20}^{(2)} \left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad (214)$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (216)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: (217)

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. (218)

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (228)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is (230)

analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_{0}^{t}\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\,(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_{+}$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ (239)

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(5)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 275

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\,((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(v_1)^{(2)}>0,(v_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{v}_1)^{(2)},,(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)}>0,(\bar{v}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(v^{(2)}\big)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,

and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)},(\nu_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},\nu^{(7)},u^{(7)}$: 361

By $(\nu_1)^{(7)}>0$, $(\nu_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0$, $(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{\nu}_1)^{(7)},,(\bar{\nu}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)}>0$, $(\bar{\nu}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0$, $(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\left((G_{43}),t\right)-(b''_{41})^{(8)}\left((G_{43}),t\right)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)}, (v_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, v^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)}, , (\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$\text{it follows } (v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)}\leq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_2)^{(1)}\right)t\right]}}\leq v^{(1)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(\bar{v}_1)^{(1)}$$

If $0<(v_1)^{(1)}\leq(\bar{v}_1)^{(1)}\leq\boxed{(v_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(v_1)^{(1)}\leq v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\leq(v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)}\leq v^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{v^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(gg) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY FIVE

Photonic Coupling Of A Semiconductor Quantum Dot And A Trapped Ion

INTRODUCTION—VARIABLES USED

Direct Photonic Coupling of a Semiconductor Quantum Dot and a Trapped Ion H. M. Meyer, R. Stockill, M. Steiner, C. Le Gall, C. Matthiesen, E. Clarke, A. Ludwig, J. Reichel, M. Atatüre, and M. Köhl Phys. Rev. Lett. 114, 123001 – Published 23 March 2015

(1) Coupling individual quantum systems lies at the heart of building scalable quantum networks. Here, authors report the first direct photonic coupling between a semiconductor quantum dots and (e&eb) a trapped ion and they demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion.
(2) Authors ameliorate the effect of (e) the 60-fold mismatch of the radiative linewidths with (e&eb) coherent photon generation and a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion.
(3) Transfer of information presented here via the classical correlations between the σz projection of (e) the quantum-dot spin and (e&eb) the internal state of the ion provides (eb) a promising step towards quantum-state transfer in (eb) a hybrid photonic network**.**

An artificial atom locked to natural atoms N. Akopian, R. Trotta, E. Zallo, S. Kumar, P. Atkinson, A. Rastelli, O. G. Schmidt, V. Zwiller (Submitted on 8 Feb 2013)

(4) Single-photon sources that emit (eb) photons at the same energy play (e&eb) a key role in the emerging (eb) concepts of quantum information, such as entanglement swapping, quantum teleportation and quantum networks.
(5) They can be realized in a variety of systems, where semiconductor quantum dots, or 'artificial atoms', are arguably among the most attractive. However, unlike 'natural atoms', no two artificial atoms are (=) alike.
(6) This peculiarity is a serious hurdle for quantum information applications that require (e) photonic quantum states with identical energies.
(7) Here we demonstrate a single artificial atom that generates (eb) photons with an absolute energy that is locked to (e&eb) an optical transition in (eb) a natural atom.
(8) Furthermore, they show that system is (=) robust and immune to drifts and fluctuations in (eb) the environment of the emitter. Demonstration is crucial for realization of a large number of universally-indistinguishable solid-state systems at arbitrary remote locations, where frequency-locked artificial atoms might become fundamental ingredients. **Cite as: arXiv: 1302.2005 [cond-mat.mes-hall] (or arXiv:1302.2005v1 [cond-mat.mes-hall] for this version)**

NOTATION

Module One

Coupling individual quantum systems lies at the heart of building scalable quantum networks.

Here, authors report the first direct photonic coupling between a semiconductor quantum dots and (e&eb) a trapped ion and they demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{13} : Category one of semiconductor quantum dots; trapped ion

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of trapped ion ;semiconductor quantum dots

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Here, authors report the first direct photonic coupling between a semiconductor quantum dots and a trapped ion and they demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{16} : Category one of quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of semiconductor quantum dots and a trapped ion and they demonstrate that single photons

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Here, authors report the first direct photonic coupling between a semiconductor quantum dots and a trapped ion and they demonstrate that single photons generated by a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{20} : Category one of semiconductor quantum dots and a trapped ion and they demonstrate that single photons generated by a quantum dot; internal state of a Yb+ ion

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of internal state of a Yb+ ion; semiconductor quantum dots and a trapped ion and they demonstrate that single photons generated by a quantum dot

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

This peculiarity is a serious hurdle for quantum information applications that require (e) photonic quantum states with identical energies

G_{24} : Category one of photonic quantum states with identical energies

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of peculiarity is a serious hurdle for quantum information applications

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and (e&eb) the internal state of the ion provides (eb) a promising step towards quantum-state transfer in (eb) a hybrid photonic network

G_{28} : Category one of Transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin; internal state of the ion provides (eb) a promising step towards quantum-state transfer in (eb) a hybrid photonic network

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of internal state of the ion provides (eb) a promising step towards quantum-state transfer in (eb) a hybrid photonic network ;Transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and the internal state of the ion provides (eb) a promising step towards quantum-state transfer in (eb) a hybrid photonic network

G_{32} : Category one of Transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and the internal state of the ion

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of promising step towards quantum-state transfer in a hybrid photonic network

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Single-photon sources that emit photons at the same energy play (e&eb) a key role in the emerging (eb) concepts of quantum information, such as entanglement swapping, quantum teleportation and quantum networks

G_{36} : Category one of Single-photon sources that emit photons at the same energy; key role in the emerging (eb) concepts of quantum information, such as entanglement swapping, quantum teleportation and quantum networks

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of key role in the emerging (eb) concepts of quantum information, such as entanglement swapping, quantum teleportation and quantum networks ;Single-photon sources that emit photons at the same energy

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

This peculiarity is a serious hurdle for quantum information applications that require (e) photonic quantum states with identical energies

G_{40} : Category one of photonic quantum states; identical energies

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of identical energies; photonic quantum states

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Here we demonstrate a single artificial atom that generates photons with an absolute energy that is locked to (e&eb) an optical transition in a natural atom.

G_{44} : Category one of single artificial atom that generates photons with an absolute energy; optical transition in a natural atom

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of optical transition in a natural atom ;single artificial atom that generates photons with an absolute energy

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}),t) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}), t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}), t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}), t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}), t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \\ +(a''_{20})^{(3,3,)}(T_{21},t) & +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) \\ +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) \\ +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \\ +(a''_{21})^{(3,3,)}(T_{21},t) & +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) \\ +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) \\ +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \\ +(a''_{22})^{(3,3,)}(T_{21},t) & +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) \\ +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) \\ +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{15} \quad (57)$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) \\ -(b''_{20})^{(3,3,)}(G_{23},t) & -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) \\ -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) \\ -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) \\ -(b''_{21})^{(3,3,)}(G_{23},t) & -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) \\ -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) \\ -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & \boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for

category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a_{20}')^{(3)} & +(a_{20}'')^{(3)}(T_{21},t) & +(a_{16}'')^{(2,2,2)}(T_{17},t) & +(a_{13}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a_{21}')^{(3)} & +(a_{21}'')^{(3)}(T_{21},t) & +(a_{17}'')^{(2,2,2)}(T_{17},t) & +(a_{14}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a_{22}')^{(3)} & +(a_{22}'')^{(3)}(T_{21},t) & +(a_{18}'')^{(2,2,2)}(T_{17},t) & +(a_{15}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* $1,2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$ *are fourth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$ *are fifth detrition coefficients for category* $1,2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$ *are sixth detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\begin{bmatrix}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\begin{bmatrix}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients*

$for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\ \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\ \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\ \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition

coefficients for category 1, 2, and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition
coefficients for category 1, 2, and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{37}'')^{(7)}(T_{37},t)}$, $\boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$ *96*

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(QQQQQQQQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(RRRRRRRR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(SSSSSSSS) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(TTTTTTTT)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(UUUUUUUU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(VVVVVVVV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}$, $(r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - \ T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(hh) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0$ *and* $T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{16})^{(2)}|G_{17}^{(1)}-G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}|G_{16}^{(1)}-G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)|G_{16}^{(1)}-G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\;e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$|(G_{19})^{(1)}-(G_{19})^{(2)}|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\,(G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t})=0$ and $\mathrm{T}_i(\mathrm{t})=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

208

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

209

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

214

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right)=(b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\hat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}'')^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right)=(b_{45}')^{(9)}$ then $T_{45}\to\infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)}-(b_i'')^{(9)}\left((G_{47})(t),t\right)<\varepsilon_9, T_{44}(t)>(m)^{(9)}$$

Then $\frac{dT_{45}}{dt}\geq(a_{45})^{(9)}(m)^{(9)}-\varepsilon_9 T_{45}$ which leads to

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1-e^{-\varepsilon_9 t})+T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t}=\frac{1}{2}$ it results

$T_{45}\geq\left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t=log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right)=(b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

$(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)},(v_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},v^{(1)},u^{(1)}$: 281

By $(v_1)^{(1)}>0$, $(v_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0$, $(u_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)}>0$, $(\bar{v}_2)^{(1)}<0$ and respectively $(\bar{u}_1)^{(1)}>0$, $(\bar{u}_2)^{(1)}<0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by
$(m_2)^{(1)}=(v_0)^{(1)},(m_1)^{(1)}=(v_1)^{(1)},\ if\ (v_0)^{(1)}<(v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$$

334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

335

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$$

336

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \le \nu^{(8)}(t) \le (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \le u^{(8)}(t) \le (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(hh) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*}) = 0$ and 512

$$\mathrm{G}_{16}^{*} = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*} = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \quad (513)$$

$$\mathrm{T}_{16}^{*} = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*} = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \quad (514)$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \quad (517)$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)} T_{44}^* \mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)} T_{45}^* \mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)} T_{46}^* \mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\right)\} = 0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)} + (b'_{42})^{(8)} - (r_{42})^{(8)})\{((\lambda)^{(8)} + (a'_{42})^{(8)} + (p_{42})^{(8)})$$

$$\left[\left(((\lambda)^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)})(q_{41})^{(8)}G^*_{41} + (a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)})s_{(41),(41)}T^*_{41} + (b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(((\lambda)^{(8)} + (a'_{41})^{(8)} + (p_{41})^{(8)})(q_{40})^{(8)}G^*_{40} + (a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)})s_{(41),(40)}T^*_{41} + (b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(((\lambda)^{(8)})^2 + (\,(a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)})\,(\lambda)^{(8)}\right)$$

$$\left(((\lambda)^{(8)})^2 + (\,(b'_{40})^{(8)} + (b'_{41})^{(8)} - (r_{40})^{(8)} + (r_{41})^{(8)})\,(\lambda)^{(8)}\right)$$

$$+\left(((\lambda)^{(8)})^2 + (\,(a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)})\,(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41} + (a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)})s_{(41),(42)}T^*_{41} + (b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\} = 0$$

$$+$$

$$(\lambda)^{(9)} + (b'_{46})^{(9)} - (r_{46})^{(9)})\{((\lambda)^{(9)} + (a'_{46})^{(9)} + (p_{46})^{(9)})$$

$$\left[\left(((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)})(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)})s_{(45),(45)}T^*_{45} + (b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(((\lambda)^{(9)} + (a'_{45})^{(9)} + (p_{45})^{(9)})(q_{44})^{(9)}G^*_{44} + (a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)})s_{(45),(44)}T^*_{45} + (b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(((\lambda)^{(9)})^2 + (\,(a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)})\,(\lambda)^{(9)}\right)$$

$$\left(((\lambda)^{(9)})^2 + (\,(b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)})\,(\lambda)^{(9)}\right)$$

$$+\left(((\lambda)^{(9)})^2 + (\,(a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)})\,(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)})s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY SIX

Chaotic Exciton-Polariton Billiard

INTRODUCTION—VARIABLES USED

Observation of non-Hermitian degeneracies in a chaotic exciton-polariton billiard T. Gao, E. Estrecho, K.Y. Bliokh, T.C.H. Liew, M.D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y.S. Kivshar, A. Truscott, R. Dall, E.A. Ostrovskaya

(1) Exciton-polaritons are (=) hybrid light-matter quasiparticles formed by (e) strongly interacting photons and (e&eb) excitons (electron-hole pairs) in (eb) semiconductor microcavities.
(2) They have emerged as a robust solid-state platform for next-generation optoelectronic applications as well as fundamental studies of quantum many-body physics. Importantly, exciton-polaritons are a profoundly open (i.e., non-Hermitian) quantum system: it requires (e) constant pumping of energy and continuously decays releasing (eb) coherent radiation.
(3) Thus, the exciton-polaritons always exist in a balanced potential landscape of (e) gain and loss.
(4) **T. Gao, E. Estrecho, K.Y. Bliokh, T.C.H. Liew, M.D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y.S. Kivshar, A. Truscott, R. Dall, E.A. Ostrovskaya** demonstrate that non-Hermiticity dramatically modifies (e&eb) the structure of modes and spectral degeneracies in exciton-polariton systems, and, therefore, will affect (e&eb) their quantum transport, localisation, and dynamical properties.
(5) Using a spatially-structured optical pump authors create (eb) a chaotic exciton-polariton billiard.
(6) Eigenmodes of this billiard exhibit multiple non-Hermitian spectral degeneracies -- exceptional points.
(7) Eigenmodes of this billiard exhibit multiple non-Hermitian spectral degeneracies -- exceptional points are known to cause (eb) remarkable wave phenomena, such as unidirectional transport, anomalous lasing/absorption, and chiral modes.
(8) By varying parameters of the billiard, (eb) they observe crossing and anti-crossing of energy levels and (eb) reveal the nontrivial topological modal structure exclusive to (e) non-Hermitian systems.
(9) They also observe the mode switching and topological Berry phase for (e) a parameter loop encircling the exceptional point. Findings pave the way for studies of non-Hermitian quantum dynamics of exciton-polaritons, which can lead to novel functionalities of polariton-based devices. **Cite as: arXiv:1504.00978 [cond-mat.quant-gas] (or arXiv:1504.00978v1 [cond-mat.quant-gas] for this version**)

NOTATION

Module One

Exciton-polaritons are hybrid light-matter quasiparticles formed by (e) strongly interacting photons and (e&eb) excitons (electron-hole pairs) in (eb) semiconductor microcavities.

G_{13} : Category one of strongly interacting photons and (e&eb) excitons (electron-hole pairs) in (eb) semiconductor microcavities

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of Exciton-polaritons are hybrid light-matter quasiparticles formed

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Exciton-polaritons are hybrid light-matter quasiparticles formed by strongly interacting photons and (e&eb)

excitons (electron-hole pairs) in semiconductor microcavities

G_{16} : Category one of Exciton-polaritons are hybrid light-matter quasiparticles formed by strongly interacting photons; excitons (electron-hole pairs) in semiconductor microcavities

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of excitons (electron-hole pairs) in semiconductor microcavities ;Exciton-polaritons are hybrid light-matter quasiparticles formed by strongly interacting photons

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They have emerged as a robust solid-state platform for next-generation optoelectronic applications as well as fundamental studies of quantum many-body physics.

Importantly, exciton-polaritons are a profoundly open (i.e., non-Hermitian) quantum system: it requires (e) constant pumping of energy and continuously decays releasing (eb) coherent radiation

G_{20} : Category one of constant pumping of energy and continuously decays releasing (eb) coherent radiation

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of exciton-polaritons are a profoundly open (i.e., non-Hermitian) quantum system

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

constant pumping of energy and continuously decays releasing (eb) coherent radiation

G_{24} : Category one of constant pumping of energy and continuously decays

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of coherent radiation

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

exciton-polaritons always exist in a balanced potential landscape of (e) gain and loss

G_{28} : Category one of exciton-polaritons always exist in a balanced potential landscape

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of gain and loss

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

T. Gao, E. Estrecho, K.Y. Bliokh, T.C.H. Liew, M.D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y.S. Kivshar, A. Truscott, R. Dall, E.A. Ostrovskaya demonstrate that non-Hermiticity dramatically modifies (e&eb) the structure of modes and spectral degeneracies in exciton-polariton systems, and, therefore, will affect (e&eb) their quantum transport, localisation, and dynamical properties

G_{32} : Category one of non-Hermiticity; structure of modes and spectral degeneracies in exciton-polariton systems, and, therefore, will affect (e&eb) their quantum transport, localisation, and dynamical properties

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of structure of modes and spectral degeneracies in exciton-polariton systems, and, therefore, will affect (e&eb) their quantum transport, localisation, and dynamical properties ;non-Hermiticity

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

T. Gao, E. Estrecho, K.Y. Bliokh, T.C.H. Liew, M.D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y.S. Kivshar, A. Truscott, R. Dall, E.A. Ostrovskaya demonstrate that non-Hermiticity dramatically modifies the structure of modes and spectral degeneracies in exciton-polariton systems, and, therefore, will affect (e&eb) their quantum transport, localisation, and dynamical properties

G_{36} : Category one of non-Hermiticity dramatically modifies the structure of modes and spectral degeneracies in exciton-polariton systems; their quantum transport, localisation, and dynamical properties

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of their quantum transport, localisation, and dynamical properties; non-Hermiticity dramatically modifies the structure of modes and spectral degeneracies in exciton-polariton systems

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Using a spatially-structured optical pump authors create (eb) a chaotic exciton-polariton billiard

G_{40} : Category one of spatially-structured optical pump

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of chaotic exciton-polariton billiard

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Eigenmodes of this billiard exhibit (eb) multiple non-Hermitian spectral degeneracies -- exceptional points

G_{44} : Category one of Eigenmodes of this billiard

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of multiple non-Hermitian spectral degeneracies -- exceptional points

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$

$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad (57)$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for

category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{16}'')^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \quad +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \quad +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) \quad +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)} & -(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)} & -(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)} & -(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)} & +(a''_{40})^{(8)}(T_{41},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45}$$

$$- \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)}G_{44}$$
$$-\left[\begin{array}{c} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right]G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)}G_{45}$$
$$-\left[\begin{array}{c} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right]G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\left[\begin{array}{c} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right]T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = \ (r_i)^{(2)}$ 105

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 107

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 108

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 110

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 111

Where we suppose

$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 112

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$$(b_i'')^{(3)}(G_{23},t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and \ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WWWWWWWW) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(XXXXXXXX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(YYYYYYYY) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(ZZZZZZZZ)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(AAAAAAAAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(BBBBBBBBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}\ (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\ \big((G_{43}), t\big) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - \ T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (145)$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad (146)$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}\,, \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}\,, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(ii) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad (180)$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that (181)

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose (182)

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad (183)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad (184)$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad (191)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.

The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right)=(b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}<1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}'')^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \quad 248$$

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t), t(t), t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{36})^{(7)}|G_{37}^{(1)} - G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)} - G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)|G_{36}^{(1)} - G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)|\; e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq$ 259

$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 261

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}), (\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}), (\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+\ (a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}\ and\ (\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)}-(a''_i)^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a'_{41})^{(8)}$$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})} \right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a'_{44})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44}, G_{46} and G_{44}, G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46}. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, *if* $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{21})^{(3)}\big((G_{23}),t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad (323)$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad (324)$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$

where $(p_i)^{(4)}$ is defined by equation

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 333

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right.$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 335

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 336

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)}$,, $(\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,

and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the

theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le v^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \le (\bar{\nu}_1)^{(7)} \le \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$, we obtain 419

$$(\nu_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(9)}(t)$:-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$ if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\,, \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions with the notations 431

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(ii) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^{*} such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*})=0$, T_{14}^{*} given by $f(T_{14}^{*})=0$ and

$$G_{13}^{*}=\frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*}=\frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*}=\frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*}=\frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*})=0$, T_{17}^{*} given by $f(\mathrm{T}_{17}^{*})=0$ and 512

$$\mathrm{G}_{16}^{*}=\frac{(\mathrm{a}_{16})^{(2)}\mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{16})^{(2)}+(\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad \mathrm{G}_{18}^{*}=\frac{(\mathrm{a}_{18})^{(2)}\mathrm{G}_{17}^{*}}{[(\mathrm{a}'_{18})^{(2)}+(\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 513$$

$$\mathrm{T}_{16}^{*}=\frac{(\mathrm{b}_{16})^{(2)}\mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{16})^{(2)}-(\mathrm{b}''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad \mathrm{T}_{18}^{*}=\frac{(\mathrm{b}_{18})^{(2)}\mathrm{T}_{17}^{*}}{[(\mathrm{b}'_{18})^{(2)}-(\mathrm{b}''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*})=0$, T_{21}^{*} given by $f(T_{21}^{*})=0$ and

$$G_{20}^{*}=\frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*}=\frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27})=0$, T_{25}^{*} given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}$, $\frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$, $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \ , \ \frac{\partial (b_i'')^{(6)}}{\partial G_j}(\ (G_{35})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}\ and\ (b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \ , \ \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big(\ (G_{39})^{**}\ \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b''_i)^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\big((b_{46}')^{(9)} - (r_{46})^{(9)}\big)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(46)(j)} T_{46}^* \mathbb{G}_j\big) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\big((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\big)\{\big((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\big)$$

$$\Big[\Big(\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)} G_{13}^*\Big)\Big]$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^*\Big)$$

$$+\Big(\big((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\big)(q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)} G_{14}^*\Big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^*\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$\Big(\big((\lambda)^{(1)}\big)^2 + \big((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\big)(\lambda)^{(1)}\Big)$$

$$+\Big(\big((\lambda)^{(1)}\big)^2 + \big((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\big)(\lambda)^{(1)}\Big)(q_{15})^{(1)} G_{15}$$

$$+\big((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\big)\big((a_{15})^{(1)}(q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G_{13}^*\big)$$

$$\Big(\big((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\big) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^*\Big)\} = 0$$

$+$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY SEVEN

Solid State Quantum Devices At The Edge Of Quantum Chaos

INTRODUCTION—VARIABLES USED

Enrico Prati 2015 J. Phys.: Conf. Ser. 626 012011 doi:10.1088/1742-6596/626/1/012011: Towards room temperature solid state quantum devices at the edge of quantum chaos for long-living quantum states OPEN ACCESS Enrico Prati

(1) Long living coherent quantum states have been observed in (eb) biological systems up to room temperature.
(2) Light harvesting in chromophoresis realized by (e) excitonic systems living at the edge of quantum chaos, where (e) energy level distribution becomes semi-Poissonian.
(3) On the other hand, artificial materials suffer the loss of (e) coherence of quantum states in quantum information processing, but semiconductor materials are known to exhibit (eb) quantum chaotic conditions, so the exploitation of similar conditions are to be considered.
(4) The advancements of nanofabrication, together with the control of implantation of individual atoms at nanometric precision, may open the experimental study of such special regime at the edge of the phase transitions for the electronic systems obtained by implanting impurity atoms in a silicon transistor. Here **Enrico Prati** reviews the recent advancements made in the field of theoretical description of the light harvesting in biological system in its connection with phase transitions at the few atoms scale and how it would be possible to achieve (eb) transition point to quantum chaotic regime.
(5) Such mechanism may thus preserve (eb) quantum coherent states at room temperature in solid state devices, to be exploited for (e) quantum information processing as well as dissipation-free quantum electronics.

Sonia Buckley et al 2012 Rep. Prog Phys. 75 126503 doi:10.1088/0034-4885/75/12/126503 Engineered quantum dot single-photon sources REVIEW ARTICLE Sonia Buckley, Kelley Rivoire and Jelena Vučković

(6) Fast, high efficiency and low error single-photon sources are required for (e) the implementation of a number of quantum information processing applications.
(7) The fastest triggered single-photon sources to date have been demonstrated using (e) epitaxially grown semiconductor quantum dots (QDs), which can be conveniently integrated with (e&eb) optical microcavities. Recent advances in QD technology, including demonstrations of high temperature and telecommunications wavelength single-photon emission, have made QD single-photon sources more practical. Here we discuss the applications of single-photon sources and their various requirements, before reviewing the progress made on a QD platform in meeting these requirements.

NOTATION

Module One

Observation of non-Hermitian degeneracies in a chaotic exciton-polariton billiard T. Gao, E.

Estrecho, K.Y. Bliokh, T.C.H. Liew, M.D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y.S. Kivshar, A. Truscott, R. Dall, E.A. Ostrovskaya

(1) Eigenmodes of this billiard exhibit multiple non-Hermitian spectral degeneracies -- exceptional points are known to cause (eb) remarkable wave phenomena, such as unidirectional transport, anomalous lasing/absorption, and chiral modes.
(2) By varying parameters of the billiard, (eb) they observe crossing and anti-crossing of energy levels and (eb) reveal the nontrivial topological modal structure exclusive to (e) non-Hermitian systems
(3) They also observe the mode switching and topological Berry phase for (e) a parameter loop encircling the exceptional point. Findings pave the way for studies of non-Hermitian quantum dynamics of exciton-polaritons, which can lead to novel functionalities of polariton-based devices. **Cite as: arXiv:1504.00978 [cond-mat.quant-gas] (or arXiv:1504.00978v1 [cond-mat.quant-gas] for this version**)

Eigenmodes of this billiard exhibit multiple non-Hermitian spectral degeneracies -- exceptional points are known to cause (eb) remarkable wave phenomena, such as unidirectional transport, anomalous lasing/absorption, and chiral modes

G_{13} : Category one of Eigenmodes of this billiard exhibit multiple non-Hermitian spectral degeneracies -- exceptional points

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of remarkable wave phenomena, such as unidirectional transport, anomalous lasing/absorption, and chiral modes

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

By varying parameters of the billiard they observe crossing and anti-crossing of energy levels and (eb) reveal the nontrivial topological modal structure exclusive to (e) non-Hermitian systems

G_{16} : Category one of varying parameters of the billiard they observe crossing and anti-crossing of energy levels

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of nontrivial topological modal structure exclusive to non-Hermitian systems

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Long living coherent quantum states have been observed in (eb) biological systems up to room temperature

G_{20} : Category one of Long living coherent quantum states

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of biological systems up to room temperature

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Light harvesting in chromophoresis realized by excitonic systems living at the edge of quantum chaos, where (e) energy level distribution becomes semi-Poissonian

G_{24} : Category one of energy level distribution becomes semi-Poissonian

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Light harvesting in chromophoresis realized by excitonic systems living at the edge of quantum chaos

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

On the other hand, artificial materials suffer the loss of coherence of quantum states in quantum information processing, but semiconductor materials are known to exhibit (eb) quantum chaotic conditions, so the exploitation of similar conditions are to be considered

G_{28} : Category one of artificial materials suffer the loss of coherence of quantum states in quantum information processing, but semiconductor materials

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of quantum chaotic conditions

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The advancements of nanofabrication, together with the control of implantation of individual atoms at nanometric precision, may open the experimental study of such special regime at the edge of the phase

transitions for the electronic systems obtained by implanting impurity atoms in a silicon transistor.

Here **Enrico Prati** reviews the recent advancements made in the field of theoretical description of the light harvesting in biological system in its connection with phase transitions at the few atoms scale and how it would be possible to achieve (eb) transition point to quantum chaotic regime

G_{32} : Category one of harvesting in biological system; phase transitions at the few atoms scale and how it would be possible to achieve (eb) transition point to quantum chaotic regime

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of phase transitions at the few atoms scale and how it would be possible to achieve (eb) transition point to quantum chaotic regime; harvesting in biological system

T_{33} : Category two of

T_{34} : Category three of

Module seven

Here **Enrico Prati** reviews the recent advancements made in the field of theoretical description of the light harvesting in biological system in its connection with phase transitions at the few atoms scale and how it would be possible to achieve (eb) transition point to quantum chaotic regime

G_{36} : Category one of light harvesting in biological system in its connection with phase transitions at the few atoms scale

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of transition point to quantum chaotic regime

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Such mechanism may thus preserve quantum coherent states at room temperature in solid state devices, to be exploited for (e) quantum information processing as well as dissipation-free quantum electronics

G_{40} : Category one of mechanism may thus preserve quantum coherent states at room temperature in solid state devices; quantum information processing as well as dissipation-free quantum electronics

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of quantum information processing as well as dissipation-free quantum electronics; mechanism may thus preserve quantum coherent states at room temperature in solid state devices

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Fast, high efficiency and low error single-photon sources are required for (e) the implementation of a number of quantum information processing applications

G_{44} : Category one of implementation of a number of quantum information processing applications

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Fast, high efficiency and low error single-photon sources

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation

coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \quad \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \quad \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (91)$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (92)$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)} & -(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)} & -(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)} & -(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & +(a''_{44})^{(9)}(T_{45},t) & +(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & +(a''_{45})^{(9)}(T_{45},t) & +(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & +(a''_{46})^{(9)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t) \end{bmatrix} G_{15}$$

Where $+(a''_{44})^{(9)}(T_{45},t)$, $+(a''_{45})^{(9)}(T_{45},t)$, $+(a''_{46})^{(9)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \tag{97}$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \tag{98}$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \le \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \le (\hat{k}_{32})^{(6)}|T_{33} - \ T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCCCCCCCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDDDDDDDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(EEEEEEEEE) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(FFFFFFFFF)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(GGGGGGGGG) $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(HHHHHHHHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\widehat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty}(a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty}(b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\widehat{M}_{40})^{(8)}t}$$ 142

$$|(b_i'')^{(8)}((G_{43})',t)-(b_i'')^{(8)}\big((G_{43}),t\big)|<(\hat{k}_{40})^{(8)}||(G_{43})-(G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $\big[(\hat{k}_{40})^{(8)},(\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}},\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+(\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)}+(b_i')^{(8)}+(\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \quad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} \text{ , } T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} \text{ , } T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^{0} + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^{0} + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^{0} + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{44})^{(9)}$, $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_{i}^{0}) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left(e^{(\tilde{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(jj) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)}\left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a_{13}')^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)},i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 19 to 24 it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\ \ \text{for t}>0$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a_{14}')^{(1)}G_{14}$ and by integrating

$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t))=(b_{14}')^{(1)}$ then $T_{14}\ \to\infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$

$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} \left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)} (\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)} (\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} \mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)} \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad (208)$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad (209)$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\hat{M}_{20})^{(3)}s_{(20)}}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)} \left(T_{21}^{(1)}, s_{(20)}\right) \left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left| (a''_{20})^{(3)} \left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)} \left(T_{21}^{(2)}, s_{(20)}\right) \right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| G_{23}{}^{(1)} - G_{23}{}^{(2)} \right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d \left(((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}) \right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[- \int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)} \right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is 218

analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}')^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad (267)$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

269

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

273

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)}+\left((\hat{P}_{44})^{(9)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$, $if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G^0_{13}e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13}e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G^0_{13}}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G^0_{15}e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+\ G_{15}^{0}e^{-(a'_{15})^{(1)}t})$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(\nu_1)^{(2)}>0,(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)}>0,(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad (311)$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad (312)$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad (313)$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad (314)$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad (315)$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

333

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^{0}e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\leq T_{34}(t)\leq \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)},(\nu_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},\nu^{(7)},u^{(7)}$: 361

By $(\nu_1)^{(7)}>0$, $(\nu_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0$, $(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

Definition of $(\bar{\nu}_1)^{(7)},,(\bar{\nu}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)}>0$, $(\bar{\nu}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0$, $(\bar{u}_2)^{(7)}<0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, ***if*** $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, ***if*** $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, ***if*** $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$, ***if*** $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t}\leq T_{38}(t)\leq$ 369

$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)},(S_2)^{(7)},(R_1)^{(7)},(R_2)^{(7)}$:- 370

Where $(S_1)^{(7)}=(a_{36})^{(7)}(m_2)^{(7)}-(a'_{36})^{(7)}$

$(S_2)^{(7)}=(a_{38})^{(7)}-(p_{38})^{(7)}$

$(R_1)^{(7)}=(b_{36})^{(7)}(\mu_2)^{(7)}-(b'_{36})^{(7)}$

$(R_2)^{(7)}=(b'_{38})^{(7)}-(r_{38})^{(7)}$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$:

$(\sigma_1)^{(8)},(\sigma_2)^{(8)},(\tau_1)^{(8)},(\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)}\leq-(a'_{40})^{(8)}+(a'_{41})^{(8)}-(a''_{40})^{(8)}(T_{41},t)+(a''_{41})^{(8)}(T_{41},t)\leq-(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)}\leq-(b'_{40})^{(8)}+(b'_{41})^{(8)}-(b''_{40})^{(8)}\big((G_{43}),t\big)-(b''_{41})^{(8)}\big((G_{43}),t\big)\leq-(\tau_1)^{(8)}$

Definition of $(\nu_1)^{(8)},(\nu_2)^{(8)},(u_1)^{(8)},(u_2)^{(8)},\nu^{(8)},u^{(8)}$: 372

By $(\nu_1)^{(8)}>0,(\nu_2)^{(8)}<0$ and respectively $(u_1)^{(8)}>0,(u_2)^{(8)}<0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_1)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_1)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$ and

Definition of $(\bar{\nu}_1)^{(8)},,(\bar{\nu}_2)^{(8)},(\bar{u}_1)^{(8)},(\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)}>0,(\bar{\nu}_2)^{(8)}<0$ and respectively $(\bar{u}_1)^{(8)}>0,(\bar{u}_2)^{(8)}<0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2+(\sigma_2)^{(8)}\nu^{(8)}-(a_{40})^{(8)}=0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2+(\tau_2)^{(8)}u^{(8)}-(b_{40})^{(8)}=0$

Definition of $(m_1)^{(8)},(m_2)^{(8)},(\mu_1)^{(8)},(\mu_2)^{(8)},(\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{44}(t) \leq G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \leq G_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)}\leq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}\leq\nu^{(1)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\bar{\nu}_1)^{(1)}$$

If $0<(\nu_1)^{(1)}\leq(\bar{\nu}_1)^{(1)}\leq\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)}\leq\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\leq(\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)}\leq\nu^{(1)}(t)\leq(m_1)^{(1)},\quad\boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21},t)\right) - (a_{21}'')^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_2)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)\leq\frac{dv^{(3)}}{dt}\leq-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2+(\sigma_1)^{(3)}v^{(3)}-(a_{20})^{(3)}\right)$$

400

From which one obtains

For $0<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(v_1)^{(3)}<(\bar{v}_1)^{(3)}$

$$v^{(3)}(t)\geq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}\quad,\quad\boxed{(C)^{(3)}=\frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$$

it follows $(v_0)^{(3)}\leq v^{(3)}(t)\leq(v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(3)}=\frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)}\leq v^{(3)}(t)\leq(\bar{v}_1)^{(3)}$

If $0<(v_1)^{(3)}<(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)}\leq\frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}\leq v^{(3)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(\bar{v}_1)^{(3)}$$

If $0<(v_1)^{(3)}\leq(\bar{v}_1)^{(3)}\leq(v_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(v_1)^{(3)}\leq v^{(3)}(t)\leq\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}\leq(v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)}\leq v^{(3)}(t)\leq(m_1)^{(3)},\quad\boxed{v^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)}-(v_0)^{(7)}}{(v_0)^{(7)}-(\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)}+(C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)} \le v^{(7)}(t) \le (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \le u^{(7)}(t) \le (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \le \frac{dv^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \le \nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
A

Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(jj) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if A

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ

is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^* such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27})=0$, T_{25}^{*} given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY EIGHT

Dispersion Mismatch, Atom Recoil, And Multiphoton Scattering

INTRODUCTION—VARIABLES USED

Protocol for hybrid entanglement between a trapped atom and a quantum dot Edo Waks and C. Monroe Phys. Rev. A 80, 062330 – Published 30 December 2009

(1) Edo Waks and C. Monroe propose a quantum optical interface between an atomic and (e&eb)

solid-state system.

(2) **They** show that quantum states in a single trapped atom can be entangled with (e&eb) the states of a semiconductor quantum dot through (e&eb) their common interaction with (e&eb) a classical laser field.

(3) The interference and detection of the resulting scattered photons can then herald (eb) the entanglement of the disparate atomic and solid-state quantum bits.

(4) Authors develop a protocol that can succeed despite a significant mismatch in (eb) the radiative characteristics of the two matter-based qubits.

(5) They study in detail a particular case of this interface applied to (e&eb) a single trapped Y171b+ ion and a cavity-coupled InAs semiconductor quantum dot.

(6) Entanglement fidelity and success rates are found to be robust to (e) a broad range of experimental nonideal effects such as **dispersion mismatch, atom recoil, and multiphoton scattering**.

(7) Authors conclude that it should be possible to produce highly entangled states of these complementary qubit systems under (e&eb) realistic experimental conditions

NOTATION

Module One

Sonia Buckley et al 2012 Rep. Prog Phys. 75 126503 doi:10.1088/0034-4885/75/12/126503 Engineered quantum dot single-photon sources REVIEW ARTICLE Sonia Buckley, Kelley Rivoire and Jelena Vučković

The fastest triggered single-photon sources to date have been demonstrated using (e) epitaxially grown semiconductor quantum dots (QDs), which can be conveniently integrated with (e&eb) optical microcavities

Recent advances in QD technology, including demonstrations of high temperature and telecommunications wavelength single-photon emission, have made QD single-photon sources more practical. Here we discuss the applications of single-photon sources and their various requirements, before reviewing the progress made on a QD platform in meeting these requirements.

G_{13} : Category one of epitaxially grown semiconductor quantum dots (QDs), which can be conveniently integrated with (e&eb) optical microcavities

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of fastest triggered single-photon sources

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

fastest triggered single-photon sources to date have been demonstrated using epitaxially grown semiconductor quantum dots (QDs), which can be conveniently integrated with (e&eb) optical microcavities

G_{16} : Category one of fastest triggered single-photon sources to date have been demonstrated using epitaxially grown semiconductor quantum dots (QDs); optical microcavities

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of optical microcavities ;fastest triggered single-photon sources to date have been demonstrated using epitaxially grown semiconductor quantum dots (QDs)

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Edo Waks and C. Monroe propose a quantum optical interface between an atomic and (e&eb) solid-state system

G_{20} : Category one of quantum optical interface between an atomic; solid-state system

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of solid-state system ;quantum optical interface between an atomic

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

They show that quantum states in a single trapped atom can be entangled with (e&eb) the states of a semiconductor quantum dot through (e&eb) their common interaction with (e&eb) a classical laser field

G_{24} : Category one of quantum states in a single trapped atom; states of a semiconductor quantum dot through (e&eb) their common interaction with (e&eb) a classical laser field

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of states of a semiconductor quantum dot through (e&eb) their common interaction with (e&eb) a classical laser field ;quantum states in a single trapped atom

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They show that quantum states in a single trapped atom can be entangled with the states of a semiconductor quantum dot through (e&eb) their common interaction with (e&eb) a classical laser field

G_{28} : Category one of quantum states in a single trapped atom can be entangled with the states of a semiconductor quantum dot; their common interaction with (e&eb) a classical laser field

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of their common interaction with (e&eb) a classical laser field; quantum states in a single trapped atom can be entangled with the states of a semiconductor quantum dot

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

They show that quantum states in a single trapped atom can be entangled with the states of a semiconductor quantum dot through their common interaction with (e&eb) a classical laser field

G_{32} : Category one of quantum states in a single trapped atom can be entangled with the states of a semiconductor quantum dot through their common interaction; classical laser field

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of classical laser field ;quantum states in a single trapped atom can be entangled with the states of a semiconductor quantum dot through their common interaction

T_{33} : Category two of

T_{34} : Category three of

Module seven

The interference and detection of the resulting scattered photons can then herald (eb) the entanglement of the disparate atomic and solid-state quantum bits

G_{36} : Category one of interference and detection of the resulting scattered photons

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of entanglement of the disparate atomic and solid-state quantum bits

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

entanglement of the disparate atomic and solid-state quantum bits

G_{40} : Category one of disparate atomic; solid-state quantum bits

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of solid-state quantum bits ;disparate atomic

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Authors develop a protocol that can succeed despite a significant mismatch in (eb) the radiative characteristics of the two matter-based qubits

G_{44} : Category one of protocol that can succeed despite a significant mismatch

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of radiative characteristics of the two matter-based qubits

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTY NINE

Trapped Ions Make Impeccable Qubits

INTRODUCTION—VARIABLES USED

Science Bulletin January 2015, Volume 60, Issue 2, pp 241-248 Date: 08 Jan 2015 Experimental digital quantum simulation of temporal–spatial dynamics of interacting fermion system Yao Lu, Guan-Ru Feng, Yan-Song Li, and Gui-Lu Long

(1) Efficient simulations of many-body quantum systems are generally difficult on classical computers due to (e) the exponential resource growth with the system size, while quantum computer has been proved to complete the same task effectively

(2) **Yao Lu, Guan-Ru Feng, Yan-Song Li, and Gui-Lu Long** studied quantum algorithms for digital simulation of (e) the dynamics of interacting quantum systems in real space.

(3) As an illustrative example **Yao Lu, Guan-Ru Feng, Yan-Song Li, and Gui-Lu Long** concentrated on a digital quantum simulation algorithm for (e) a two-fermion system with Coulomb interaction.

(4) Experimentally realized algorithm was on (eb) a three-qubit NMR device and interesting phenomena such as the moving toward or against each other of particles under (e&eb) attractive or

repulsive interaction has been clearly observed. This experiment demonstrated the very promising potential of quantum computers, even of small scale, to address the simulation of complex quantum systems.

Quantum Information Processing July 2015, Volume 14, and Issue 7, pp 2551-2562 Date: 05 May 2015 Entanglement and quantum teleportation in a three-qubit Heisenberg chain with three-site interactions Yu-Xing Xi, Wei-Wen Cheng, Yan-Xia Huang

(5) The thermal entanglement of a three-qubit XXZ Heisenberg model with (e&eb) three-site interactions in an external magnetic field, and the quantum teleportation via (e&eb) this model in thermal equilibrium state are investigated.
(6) It is found that entanglement and average fidelity depend on (∝) temperature, magnetic field, and anisotropy parameter JZ.
(7) Only ferromagnetic system is suitable for (e) quantum teleportation.
(8) XZX+YZY interaction is in favor of (e) entanglement, average fidelity, and critical temperatures, while XZY −YZX interaction against all of (e) them.
(9) Moreover, **Yu-Xing Xi, Wei-Wen Cheng, Yan-Xia Huang** also find entanglement does not (e) fully reflect average fidelity in virtue of (e) study the relation between entanglement and (e&eb) average fidelity.

NOTATION

Module One

Efficient simulations of many-body quantum systems are generally difficult on classical computers due to (e) the exponential resource growth with the system size, while quantum computer has been proved to complete the same task effectively

G_{13} : Category one of exponential resource growth with the system size, while quantum computer has been proved to complete the same task effectively

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of efficient simulations of many-body quantum systems are generally difficult on classical computers

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Yao Lu, Guan-Ru Feng, Yan-Song Li, and Gui-Lu Long studied quantum algorithms for digital simulation of (e) the dynamics of interacting quantum systems in real space

G_{16} : Category one of dynamics of interacting quantum systems in real space

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantum algorithms for digital simulation

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

As an illustrative example **Yao Lu, Guan-Ru Feng, Yan-Song Li, and Gui-Lu Long** concentrated on a digital quantum simulation algorithm for (e) a two-fermion system with Coulomb interaction

G_{20} : Category one of two-fermion system with Coulomb interaction

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of digital quantum simulation algorithm

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The thermal entanglement of a three-qubit XXZ Heisenberg model with (e&eb) three-site interactions in an external magnetic field, and the quantum teleportation via (e&eb) this model in thermal equilibrium state are investigated

G_{24} : Category one of thermal entanglement of a three-qubit XXZ Heisenberg model; three-site interactions in an external magnetic field, and the quantum teleportation via (e&eb) this model in thermal equilibrium state are investigated

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of three-site interactions in an external magnetic field, and the quantum teleportation via (e&eb) this model in thermal equilibrium state are investigated ;thermal entanglement of a three-qubit XXZ Heisenberg model

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

thermal entanglement of a three-qubit XXZ Heisenberg model with three-site interactions in an external magnetic field, and the quantum teleportation via (e&eb) this model in thermal equilibrium state are investigated

G_{28} : Category one of thermal entanglement of a three-qubit XXZ Heisenberg model with three-site interactions in an external magnetic field, and the quantum teleportation; model in thermal equilibrium state

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of model in thermal equilibrium state; thermal entanglement of a three-qubit XXZ Heisenberg model with three-site interactions in an external magnetic field, and the quantum teleportation

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

It is found that entanglement and average fidelity depend on ($\propto$) temperature, magnetic field, and anisotropy parameter JZ

G_{32} : Category one of entanglement and average fidelity

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of (Variable) X temperature, magnetic field, and anisotropy parameter JZ

T_{33} : Category two of

T_{34} : Category three of

Module seven

Only ferromagnetic system is suitable for (e) quantum teleportation

G_{36} : Category one of quantum teleportation; ferromagnetic system

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of ferromagnetic system ; quantum teleportation

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

XZX+YZY interaction is in favor of (e) entanglement, average fidelity, and critical temperatures,

while XZY −YZX interaction against all of (e) them

G_{40} : Category one of entanglement, average fidelity, and critical temperatures

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of XZX+YZY interaction is in favor

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

XZY −YZX interaction against all of (e) them

G_{44} : Category one of XZY −YZX interaction; entanglement, average fidelity, and critical temperatures

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of entanglement, average fidelity, and critical temperatures ;XZY −YZX interaction

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY

Hilbert Space Structure In Quantum Gravity

INTRODUCTION—VARIABLES USED

Hilbert space structure in quantum gravity: an algebraic perspective Steven B. Giddings (Submitted on 27 Mar 2015 (v1), last revised 18 Aug 2015 (this version, v2))

(1) If quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory is through (e&eb) identifying the relevant quantum structures rather than by (e) quantizing classical spacetime.
(2) Viable approach to the theory is through (e&eb) identifying the relevant quantum structures viewpoint is supported by (e) difficulties of such quantization, and by (e) the apparent lack of (e) a fundamental role for (e) locality.
(3) In finite or discrete quantum systems, important structure is provided by (e) tensor factorizations of the Hilbert space.
(4) However, even in local quantum field theory properties of the generic type III von Neumann algebras and of long range gauge fields indicate (eb) that factorization of the Hilbert space is problematic.
(5) Instead it is better to focus on the structure of the algebra of observables, and in particular on its subalgebras corresponding to (e&eb) regions.

(6) Paper suggests that study of analogous algebraic structure in gravity gives (eb) an important perspective on the nature of the quantum theory.
(7) Significant departures from the subalgebra structure of local quantum field theory are found, working in (eb) the correspondence limit of long-distances/low-energies.
(8) Particularly, there are obstacles to identifying commuting algebras of (e) localized operators.
(9) In addition to suggesting important properties of the algebraic structure, this and related observations pose challenges (e) to proposals of a fundamental role for entanglement. **Cite as: arXiv:1503.08207 [hep-th] (or arXiv:1503.08207v2 [hep-th] for this version)**

NOTATION

Module One

If quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory is through (e&eb) identifying the relevant quantum structures rather than by (e) quantizing classical spacetime

G_{13} : Category one of quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory; identifying the relevant quantum structures rather than by (e) quantizing classical spacetime

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of identifying the relevant quantum structures rather than by (e) quantizing classical spacetime ;quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

If quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory is through identifying the relevant quantum structures rather than by (e) quantizing classical spacetime

G_{16} : Category one of quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory is through identifying the relevant quantum structures; quantizing classical spacetime

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantizing classical spacetime; quantum gravity respects the principles of quantum mechanics, suitably generalized, it may be that a more viable approach to the theory is through identifying the relevant quantum structures

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Viable approach to the theory is through identifying the relevant quantum structures viewpoint is supported by (e) difficulties of such quantization, and by (e) the apparent lack of (e) a fundamental role for (e) locality

G_{20} : Category one of difficulties of such quantization, and by (e) the apparent lack of (e) a fundamental role for (e) locality

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of approach to the theory is through identifying the relevant quantum structures viewpoint

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Viable approach to the theory is through identifying the relevant quantum structures viewpoint is supported by difficulties of such quantization, and by (e) the apparent lack of a fundamental role for (e) locality

G_{24} : Category one of apparent lack of a fundamental role for locality

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of approach to the theory is through identifying the relevant quantum structures viewpoint is supported by difficulties of such quantization

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

important structure in finite or discrete quantum systems is provided by (e) tensor factorizations of the Hilbert space

G_{28} : Category one of tensor factorizations of the Hilbert space

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of important structure in finite or discrete quantum systems

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

However, even in local quantum field theory properties of the generic type III von Neumann algebras and of long range gauge fields indicate (eb) that factorization of the Hilbert space is problematic

G_{32} : Category one of local quantum field theory properties of the generic type III von Neumann algebras and of long range gauge fields i

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of factorization of the Hilbert space is problematic

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Instead it is better to focus on the structure of the algebra of observables, and in particular on its subalgebras corresponding to (e&eb) regions

G_{36} : Category one of subalgebras; regions

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of regions; subalgebras

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Significant departures from the subalgebra structure of local quantum field theory are found, working in (eb) the correspondence limit of long-distances/low-energies

G_{40} : Category one of departures from the subalgebra structure of local quantum field theory

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of correspondence limit of long-distances/low-energies

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Particularly, there are obstacles to identifying commuting algebras of (e) localized operators

G_{44} : Category one of localized operators

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of commuting algebras

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY ONE

Entanglement And State Manipulation Of Two Spin Coupled Qubits

INTRODUCTION—VARIABLES USED

Fine Entanglement and State Manipulation of Two Spin Coupled Qubits: A Lie Theoretic Overview Roderick Vance (Submitted on 18 Feb 2015)

(1) By building on the work in Kuzmak & Tkachuk, "Preparation of quantum states of two spin-12 particles in the form of the Schmidt decomposition", Physics Letters A, {\bf 378}, pp1469-1474, which outlined the control of (e) the degree of entanglement within this system, it is proven that any SU(4) state manipulation operator can be realised for (e) this system using (e) a sequence of pulsed magnetic fields in either two linearly independent directions if (e) the gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios.

(2) To achieve this goal, an elementary Lie theoretic proof of the fact that the group of transformations generated by (e) finite products of exponentials of (e) a set of Lie algebra vectors is equal to (=) the Lie group generated by the smallest Lie algebra containing (e) those vectors is (=) rewritten into an explicit algorithm. A numerical example as well as the proof of the algorithm's effectiveness is given. **Cite as: arXiv: 1502.05200 [quant-ph] (or arXiv: 1502.05200v1 [quant-ph] for this version)**

Published Online September 1 2005 Science 30 September 2005: Vol. 309 no. 5744 pp. 2180-2184 DOI: 10.1126/science.1116955 Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots J. R. Petta1, A. C. Johnson1, J. M. Taylor1, E. A. Laird1, A. Yacoby2, M. D. Lukin1, C. M. Marcus1, M. P. Hanson3, A. C. Gossard3

(3) **J. R. Petta1, A. C. Johnson1, J. M. Taylor1, E. A. Laird1, A. Yacoby2, M. D. Lukin1, C. M. Marcus1, M. P. Hanson3, A. C. Gossard3** demonstrated coherent control of a quantum two-level

system based on (e) two-electron spin states in a double quantum dot, allowing (eb) state preparation, coherent manipulation, and projective readout.

(4) These techniques are based on rapid electrical control of (e) the exchange interaction.

(5) Separating and later recombining a singlet spin state provided (eb) a measurement of the spin dephasing time, T2*, of ~10 nanoseconds, limited by (e) hyperfine interactions with the gallium arsenide host nuclei. Rabi oscillations of two-electron spin states were demonstrated, and spin-echo pulse sequences were used to (e) suppress hyperfine-induced dephasing. Using these quantum control techniques, a coherence time for two-electron spin states exceeding 1 microsecond was observed.

Letters to Nature 431, 159-162 (9 September 2004) | doi: 10.1038/nature02831; Received 25 May 2004; Accepted 5 July 2004 Coherent dynamics of a flux qubit coupled to a harmonic oscillator I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1

(6) In the emerging field of quantum computation1 and quantum information, superconducting devices are promising candidates for the implementation of solid-state quantum bits (qubits). Single-qubit operations2, 3, 4, 5, 6, direct coupling between two qubits7, 8, 9, 10 and (e&eb) the realization of a quantum gate11 have been reported.

(7) However, complex manipulation of entangled states—such as the coupling of a two-level system to (e&eb) a quantum harmonic oscillator, as demonstrated in ion/atom-trap experiments12, 13 and cavity quantum electrodynamics14—has yet to be achieved for (e) superconducting devices.

(8) Here authors demonstrate entanglement between a superconducting flux qubit (a two-level system) and (e&eb) a superconducting quantum interference device (SQUID).

(9) Superconducting quantum interference device (SQUID) provides the measurement system for (e) detecting the quantum states; it is also an effective inductance that, in parallel with an external shunt capacitance, acts (e&eb) as a harmonic oscillator. **I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1** achieve generation and control of the entangled state by (e) performing microwave spectroscopy and detecting the resultant Rabi oscillations of the coupled system.

Coherent manipulation, measurement and entanglement of individual solid-state spins using optical fields W. B. Gao, A. Imamoglu, H. Bernien & R. Hanson Nature Photonics 9, 363–373 (2015) doi:10.1038/nphoton.2015.58 Published online 28 May 2015

(10) Realization of a quantum interface between stationary and flying qubits is a requirement for (e) long-distance quantum communication and distributed quantum computation.

(11) The prospects for integrating many qubits on a single chip render (eb) solid-state spins promising candidates for (e) stationary qubits. Certain solid-state systems, including quantum dots and nitrogen–vacancy centres in diamond, exhibit spin-state-dependent optical transitions, allowing (eb) for fast initialization, manipulation and measurement of the spins using laser excitation.

(12) Recent progress has brought spin photonics research in these materials into the quantum realm, allowing the demonstration of spin–photon entanglement, which in turn has enabled (eb) distant spin entanglement as well as quantum teleportation.

(13) Advances in the fabrication of photonic nanostructures hosting spin qubits suggest (eb) that chips incorporating a high-efficiency spin–photon interface in a quantum photonic network are within reach.

NOTATION

Module One

By building on the work in Kuzmak & Tkachuk, "Preparation of quantum states of two spin-12 particles in the form of the Schmidt decomposition", Physics Letters A, {\bf 378}, pp1469-1474, which outlined the control of (e) the degree of entanglement within this system, it is proven that any SU(4) state manipulation operator can be realised for (e) this system using (e) a sequence of pulsed magnetic fields in either two linearly independent directions if (e) the gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios

G_{13} : Category one of sequence of pulsed magnetic fields in either two linearly independent directions if (e) the gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of any SU(4) state manipulation operator can be realised for this system

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

By building on the work in Kuzmak & Tkachuk, "Preparation of quantum states of two spin-12 particles in the form of the Schmidt decomposition", Physics Letters A, {\bf 378}, pp1469-1474, which outlined the control of (e) the degree of entanglement within this system, it is proven that any SU(4) state manipulation operator can be realised for this system using (e) a sequence of pulsed magnetic fields in either two linearly independent directions if (e) the gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios

G_{16} : Category one of sequence of pulsed magnetic fields in either two linearly independent directions if (e) the gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of any SU(4) state manipulation operator can be realised for this system

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

By building on the work in Kuzmak & Tkachuk, "Preparation of quantum states of two spin-12 particles in the form of the Schmidt decomposition", Physics Letters A, {\bf 378}, pp1469-1474, which outlined the control of (e) the degree of entanglement within this system, it is proven that any SU(4) state manipulation operator can be realised for this system using a sequence of pulsed magnetic fields in either two linearly independent directions if (e) the gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios

G_{20} : Category one of gyromagnetic ratios are unequal or three directions for equal gyromagnetic ratios

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of any SU(4) state manipulation operator can be realised for this system using a sequence of pulsed magnetic fields in either two linearly independent directions

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

To achieve this goal, an elementary Lie theoretic proof of the fact that the group of transformations generated by finite products of exponentials of a set of Lie algebra vectors is equal to (=) the Lie group generated by the smallest Lie algebra containing (e) those vectors is (=) rewritten into an explicit algorithm. A numerical example as well as the proof of the algorithm's effectiveness is given.

Cite as: arXiv: 1502.05200 [quant-ph] (or arXiv: 1502.05200v1 [quant-ph] for this version)

G_{24} : Category one of Lie theoretic proof of the fact that the group of transformations generated by finite products of exponentials of a set of Lie algebra vectors

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Lie group generated by the smallest Lie algebra containing those vectors is rewritten into an explicit algorithm

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

J. R. Petta1, A. C. Johnson1, J. M. Taylor1, E. A. Laird1, A. Yacoby2, M. D. Lukin1, C. M. Marcus1, M. P. Hanson3, A. C. Gossard3 demonstrated coherent control of a quantum two-level system based on (e) two-electron spin states in a double quantum dot, allowing (eb) state preparation, coherent manipulation, and projective readout

G_{28} : Category one of two-electron spin states in a double quantum dot, allowing (eb) state preparation, coherent manipulation, and projective readout

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of coherent control of a quantum two-level system

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

J. R. Petta1, A. C. Johnson1, J. M. Taylor1, E. A. Laird1, A. Yacoby2, M. D. Lukin1, C. M. Marcus1, M. P. Hanson3, A. C. Gossard3 demonstrated coherent control of a quantum two-level system based on two-electron spin states in a double quantum dot, allowing (eb) state preparation, coherent manipulation, and projective readout

G_{32} : Category one of coherent control of a quantum two-level system based on two-electron spin states in a double quantum dot,

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of state preparation, coherent manipulation, and projective readout

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

These techniques are based on rapid electrical control of (e) the exchange interaction

G_{36} : Category one of exchange interaction

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of techniques are based on rapid electrical control

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Separating and later recombining a singlet spin state provided (eb) a measurement of the spin dephasing time, T2*, of ~10 nanoseconds, limited by (e) hyperfine interactions with the gallium arsenide host nuclei.

Rabi oscillations of two-electron spin states were demonstrated, and spin-echo pulse sequences were used to (e) suppress hyperfine-induced dephasing. Using these quantum control techniques, a coherence time for two-electron spin states exceeding 1 microsecond was observed.

G_{40} : Category one of Separating and later recombining a singlet spin state

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of measurement of the spin dephasing time, T2*, of ~10 nanoseconds, limited by (e) hyperfine interactions with the gallium arsenide host nuclei

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Rabi oscillations of two-electron spin states were demonstrated, and spin-echo pulse sequences were used to (e) suppress hyperfine-induced dephasing. Using these quantum control techniques, a coherence time for two-electron spin states exceeding 1 microsecond was observed

G_{44} : Category one of suppress hyperfine-induced dephasing. Using these quantum control techniques, a coherence time for two-electron spin states exceeding 1 microsecond was observed

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Rabi oscillations of two-electron spin states were demonstrated, and spin-echo pulse sequences

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY TWO

Searching For A Needle In A Haystack

INTRODUCTION—VARIABLES USED

Quantum Mechanics Helps in Searching for a Needle in a Haystack Lov K. Grover Phys. Rev. Lett. 79, 325 – Published 14 July 1997

(1) Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing N names arranged in (eb) completely random order. To find someone's phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to (e) access the database a minimum of 0.5N times.
(2) Quantum mechanical systems can be in a superposition of (e) states and simultaneously examine multiple names.
(3) By properly adjusting the phases of various operations, successful computations reinforce (eb) each other while others interfere (e&eb) randomly. As a result, the desired phone number can be obtained in (eb) only O (N−−√) accesses to the database.

Frontiers of Computer Science in China July 2007, Volume 1, Issue 3, pp 247-271 Search an unsorted

database with quantum mechanics Guilu Long, Yang Liu

(4) In this article, **Guilu Long, Yang Liu** review quantum search algorithms for (e) unsorted database search problem. Unsorted database search is a very important problem in science and technology. In a quantum computer, a marked state can be found with very high probability using (e) the Grover's algorithm, or exactly with the Long algorithm.

(5) Guilu Long, Yang Liu review the Grover algorithm and related generalizations. In particular, **Guilu Long, Yang Liu** review the phase matching conditions in (eb) quantum search algorithm. Several issues that may cause confusion about the quantum search algorithm are also clarified.

NOTATION

Module One

Letters to Nature 431, 159-162 (9 September 2004) | doi: 10.1038/nature02831; Received 25 May 2004; Accepted 5 July 2004 Coherent dynamics of a flux qubit coupled to a harmonic oscillator I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1

(1) In the emerging field of quantum computation1 and quantum information, superconducting devices are promising candidates for the implementation of solid-state quantum bits (qubits). Single-qubit operations2, 3, 4, 5, 6, direct coupling between two qubits7, 8, 9, 10 and (e&eb) the realization of a quantum gate11 have been reported.

(2) However, complex manipulation of entangled states—such as the coupling of a two-level system to (e&eb) a quantum harmonic oscillator, as demonstrated in ion/atom-trap experiments12, 13 and cavity quantum electrodynamics14—has yet to be achieved for (e) superconducting devices.

(3) Here authors demonstrate entanglement between a superconducting flux qubit (a two-level system) and (e&eb) a superconducting quantum interference device (SQUID).

(4) Superconducting quantum interference device (SQUID) provides the measurement system for (e) detecting the quantum states; it is also an effective inductance that, in parallel with an external shunt capacitance, acts (e&eb) as a harmonic oscillator. **I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1** achieve generation and control of the entangled state by (e) performing microwave spectroscopy and detecting the resultant Rabi oscillations of the coupled system.

In the emerging field of quantum computation1 and quantum information, superconducting devices are promising candidates for the implementation of solid-state quantum bits (qubits).

Single-qubit operations2, 3, 4, 5, 6, direct coupling between two qubits7, 8, 9, 10 and (e&eb) the realization of a quantum gate11 have been reported

G_{13} : Category one of Single-qubit operations2, 3, 4, 5, 6, direct coupling between two qubits7, 8, 9, 10; realization of a quantum gate11

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of realization of a quantum gate11 ;Single-qubit operations2, 3, 4, 5, 6, direct coupling between two qubits7, 8, 9, 10

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

However, complex manipulation of entangled states—such as the coupling of a two-level system to (e&eb) a quantum harmonic oscillator, as demonstrated in ion/atom-trap experiments12, 13 and cavity quantum electrodynamics14—has yet to be achieved for (e) superconducting devices

Here authors demonstrate entanglement between a superconducting flux qubit (a two-level system) and (e&eb) a superconducting quantum interference device (SQUID)

G_{16} : Category one of superconducting flux qubit (a two-level system); superconducting quantum interference device (SQUID)

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of superconducting quantum interference device (SQUID);superconducting flux qubit (a two-level system)

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Superconducting quantum interference device (SQUID) provides the measurement system for (e) detecting the quantum states;

Superconducting quantum interference device (SQUID) is also an effective inductance that, in parallel with an external shunt capacitance, acts (e&eb) as a harmonic oscillator. **I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1** achieve generation and control of the entangled state by (e) performing microwave spectroscopy and detecting the resultant Rabi oscillations of the coupled system

G_{20} : Category one of detecting the quantum states

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Superconducting quantum interference device (SQUID) provides the measurement system

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Superconducting quantum interference device (SQUID) is also an effective inductance that, in parallel with an external shunt capacitance, acts (e&eb) as a harmonic oscillator

I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1 achieve generation and control of the entangled state by (e) performing microwave spectroscopy and

detecting the resultant Rabi oscillations of the coupled system

G_{24} : Category one of Superconducting quantum interference device (SQUID) is also an effective inductance that, in parallel with an external shunt capacitance; harmonic oscillator

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of harmonic oscillator; Superconducting quantum interference device (SQUID) is also an effective inductance that, in parallel with an external shunt capacitance

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

I. Chiorescu1,4, P. Bertet1, K. Semba1,2, Y. Nakamura1,3, C. J. P. M. Harmans1 & J. E. Mooij1
achieve generation and control of the entangled state by (e) performing microwave spectroscopy and detecting the resultant Rabi oscillations of the coupled system

G_{28} : Category one of performing microwave spectroscopy and detecting the resultant Rabi oscillations of the coupled system

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of generation and control of the entangled state

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing N names arranged in (eb) completely random order.

To find someone's phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to (e) access the database a minimum of 0.5N times

G_{32} : Category one of access the database a minimum of 0.5N times

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of classical algorithm (whether deterministic or probabilistic) to find someone's phone number with a probability of 50%,

T_{33} : Category two of

T_{34} : Category three of

Module seven

Successful computations by properly adjusting the phases of various operations reinforce (eb) each other while others interfere (e&eb) randomly.

As a result, the desired phone number can be obtained in (eb) only O (N−−√) accesses to the database

G_{36} : Category one of successful computations by properly adjusting the phases of various operations

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of each other while others interfere (e&eb) randomly.

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

desired phone number can be obtained in (eb) only O (N−−√) accesses to the database

G_{40} : Category one of desired phone number can be obtained

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of O (N−−√) accesses to the database

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

In this article, **Guilu Long, Yang Liu** review quantum search algorithms for (e) unsorted database search problem.

Unsorted database search is a very important problem in science and technology.

In a quantum computer, a marked state can be found with very high probability using (e) the Grover's algorithm, or exactly with the Long algorithm

G_{44} : Category one of Grover's algorithm, or exactly with the Long algorithm

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of marked state can be found with very high probability

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY THREE

Quantum-Accelerated Fractal Image Compression

INTRODUCTION—VARIABLES USED

Quantum-Accelerated Fractal Image Compression: An Interdisciplinary Approach Songlin Du ; Sch. of Inf. Sci. & Eng., Lanzhou Univ., Lanzhou, China ; Yaping Yan ; Yide Ma

(1) **Fractal image compression (FIC)** is one of the most widely approved image compression approaches for (e) its high compression ratio and quality of retrieved images.

(2) However, FIC suffers from high computational cost in searching local self-similarities in (eb) natural image. Although many papers aiming at speeding up FIC have been published, they use pre-processing tools or approximation methods. **Reducing the intrinsic computational complexity of (e) FIC is still an open problem.** Since quantum mechanics based Grover's quantum search algorithm (QSA) is able to achieve (eb) square-root speedup over classical algorithms in unsorted database searching, **Lanzhou, China ; Yaping Yan ; Yide Ma** propose an interdisciplinary approach by using Grover's QSA to reduce (e) the intrinsic computational complexity of FIC in this letter.

(3) In particular, both domain blocks and range blocks are represented as (=) quantum states, and then Grover's QSA is employed to (e) search the most similar domain block for each range block under the criterion of (e) maximizing quantum fidelity between (e&eb) these two kinds of quantum states.

(4) Without sacrificing compression ratio, experimental results show (eb) that the execution time of the proposed method is 100 times shorter than that of (e) the baseline FIC.

(5) Moreover, retrieved images from proposal are also less (e) distorted than those from other state-of-the-art FIC approaches. **Published in: Signal Processing Letters, IEEE (Volume:22 , Issue: 4) Page(s): 499 – 503 ISSN : 1070-9908 INSPEC Accession Number: 14681628 DOI: 10.1109/LSP.2014.2363689 Date of Publication : 17 October 2014 Date of Current Version : 23 October 2014 Issue Date : April 2015 Sponsored by : IEEE Signal Processing Society Publisher: IEEE**

NOTATION

Module One

Fractal image compression (FIC) is one of the most widely approved image compression approaches for (e) its high compression ratio and quality of retrieved images

G_{13} : Category one of its high compression ratio and quality of retrieved images

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of **Fractal image compression (FIC)** is one of the most widely approved image compression approaches

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

However, FIC suffers from high computational cost in searching local self-similarities in (eb) natural image. Although many papers aiming at speeding up FIC have been published, they use pre-processing tools or approximation methods. **Reducing the intrinsic computational complexity of (e) FIC is still an open problem.** Since quantum mechanics based Grover's quantum search algorithm (QSA) is able to achieve (eb) square-root speedup over classical algorithms in unsorted database searching, **Lanzhou, China ; Yaping Yan ; Yide Ma** propose an interdisciplinary approach by using Grover's QSA to reduce (e) the intrinsic computational complexity of FIC in this letter

In particular, both domain blocks and range blocks are represented as (=) quantum states, and then Grover's QSA is employed to (e) search the most similar domain block for each range block under the criterion of (e) maximizing quantum fidelity between (e&eb) these two kinds of quantum states

G_{16} : Category one of both domain blocks and range blocks

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one ofquantum states, and then Grover's QSA is employed to (e) search the most similar domain block for each range block under the criterion of (e) maximizing quantum fidelity between (e&eb) these two kinds of quantum states

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

In particular, both domain blocks and range blocks are represented as quantum states, and then Grover's QSA is employed to (e) search the most similar domain block for each range block under the criterion of (e) maximizing quantum fidelity between (e&eb) these two kinds of quantum states

G_{20} : Category one of search the most similar domain block for each range block under the criterion of (e) maximizing quantum fidelity between (e&eb) these two kinds of quantum states

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of both domain blocks and range blocks are represented as quantum states, and then Grover's QSA is employed

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In particular, both domain blocks and range blocks are represented as quantum states, and then Grover's QSA is employed to search the most similar domain block for each range block under the criterion of (e) maximizing quantum fidelity between (e&eb) these two kinds of quantum states

G_{24} : Category one of maximizing quantum fidelity between these two kinds of quantum states

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of both domain blocks and range blocks are represented as quantum states, and then Grover's QSA is employed to search the most similar domain block for each range block under the criterion

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Moreover, retrieved images from proposal are also less (e) distorted than those from other state-of-the-art FIC approaches.

Published in: Signal Processing Letters, IEEE (Volume:22 , Issue: 4) Page(s): 499 – 503 ISSN : 1070-9908 INSPEC Accession Number: 14681628 DOI: 10.1109/LSP.2014.2363689 Date of Publication : 17 October 2014 Date of Current Version : 23 October 2014 Issue Date : April 2015 Sponsored by : IEEE Signal Processing Society Publisher: IEEE

G_{28} : Category one of distorted than those from other state-of-the-art FIC approaches

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of retrieved images from proposal

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Coherent manipulation, measurement and entanglement of individual solid-state spins using optical fields W. B. Gao, A. Imamoglu, H. Bernien & R. Hanson Nature Photonics 9, 363–373 (2015) doi:10.1038/nphoton.2015.58 Published online 28 May 2015

(1) Realization of a quantum interface between stationary and flying qubits is a requirement for (e) long-distance quantum communication and distributed quantum computation.
(2) The prospects for integrating many qubits on a single chip render (eb) solid-state spins promising

candidates for (e) stationary qubits. Certain solid-state systems, including quantum dots and nitrogen–vacancy centres in diamond, exhibit spin-state-dependent optical transitions, allowing (eb) for fast initialization, manipulation and measurement of the spins using laser excitation.

(3) Recent progress has brought spin photonics research in these materials into the quantum realm, allowing the demonstration of spin–photon entanglement, which in turn has enabled (eb) distant spin entanglement as well as quantum teleportation.

(4) Advances in the fabrication of photonic nanostructures hosting spin qubits suggest (eb) that chips incorporating a high-efficiency spin–photon interface in a quantum photonic network are within reach.

Realization of a quantum interface between stationary and flying qubits is a requirement for (e) long-distance quantum communication and distributed quantum computation

G_{32} : Category one of long-distance quantum communication and distributed quantum computation

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Realization of a quantum interface between stationary and flying qubits

T_{33} : Category two of

T_{34} : Category three of

Module seven

The prospects for integrating many qubits on a single chip render (eb) solid-state spins promising candidates for (e) stationary qubits. Certain solid-state systems, including quantum dots and nitrogen–vacancy centres in diamond, exhibit spin-state-dependent optical transitions, allowing (eb) for fast initialization, manipulation and measurement of the spins using laser excitation

G_{36} : Category one of prospects for integrating many qubits on a single chip

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of solid-state spins promising candidates for (e) stationary qubits. Certain solid-state systems, including quantum dots and nitrogen–vacancy centres in diamond, exhibit spin-state-dependent optical transitions, allowing (eb) for fast initialization, manipulation and measurement of the spins using laser excitation

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Prospects for integrating many qubits on a single chip render solid-state spins promising candidates for

stationary qubits.

Certain solid-state systems, including quantum dots and nitrogen–vacancy centres in diamond, exhibit spin-state-dependent optical transitions, allowing (eb) for fast initialization, manipulation and measurement of the spins using laser excitation

G_{40} : Category one of Certain solid-state systems, including quantum dots and nitrogen–vacancy centres in diamond, exhibit spin-state-dependent optical transitions

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of fast initialization, manipulation and measurement of the spins using laser excitation

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Recent progress has brought spin photonics research in these materials into the quantum realm, allowing the demonstration of spin–photon entanglement, which in turn has enabled (eb) distant spin entanglement as well as quantum teleportation

G_{44} : Category one of spin photonics research in these materials into the quantum realm, allowing the demonstration of spin–photon entanglement,

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of spin entanglement as well as quantum teleportation

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY FOUR

Large-Scale Quantum Computation

INTRODUCTION—VARIABLES USED

Surface codes: Towards practical large-scale quantum computation Austin G. Fowler, Matteo Mariantoni, John M. Martinis, and Andrew N. Cleland Phys. Rev. A 86, 032324 – Published 18 September 2012

(1) This article provides an introduction to surface code quantum computing. **Austin G. Fowler, Matteo Mariantoni, John M. Martinis, and Andrew N. Cleland** first estimate the size and speed of (e) a surface code quantum computer.
(2) They then introduce the concept of the stabilizer, using (e) two qubits, and extend this concept to stabilizers acting on (e&eb) a two-dimensional array of physical qubits, on which they implement (e&eb) the surface code.
(3) **Austin G. Fowler, Matteo Mariantoni, John M. Martinis, and Andrew N. Cleland** next describe how logical qubits are formed in (eb) the surface code array and give numerical estimates of (e) their fault tolerance.
(4) Authors outline how logical qubits are (=) physically moved on the array, how qubit braid transformations (e&eb) are (=) constructed, and how a braid between two logical qubits is equivalent to (=) a controlled-not gate
(5) They then describe the single-qubit Hadamard, Sˆ and Tˆ operators, completing (e) the set of required gates for a universal quantum computer.
(6) Austin G. Fowler, Matteo Mariantoni, John M. Martinis, and Andrew N. Cleland conclude by briefly discussing physical implementations of (e) the surface **code. They also** include a number of Appendices in which they provide supplementary information to the main text.

NOTATION

Module One

This article provides an introduction to surface code quantum computing.
Austin G. Fowler, Matteo Mariantoni, John M. Martinis, and Andrew N. Cleland first estimate the size and speed of (e) a surface code quantum computer

G_{13} : Category one of surface code quantum computer

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of size and speed

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

They then introduce the concept of the stabilizer, using two qubits, and extend this concept to stabilizers acting on (e&eb) a two-dimensional array of physical qubits, on which they implement (e&eb) the surface code

G_{16} : Category one of concept of the stabilizer, using two qubits, and extend this concept to stabilizers; two-dimensional array of physical qubits, on which they implement (e&eb) the surface code

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of two-dimensional array of physical qubits, on which they implement (e&eb) the surface code ;concept of the stabilizer, using two qubits, and extend this concept to stabilizers

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They then introduce the concept of the stabilizer, using two qubits, and extend this concept to stabilizers acting on a two-dimensional array of physical qubits, on which they implement (e&eb) the surface code

G_{20} : Category one of concept of the stabilizer, using two qubits, and extend this concept to stabilizers acting on a two-dimensional array of physical qubits, on which they implement; surface code

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of surface code; concept of the stabilizer, using two qubits, and extend this concept to stabilizers acting on a two-dimensional array of physical qubits, on which they implement

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Authors outline how logical qubits are physically moved on the array, how qubit braid transformations (e&eb) are (=) constructed, and how a braid between two logical qubits is equivalent to (=) a controlled-not

G_{24} : Category one of qubit braid; logical qubits

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of logical qubits; qubit braid

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

braid between two logical qubits is equivalent to (=) a controlled-not

G_{28} : Category one of braid between two logical qubits

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of controlled-not gate

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Signatures of chaos in the dynamics of quantum discord Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose Phys. Rev. E 91, 032906 – Published 10 March 2015

(1) **Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose** identify signatures of chaos in (eb) the dynamics of discord in (eb) a multiqubit system collectively modelled as (=) a quantum kicked top.
(2) Evolution of discord between any two qubits is (=) quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity is (=) lost.
(3) As the initial wave function is varied from the regular regions to the chaotic sea, a contour plot of the time-averaged discord remarkably reproduces the structures of the classical stroboscopic map.
(4) They also find surprisingly opposite behavior of two-qubit discord versus entanglement of the two qubits as measured by the concurrence.
(5) Results provide evidence of signatures of chaos in dynamically generated discord. http://dx.doi.org/10.1103/PhysRevE.91.032906©2015 American Physical Society

Vaibhav Madhok, Vibhu Gupta, Denis-Alexandre Trottier, and Shohini Ghose identify signatures of chaos in the dynamics of discord in (eb) a multiqubit system collectively modelled as (=) a quantum kicked top

G_{32} : Category one of signatures of chaos in the dynamics of discord

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of multiqubit system collectively modelled as a quantum kicked top

T_{33} : Category two of

T_{34} : Category three of

Module seven

Evolution of discord between any two qubits is (=) quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity is (=) lost

G_{36} : Category one of Evolution of discord between any two qubits

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity is (=) lost

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

evolution of discord between any two qubits is quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity is (=) lost

G_{40} : Category one of evolution of discord between any two qubits is quasiperiodic in regular regions, while in chaotic regions the quasiperiodicity

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of lost

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

They also find surprisingly opposite behavior of two-qubit discord versus entanglement of the two qubits as measured by the concurrence

G_{44} : Category one of opposite behavior of two-qubit discord; entanglement of the two qubits as measured by the concurrence

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of entanglement of the two qubits as measured by the concurrence ;opposite behavior of two-qubit discord

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY FIVE

Quantum Circuit Representation Of Grover's Algorithm

INTRODUCTION—VARIABLES USED

Grover's Algorithm (Source: Wikipedia)

Setup

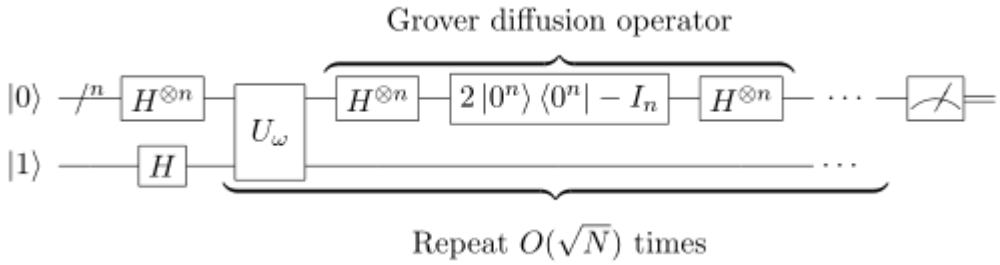
(1) Consider an unsorted database with N entries. The algorithm requires (e) an N-dimensional state space H, which can be supplied by (e) n=log2 N qubits.
(2) Consider the problem of determining the index of (e) the database entry which satisfies (eb) some search criterion.
(3) Let f be the function which maps database entries to (e&eb) 0 or 1, where f (ω) =1 if and only if (e) ω satisfies the search criterion.
(4) We are provided with (quantum black box) access to (e) a subroutine in the form of a unitary operator, Uω, which acts as follows (for the ω for which f (ω) =1):

$$U_\omega|\omega\rangle = -|\omega\rangle$$

$$U_\omega|x\rangle = |x\rangle \qquad \text{for all } x \neq \omega$$

Our goal is to identify the index $|\omega\rangle$.

Algorithm steps



Quantum circuit representation of Grover's algorithm

The steps of Grover's algorithm are given as follows.

(5) Let $|s\rangle$ denote the uniform superposition over all states

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle .$$

(6) Then the operator

$$U_s = 2\,|s\rangle\langle s| - I$$

is known as the Grover diffusion operator.

Here is the algorithm:

(7) Initialize the system to the state

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle$$

(8) Perform the following "Grover iteration" r (N) times. The function r (N), which is asymptotically O (N½), is described below.

Apply the operator U_{ω}.

Apply the operator U_{s}.

(9) Perform the measurement Ω. The measurement result will be (=) eigenvalue λω with probability approaching 1 for N≫1. From λω, ω may be obtained.

NOTATION

Module One

Non-Abelian anyons and topological quantum computation Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma Rev. Mod Phys. 80, 1083 – Published 12 September 2008

(1) Topological quantum computation has emerged as one of the most exciting approaches to constructing (eb) a fault-tolerant quantum computer. Proposal relies on the existence of (e) topological states of matter whose quasiparticle excitations are (=) neither bosons nor fermions, but are (=) particles known as non-Abelian anyons, meaning that they obey (e&eb) non-Abelian braiding statistics.
(2) Quantum information is stored in (eb) states with multiple quasiparticles, which have (e) a topological degeneracy.
(3) Unitary gate operations that are necessary for (e) quantum computation are carried out by (e) braiding quasiparticles and then measuring (eb) the multiquasiparticle states.
(4) fault tolerance of a topological quantum computer arises from (e) the nonlocal encoding of the quasiparticle states, which makes them immune to (e) errors caused by (er-) local perturbations.
(5) To date, the only such topological states thought to have been found in nature are (=) fractional quantum Hall states, most prominently the ν=5⁄2 state, although several other prospective candidates have been proposed in systems as disparate as ultracold atoms in (eb) optical lattices and thin-film superconductors.
(6) In this review article, current research in this field is described, focusing on the general theoretical concepts of non-Abelian statistics as it relates to (e&eb) topological quantum computation, on understanding non-Abelian quantum Hall states, on proposed experiments to (e) detect non-Abelian anyons, and on (eb) proposed architectures for (e) a topological quantum computer.
(7) Both the mathematical underpinnings of topological quantum computation and the physics of the subject are addressed, using (e) the ν=5⁄2 fractional quantum Hall state as (=) the archetype of a non-Abelian topological state enabling (eb) fault-tolerant quantum computation. **http://dx.doi.org/10.1103/RevModPhys.80.1083**

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle$$

G_{13} : Category one of LHS of the above equation

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of RHS of the above equation

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Topological quantum computation has emerged as one of the most exciting approaches to constructing (eb) a fault-tolerant quantum computer.

Proposal relies on the existence of topological states of matter whose quasiparticle excitations are (=) neither bosons nor fermions, but are (=) particles known as non-Abelian anyons, meaning that they obey (e&eb) non-Abelian braiding statistics

G_{16} : Category one of Proposal relies on the existence of topological states of matter whose quasiparticle excitations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of neither bosons nor fermions, but are (=) particles known as non-Abelian anyons, meaning that they obey (e&eb) non-Abelian braiding statistics

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Proposal relies on the existence of topological states of matter whose quasiparticle excitations are neither bosons nor fermions, but are particles known as non-Abelian anyons, meaning that they obey (e&eb) non-Abelian braiding statistics

G_{20} : Category one of Proposal relies on the existence of topological states of matter whose quasiparticle excitations are neither bosons nor fermions, but are particles known as non-Abelian anyons; non-Abelian braiding statistics

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of non-Abelian braiding statistics; Proposal relies on the existence of topological states of matter whose quasiparticle excitations are neither bosons nor fermions, but are particles known as non-Abelian anyons

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Quantum information is stored in (eb) states with multiple quasiparticles, which have (e) a topological degeneracy

G_{24} : Category one of Quantum information is stored

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of states with multiple quasiparticles, which have (e) a topological degeneracy

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Quantum information is stored in states with multiple quasiparticles, which have (e) a topological degeneracy

G_{28} : Category one of topological degeneracy

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Quantum information is stored in states with multiple quasiparticles

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Unitary gate operations that are necessary for quantum computation are carried out by (e) braiding quasiparticles and then measuring (eb) the multiquasiparticle states

G_{32} : Category one of braiding quasiparticles and then measuring (eb) the multiquasiparticle states

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Unitary gate operations that are necessary for quantum computation are carried out

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Unitary gate operations that are necessary for quantum computation are carried out by braiding quasiparticles and then measuring (eb) the multiquasiparticle states

G_{36} : Category one of Unitary gate operations that are necessary for quantum computation are carried out by braiding quasiparticles

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of multiquasiparticle states

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

fault tolerance of a topological quantum computer arises from (e) the nonlocal encoding of the quasiparticle states, which makes them immune to (e) errors caused by (er-) local perturbations

G_{40} : Category one of nonlocal encoding of the quasiparticle states, which makes them immune to (e) errors caused by (er-) local perturbations

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of fault tolerance of a topological quantum computer

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

fault tolerance of a topological quantum computer arises from the nonlocal encoding of the quasiparticle states, which makes them immune to (e) errors caused by (er-) local perturbations

G_{44} : Category one of errors caused by (er-) local perturbations

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of fault tolerance of a topological quantum computer arises from the nonlocal encoding of the quasiparticle states, which makes them immune

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients

for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) & \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) & \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) \quad +(a''_{32})^{(6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) \quad +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) \quad +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{30} \quad (81)$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3
$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3
$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation

coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)}-(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad 82$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)}-(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad 83$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)}-(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt}=(a_{32})^{(6)}G_{33} \quad 85$$

$$-\begin{bmatrix}(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t)\\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,5)}(G_{31},t)}$ $are\ second\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{24}'')^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4,4,4,)}(G_{27},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}, \boxed{(a_{37}'')^{(7)}(T_{37},t)}, \boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \quad (98)$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: (99)

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: (100)

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: (101)

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\left((G_{19}), t\right)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \qquad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty} (b_i'')^{(4)}\big((G_{27}), t\big) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}\big[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}\big] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\big((G_{35}), t\big) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(IIIIIIIII) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(JJJJJJJJJ)The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(KKKKKKKKK) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(LLLLLLLLL)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(MMMMMMMMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(NNNNNNNNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \ \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \ , \ G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \ , \ T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \leq (\hat{Q}_{24})^{(4)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(- \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{\left(- \frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(kk) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\widehat{P}_{13})^{(1)}$ and $(\widehat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)} - G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t} \leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ and $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ and $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205

The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right)=(b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}<1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad (208)$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad (209)$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a_{20}')^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a_{20}'')^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad (214)$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a_{20}')^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t), t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}), (\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}), (\widetilde{T_{27}})\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)}\left|G_{25}^{(1)} - G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)} +$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t} \leq \quad 226$$

$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ and $(b''_i)^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 228

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(4)}-(a''_i)^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(4)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1-e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}), (\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}), (\widetilde{T_{31}})\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$

$$\int_0^t\{(a_{28}')^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a_{28}'')^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)}t} \leq \frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right) d\left(((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)})\right) \quad (237)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(5)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a'_{30})^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. (241)

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. (242)

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}})\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$|\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{32})^{(6)}|G_{33}^{(1)} - G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)} +$

$\int_0^t\{(a_{32}')^{(6)}|G_{32}^{(1)} - G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} +$

$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}} +$

$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\} ds_{(32)}$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}t} \leq \frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right) \quad (248)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also G_{33} *and* G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. (252)

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\big((G_{35})(t),t\big) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t),t(t),t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\tilde{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})}$: $\left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)}-G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)}-G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on (G_{39})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{40})^{(8)} \left|G_{41}^{(1)} - G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)} +$$
$$\int_0^t \{(a'_{40})^{(8)} \left|G_{40}^{(1)} - G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}} +$$
$$G_{40}^{(2)} \left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)} (\widehat{k}_{40})^{(8)}\right) d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ *and* $(\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}, i = 40{,}41{,}42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(8)} - (a''_i)^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(8)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

279 A

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(\left((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$$G_{45}\le\left((\widehat{M}_{44})^{(9)}\right)_2=G_{45}^0+2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$

$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t} \right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right.$ 286

$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})} [e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}] + G_{15}^0 e^{-(a'_{15})^{(1)} t} \right)$

$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$ 287

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$ 288

$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le$ 289

$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t} \right] + T_{15}^0 e^{-(R_2)^{(1)} t}$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t} \right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right.$ 312

$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} [e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}] + G_{18}^0 e^{-(a'_{18})^{(2)} t} \right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t} \right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t}-e^{-(a'_{22})^{(3)}t}\right]+\ G_{22}^{0}e^{-(a'_{22})^{(3)}t})$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le\frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t}-e^{-(b'_{22})^{(3)}t}\right]+T_{22}^{0}e^{-(b'_{22})^{(3)}t}\le T_{22}(t)\le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t}-e^{-(R_2)^{(3)}t}\right]+T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)},(S_2)^{(3)},(R_1)^{(3)},(R_2)^{(3)}$:- 328

Where $(S_1)^{(3)}=(a_{20})^{(3)}(m_2)^{(3)}-(a'_{20})^{(3)}$

$(S_2)^{(3)}=(a_{22})^{(3)}-(p_{22})^{(3)}$

$(R_1)^{(3)}=(b_{20})^{(3)}(\mu_2)^{(3)}-(b'_{20})^{(3)}$

$(R_2)^{(3)}=(b'_{22})^{(3)}-(r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$:

$(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)}\le-(a'_{24})^{(4)}+(a'_{25})^{(4)}-(a''_{24})^{(4)}(T_{25},t)+(a''_{25})^{(4)}(T_{25},t)\le-(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)}\le-(b'_{24})^{(4)}+(b'_{25})^{(4)}-(b''_{24})^{(4)}\left((G_{27}),t\right)-(b''_{25})^{(4)}\left((G_{27}),t\right)\le-(\tau_1)^{(4)}$

Definition of $(v_1)^{(4)},(v_2)^{(4)},(u_1)^{(4)},(u_2)^{(4)},v^{(4)},u^{(4)}$: 329

By $(v_1)^{(4)}>0$, $(v_2)^{(4)}<0$ and respectively $(u_1)^{(4)}>0$, $(u_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2+(\sigma_1)^{(4)}v^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_1)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)}>0$, $(\bar{v}_2)^{(4)}<0$ and respectively $(\bar{u}_1)^{(4)}>0$, $(\bar{u}_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2+(\sigma_2)^{(4)}v^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_2)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$

Definition of $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)},\ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t} \tag{332}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \tag{333}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \tag{334}$$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \tag{335}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \tag{336}$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)},$ ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$

and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)},$ ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},$ ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)},$ ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, ***if*** $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, ***if*** $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, ***if*** $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{40}(t) \leq G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{41}(t) \leq \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \leq G_{42}(t) \leq \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \leq T_{40}(t) \leq T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \leq T_{40}(t) \leq \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \leq T_{42}(t) \leq \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G^0_{44} e^{(S_1)^{(9)} t}$$

(

$$\frac{(a_{46})^{(9)} G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \Big] + G^0_{46} e^{-(S_2)^{(9)} t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G_{44}^{0}}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+\ G_{46}^{0}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^{0}e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^{0}e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)}=(a_{44})^{(9)}(m_2)^{(9)}-(a'_{44})^{(9)}$

$(S_2)^{(9)}=(a_{46})^{(9)}-(p_{46})^{(9)}$

$(R_1)^{(9)}=(b_{44})^{(9)}(\mu_2)^{(9)}-(b'_{44})^{(9)}$

$(R_2)^{(9)}=(b'_{46})^{(9)}-(r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a'_{13})^{(1)}-(a'_{14})^{(1)}+(a''_{13})^{(1)}(T_{14},t)\right)-\ (a''_{14})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\le\frac{dv^{(1)}}{dt}\le-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\ge\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)}\le\nu^{(1)}(t)\le(\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$, we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

408

Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)\le\frac{d\nu^{(6)}}{dt}\le-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)},(\nu_0)^{(6)}$:-

For $0<\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}<(\nu_1)^{(6)}<(\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t)\ge\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}\quad,\quad\boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)}\le\nu^{(6)}(t)\le(\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t)\le\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)}\le\nu^{(6)}(t)\le(\bar{\nu}_1)^{(6)}$

If $0<(\nu_1)^{(6)}<(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}<(\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)}\le\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}\le\nu^{(6)}(t)\le$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\le(\bar{\nu}_1)^{(6)}$$

If $0<(\nu_1)^{(6)}\le(\bar{\nu}_1)^{(6)}\le\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)}\le\nu^{(6)}(t)\le\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\le(\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)}\le\nu^{(6)}(t)\le(m_1)^{(6)},\quad\boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)}\le u^{(6)}(t)\le(\mu_1)^{(6)},\quad\boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .
Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then
$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

416

Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \le \frac{d\nu^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \ge \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \le \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \le \nu^{(7)}(t) \le (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \le \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \le \nu^{(7)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \le (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

424
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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \le \frac{d\nu^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\ ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 427$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 428$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0\ , \quad 429$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 430$$

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(kk) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T_{14}^* :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^{*} :- 494

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^{*} for which $f(\mathrm{T}_{17}^{*})=0$. With this value , we obtain from the three first equations

$$G_{16}=\frac{(a_{16})^{(2)}\mathrm{G}_{17}}{[(a_{16}')^{(2)}+(a_{16}'')^{(2)}(\mathrm{T}_{17}^{*})]} \quad , \quad G_{18}=\frac{(a_{18})^{(2)}\mathrm{G}_{17}}{[(a_{18}')^{(2)}+(a_{18}'')^{(2)}(\mathrm{T}_{17}^{*})]} \qquad 495$$

Definition and uniqueness of T_{21}^{*} :- 496

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^{*} for which $f(T_{21}^{*})=0$. With this value , we obtain from the three first equations

$$G_{20}=\frac{(a_{20})^{(3)}G_{21}}{[(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}=\frac{(a_{22})^{(3)}G_{21}}{[(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21}^{*})]}$$

Definition and uniqueness of T_{25}^{*} :- 497

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^{*} for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a_{24}')^{(4)}+(a_{24}'')^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a_{26}')^{(4)}+(a_{26}'')^{(4)}(T_{25}^{*})]}$$

Definition and uniqueness of T_{29}^{*} :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^{*} for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a_{28}')^{(5)}+(a_{28}'')^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a_{30}')^{(5)}+(a_{30}'')^{(5)}(T_{29}^{*})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a_{32}')^{(6)}+(a_{32}'')^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a_{34}')^{(6)}+(a_{34}'')^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T^*_{41} for which $f(T^*_{41}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$$

Definition and uniqueness of T^*_{45} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T^*_{45} for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is

a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G^*_{14} given by $\varphi(G^*) = 0$, T^*_{14} given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G^*_{17} given by $\varphi((G_{19})^*) = 0$, T^*_{17} given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G^*_{21} given by $\varphi((G_{23})^*) = 0$, T^*_{21} given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G^*_{25} given by $\varphi(G_{27}) = 0$, T^*_{25} given by $f(T^*_{25}) = 0$ and

$$G^*_{24} = \frac{(a_{24})^{(4)}G^*_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G^*_{26} = \frac{(a_{26})^{(4)}G^*_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*})=0$, T_{29}^{*} given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*})=0$, T_{41}^{*} given by $f(T_{41}^{*})=0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^* \mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}} (T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j} ((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} G_{16}^* \mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)} G_{17}^* \mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)} G_{18}^* \mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)} T_{16}^* \mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)} T_{17}^* \mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)} T_{18}^* \mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}} (T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j} ((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)} G_{20}^* \mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)} G_{21}^* \mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)} G_{22}^* \mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)} T_{20}^* \mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)} T_{21}^* \mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}')^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)} T_{22}^* \mathbb{G}_j\right) \tag{546}$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \ , \ \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

586 A

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\,(G_{47})^*\,) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\,(a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left((\lambda)^{(1)}\right)^2+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\Big)\}=0$$

$$+$$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\Big)\right]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\Big)\right]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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